Comparison and evaluation between FBMC and OFDM systems

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***Abstract*—This paper studies the analytical bit error probability (BEP) of filter bank based multicarrier (FBMC) transmission and conventional cyclic prefix (CP) based orthogonal frequency division multiplexing (OFDM) systems. We first compare and evaluate these two schemes under the additive white Gaussian noise (AWGN) channel, and then extend the results to the Rayleigh channel. Closed form expressions of the bit error probabilities for both two methods are derived and validated in this research. The results reveal that the performance of these two methods are the same when the perfect reconstruction conditions of them are satisfied. However, the FBMC technique has less out-of-band power leakage as a result of lower side lobes. Meanwhile, the omission of CP improves the bandwidth efficiency of the system with a increase in the equalization complexity.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) has been probably the most successful multi-carrier modulation (MCM) scheme for the wired or wireless communication in the past two decades and it is employed in the standard of today's 4G network. Meanwhile, OFDM still plays an important role in many current applications such as asymmetric digital subscriber line (ADSL), digital video broadcasting (DVB) and wireless local area networks (WLANs). However, with the increasing number of users and devices, the need for energy efficiency and sophisticated spectrum utilization at low costs drawbacks of OFDM become apparent such as the cyclic prefix (CP) leading to a decrease in the system efficiency. Consequently, other MCM methods have been proposed and evaluated recently and latest research indicates a great potential for these methods which may have a big influence on next generation communication systems and may replace the conventional OFDM [1], [2].

One important, new emerging MCM method is the filter bank based multicarrier (FBMC) transmission, which introduces filter banks to the OFDM system and discards the CP. In this sense, the FBMC is an evolution of OFDM. The employed filter bank will import extract flexibility to the system to cope with some drawbacks of OFDM. Intuitively, the filter bank can be designed with different properties to satisfy the communication requirements. For example, for the OFDM system, CP is used to eliminate inter symbol interference (ISI). In order to remove this interference completely, the length of CP must be no shorter than the length of the impulse response of the corresponding channel. However, this CP-redundancy decreases both spectral and power efficiency. For the FBMC, instead of CP, filter banks can be designed and applied to reduce the out-of-band power leakage and increase the spectral efficiency with a cost of computational complexity.

In the past research activities, most comparisons of bit error rate (BER) between these two systems are based on simulations. However, the analytical bit error probability (BEP) expressions of them have not been derived and evaluated yet. In this contribution, we first derive the analytical BEP expressions for both OFDM and FBMC under the additive white Gaussian noise (AWGN) channel. Then, by combining these analytical expressions with the characteristic of the Rayleigh channel, the close-form BEP expressions under Rayleigh Channel are also able to be obtained.

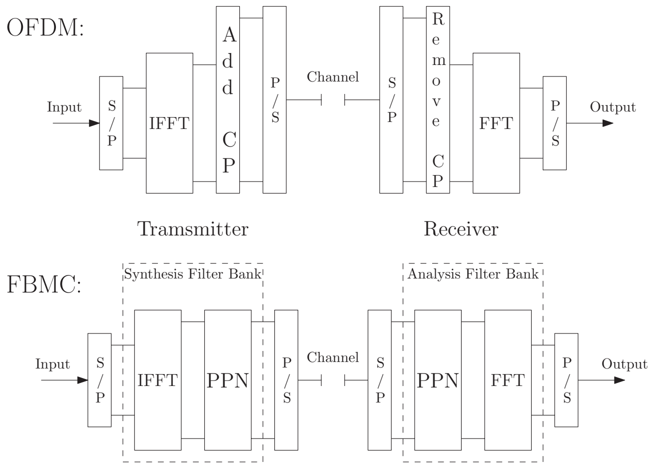

Fig. 1. OFDM and FBMC systems [3]

The rest of this paper is organized as follows. In Section II, we introduce the basic knowledge of OFDM and FBMC briefly. The system models under the AWGN channel are introduced and analysed in Section III. Also, the BEP expression of these two techniques are obtained in this section too. In Section IV, we incorporate the Rayleigh channel into the models and the analytical BEP expressions of this scenario are derived. The simulation results and the evaluation based on these are provided in Section V. Finally, we conclude our results and give some potential future work in Section VI.

II. Introduction to OFDM and FBMC

In this section, the basic concepts for these two multicarrier systems are introduced. The basic structure of them are shown in Figure 1.

A. *OFDM*

In the transmitter of the OFDM system, a bit stream is usually mapped to a quadrature amplitude modulation (QAM) symbol, called X, as shown in Figure 2. And then this constellation symbol stream passes through a serial-to-parallel converter, form a set of M parallel QAM symbols $X[0], X[1], ..., X[M-1]$ representing the symbols transmitted

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on each subcarriers. By modulating these symbols with the inverse fast Fourier transform (IFFT) at the transmitting side, a discrete baseband OFDM symbol can be generated, which can be written as

$$x[n] = \mathcal{F}^{-1}\{X[i]\} = \frac{1}{\sqrt{M}} \sum_{i=0}^{M-1} X[i] e^{j\frac{2\pi}{M}ni}, \ \ 0 \leq n \leq M-1, \tag{1}$$

where, M is the number of subcarriers, $X[i]$ is a complex QAM symbol transmitted on the ith subcarrier.

Obviously, the QAM symbol can be obtained at the receiver side utilizing Fast Fourier transform (FFT). In the perfect reconstruction situation, the recovered signal is

$$\hat{X}[i] = \mathcal{F}\{x[n]\} = \frac{1}{\sqrt{M}} \sum_{n=0}^{M-1} x[n] e^{-j\frac{2\pi}{M}ni}, \quad 0 \leq i \leq M-1. \tag{2}$$

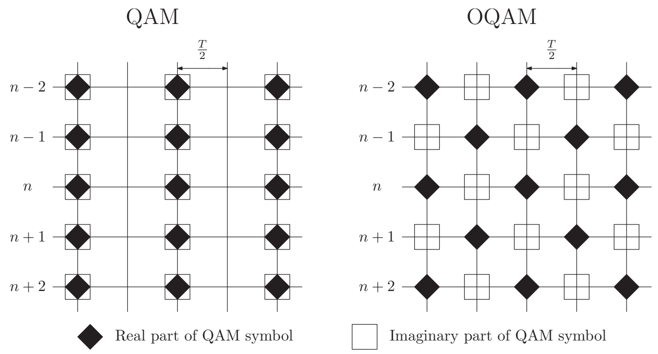


Fig. 2. OFDM and FBMC symbol mapping on carriers

B. FBMC/OQAM

There are two kinds of implementations of FBMC, the frequency spreading filter bank multicarrier (FS-FBMC) and the poly-phase network filter bank multicarrier (PPN-FBMC). The latter is selected as the model here as it reduces the high complexity which is introduced by the extra filtering operations at the transmitter and receiver. The simple PPN for the transmitter is shown in Figure 3.

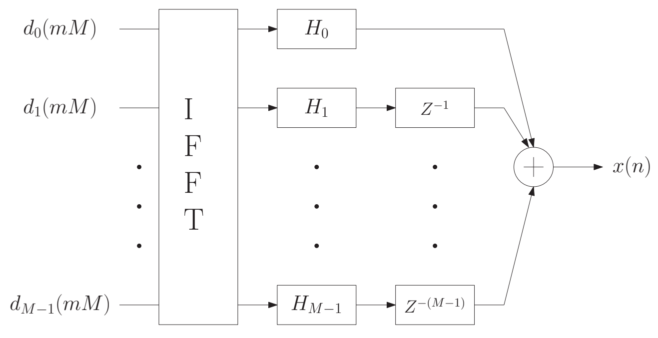


Fig. 3. PPN-IFFT FBMC transmitter [4]

In this work, the PPN filter applied is from PHYDYAS [4] with the overlapping factor $K = 4$. The frequency domain coefficients of it are

$$P_0 = 1; \ P_{\pm 1} = 0.97196; \ P_{\pm 2} = \frac{\sqrt{2}}{2}; \ P_{\pm 3} = 0.235147.$$

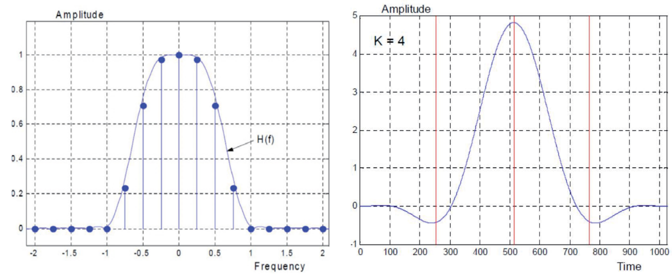
And they satisfy the equation

$$\frac{1}{K} \sum_{k=-K+1}^{K-1} |P_k|^2 = 1. \tag{3}$$

Based on these provided coefficients, the frequency response of the filter with M subcarriers is obtained through the equation

$$P(f) = \sum_{k=-(K-1)}^{K-1} P_k \frac{\sin\left(\pi\left(f - \frac{k}{MK}\right)MK\right)}{MK \sin\left(\pi\left(f - \frac{k}{MK}\right)\right)}. \tag{4}$$

As shown in Figure 4(a), a highly frequency selective filter is acquired with almost no out of band leakage. Therefore,



(a) Prototype filter frequency response (b) Prototype filter impulse response

Fig. 4. Prototype filter [4]

the impulse response $p[m]$ of the prototype filter is given by applying the IFFT to the frequency response, which is

$$p[m] = 1 + 2\sum_{k=1}^{K-1} (-1)^k P_k \cos\left(\frac{2\pi k}{MK} m\right) \tag{5}$$
$$p[0] = 0,$$

where $m = 1, \ldots, L_p - 1$, here $L_p = MK$ is the length of the filter p. The $p[0]$ applied here is to make the number of coefficients an odd number, thus, the delay of the filter can be adjusted to an integer multiple of the sample periods [5].

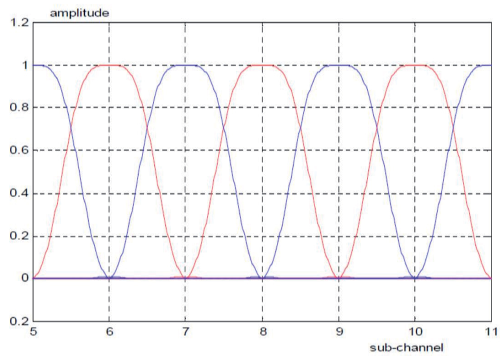


Fig. 5. Filter bank generated from prototype filter [6]

With this prototype filter, a filter bank of the system is obtained with frequency shifts, as shown in Figure 5. Obviously, the subcarriers with odd (or even) index are not overlapped. Only the neighbour subcarriers have influences in a certain subcarrier. On account of this feature, one can exploit only odd or even subcarriers for communication with QAM symbols. However, by utilizing only alternate subcarriers, the bit rate comes down by half as half of the capacity is left unused. To gain the full capacity, orthogonality is necessary for the neighbouring sub-channels. This orthogonality is satisfied with offset quadrature amplitude modulation (OQAM). The OQAM

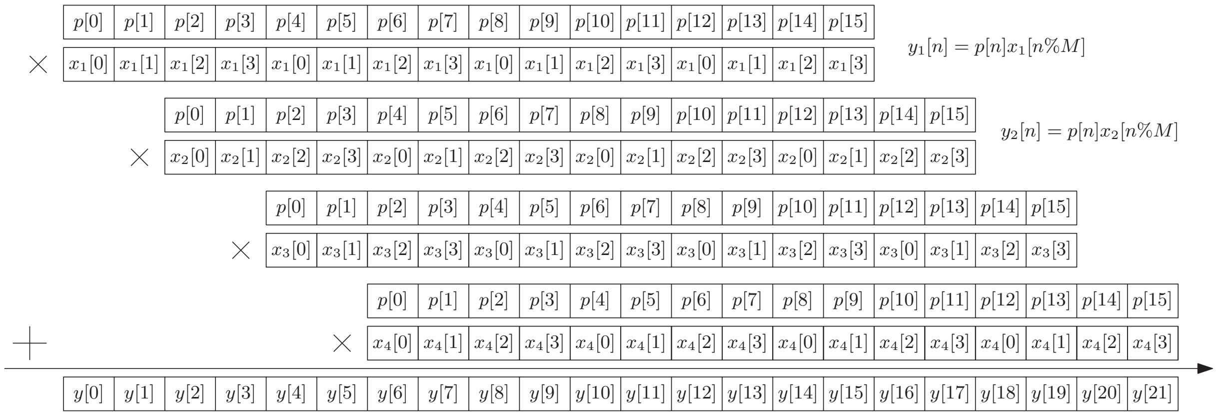


Fig. 6. Filter bank generated from prototype filter

staggered the in-phase and quadrature components by half a symbol period. Figure 2 implies the difference between QAM and OQAM symbols intuitively. In each sub-channel either the real or the imaginary part is transmitted avoiding interference between neighbouring sub-channels. Finally in order to keep up the bit rate, the FFT is run at twice the rate as in OFDM.

Now, we consider the signal at the transmitter side of the FBMC. As show in Figure 1, after the serial-to-parallel conversion, the OQAM symbols are passed into the IFFT block. Be denoting $X_\rho[i]$ as the symbol on the ith carrier of ρth frame, we obtain the IFFT block output $x_\rho[n]$,

$$x_\rho[n] = \mathcal{F}^{-1}\{X_\rho[i]\} = \frac{1}{\sqrt{M}} \sum_{i=0}^{M-1} X_\rho[i] e^{j2\pi ni/M} \ . \tag{6}$$

These symbols will be treated with the prototype filter before transmitting. Figure 6 shows how this symbol is constructed, in which both the overlapping factor and the carrier size are equal to 4. Obviously, the procedure to form such a symbol can be accomplished by two steps:

1) Treat each frame with the prototype filter. As we can see from Figure 6, the output of IFFT $x_\rho[n]$ is duplicated K times and then multiplied with the impulse response of the prototype filter. In order to simplify this procedure, we first define the modulo operation as $a\%b = a - \left\lfloor \frac{a}{b} \right\rfloor \times b$, where a, b are integers and $\lfloor x \rfloor$ means the largest integer not greater than x. Now, each filtered frame symbols $y_{\rho,n}$ is

$$y_\rho[n] = p[n] x_\rho[n\%M] \ . \tag{7}$$

2) Overlap the frames with half symbol duration shift. From the previous step, we get the filtered frames. Now they must be shifted with half symbol period $M/2$ one by one. Thus, the final transmitting signal is obtained by

$$y[n] = \sum_{\rho=1}^{K} y_\rho[n - \frac{\rho-1}{2} M] \ . \tag{8}$$

Because of the shift, the total length of y_n will be extended, which is equal to $L_p + (K-1)M/2 = (3K-1)M/2$.

At the receiver side of the FBMC, a two steps process is employed to recover the signal which will be passed into the FFT block, as shown in Figure 7. Let $r[n]$ denotes the received signal, then:

1) Divide the signal into K frames and apply the prototype filter to each frame. It is easy to see that for ρth frame we have $r_\rho[n] = r[n + \frac{\rho-1}{2} M]$. Therefore, by multiplying the ρth frame with the impulse response of the prototype filter, we have

$$s_\rho[n] = p[n] r_\rho[n]. \tag{9}$$

2) Resize the signal from the last step and sum up. Recall that the symbol after IFFT is duplicated K times at the transmitter side. Therefore, these K symbols must be reformed and summed up back to one symbol again. We use $\hat{s}_\rho[n]$ to denote this symbol, then

$$\hat{s}_\rho[n] = \sum_{k=0}^{K-1} s_\rho[n + kM] \quad n = 0, 1, ..., M-1. \tag{10}$$

At this point, we can pass the symbol $\hat{s}_\rho[n]$ to the FFT block frame by frame to recreate the OQAM symbol, which is

$$\hat{X}_\rho[i] = \mathcal{F}\{\hat{s}_\rho[n]\} = \frac{1}{\sqrt{M}} \sum_{n=0}^{M-1} \hat{s}_\rho[n] e^{-j2\pi ni/M} \ . \tag{11}$$

III. Systems under AWGN channel

In this section, we compare the performance of these two systems under the AWGN channel and acquire the analytical BEP expressions for them simultaneously. The Gaussian noise is defined as $w \sim \mathcal{N}(0, N_0)$.

A. OFDM based model

In the AWGN channel, the CP component of the OFDM system can be omitted as there is no ISI occurred. Hence, by denoting the received signal as $r[n] = x[n] + w[n]$ and combing equation (1) with equation (2) , we have

$$\begin{aligned} \hat{X}[i] &= \mathcal{F}\{r[n]\} = \mathcal{F}\{x[n] + w[n]\} \\ &= \mathcal{F}\{x[n]\} + \mathcal{F}\{w[n]\} \\ &= \mathcal{F}\{\mathcal{F}^{-1}\{X[i]\}\} + \mathcal{F}\{w[n]\} \\ &= X[i] + W[i]. \end{aligned} \tag{12}$$

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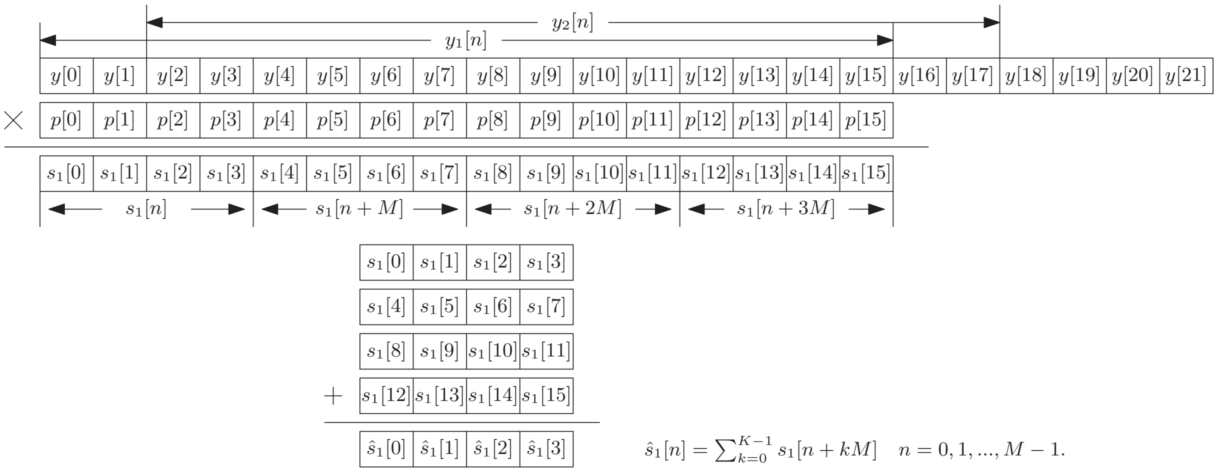

Fig. 7. FBMC receiver

From equation (12) , we can see that $W[i]$ is the noise term which is corrupting the QAM signal. Thus, the power of it should be obtained for the BER expression, which is

$$\begin{aligned} E\left\{W^2[i]\right\} &= E\left\{\left(\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} w[n]e^{-j\frac{2\pi}{N}ni}\right)^2\right\} \\ &= \frac{1}{N}\sum_{n=0}^{N-1} E\left\{w^2[n]\right\} e^{-j\frac{4\pi}{N}ni} = N_0. \end{aligned} \tag{13}$$

B. FBMC based model

Similar to the OFDM model, in the AWGN channel, the received signal of the FBMC system can be expressed as

$$\begin{aligned} r[n] &= y[n] + w[n] = \sum_{\rho=1}^{K} y_\rho[n - \frac{\rho-1}{2}M] + w[n] \\ &= \sum_{\rho=1}^{K} p[n - \frac{\rho-1}{2}M] x_\rho[(n - \frac{\rho-1}{2}M)\%M] + w[n]. \end{aligned} \tag{14}$$

After acquiring the sequence, the receiver of FBMC will conduct the two steps discussed in Section II. By resizing this signal and adding them together in the way of Equation (9) and Equation (10), the signal enter the FFT block is

$$\begin{aligned} \hat{s}_\rho[n] &= \sum_{k=0}^{K-1} s_\rho[n+kM] = \sum_{k=0}^{K-1} p[n+kM] r_\rho[n+kM] \\ &= \sum_{k=0}^{K-1} p[n+kM] r[n+kM+\frac{\rho-1}{2}M] \ , \end{aligned} \tag{15}$$

in which $r[n+kM+\frac{\rho-1}{2}M]$ can be simplified as

$$\begin{aligned} & r[n+kM+\frac{\rho-1}{2}M] \\ &= \sum_{\rho=1}^{K} p[n+kM+\frac{\rho-1}{2}M - \frac{\rho-1}{2}M] \cdot \\ &\quad x_\rho[\left(n+kM+\frac{\rho-1}{2}M - \frac{\rho-1}{2}M\right)\%M] \\ &\quad + w[n+kM+\frac{\rho-1}{2}M] \\ &= \sum_{\rho=1}^{K} p[n+kM] x_{\rho,n\%M} + w_{n+kM+\frac{\rho-1}{2}M}. \end{aligned} \tag{16}$$

Thus, by substituting Equation (16) into Equation (15), we have,

$$\begin{aligned} \hat{s}_\rho[n] &= \sum_{k=0}^{K-1} p[n+kM] \\ &\quad \left(\sum_{\rho=1}^{K} p[n+kM] x_\rho[n\%M] + w[n+kM+\frac{\rho-1}{2}M]\right) \\ &= \underbrace{\sum_{k=0}^{K-1} p^2[n+kM]}_{\hat{p}^2[n]} \sum_{\rho=1}^{K} x_\rho[n\%M] \\ &\quad + \underbrace{\sum_{k=0}^{K-1} p[n+kM] w[n+kM+\frac{\rho-1}{2}M]}_{\hat{w}_\rho[n]} \\ &= \hat{p}^2[n] \sum_{\rho=1}^{K} x_\rho[n\%M] + \hat{w}_\rho[n] \\ &= \hat{p}^2[n] x_\rho[n] + \hat{w}_\rho[n]. \end{aligned} \tag{17}$$

Therefore, the demodulator received signal should be,

$$\begin{aligned} \hat{X}_\rho[i] &= \mathcal{F}\{\hat{s}_\rho[n]\} = \mathcal{F}\{\hat{p}^2[n] x_\rho[n] + \hat{w}_\rho[n]\} \\ &= \mathcal{F}\{\hat{p}^2[n] x_\rho[n]\} + \mathcal{F}\{\hat{w}_\rho[n]\}. \end{aligned} \tag{18}$$

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Where, $\mathcal{F}\{\hat{p}^2[n]x_\rho[n]\}$ can be expressed as

$$\begin{aligned}
&\mathcal{F}\{\hat{p}^2[n]x_\rho[n]\}\\
&=\frac{1}{\sqrt{M}}\sum_{n=0}^{M-1}\hat{p}^2[n]\mathcal{F}^{-1}\{X_\rho[i]\}e^{-j2\pi ni/M}\\
&=\frac{1}{\sqrt{M}}\sum_{n=0}^{M-1}\hat{p}^2[n]\frac{1}{\sqrt{M}}\sum_{i=0}^{M-1}X_\rho[i]e^{j2\pi ni/M}e^{-j2\pi ni/M}\\
&=X_\rho[i]\frac{1}{M}\sum_{n=0}^{M-1}\hat{p}^2[n]\\
&=X_\rho[i]\frac{1}{M}\sum_{n=0}^{M-1}\sum_{k=0}^{K-1}p^2[n+kM]\\
&=X_\rho[i]\frac{1}{M}\sum_{n=0}^{Lp-1}p^2[n].
\end{aligned}\tag{19}$$

As in equation (19), the term $\frac{1}{M}\sum_{n=0}^{Lp-1}p^2[n]=1$. Thus, we can recover the signal with the added noise, which is,

$$\hat{X}_\rho[i]=X_\rho[i]+\mathcal{F}\{\hat{w}_\rho[n]\}=X_\rho[i]+\hat{W}_\rho[i].\tag{20}$$

The equation above shows that the noise term at the OQAM demodulator is $\hat{W}_\rho[i]$. As a result, the power of it should be obtained to get the analytical BER expression.

$$\begin{aligned}
E\left\{\hat{W}_\rho^2[i]\right\}&=E\left\{\mathcal{F}^2\{\hat{w}_\rho[n]\}\right\}\\
&=E\left\{\left(\frac{1}{\sqrt{M}}\sum_{n=0}^{M-1}\hat{w}_\rho[n]e^{-j2\pi ni/M}\right)^2\right\}\\
&=\frac{1}{M}\sum_{n=0}^{M-1}E\left\{\hat{w}_\rho^2[n]\right\}e^{-j4\pi ni/M}\\
&=\frac{1}{M}\sum_{n=0}^{M-1}E\left\{\hat{w}_\rho^2[n]\right\}.
\end{aligned}\tag{21}$$

It can be derived as

$$\begin{aligned}
&E\left\{\hat{w}_\rho^2[n]\right\}\\
&=E\left\{\left(\sum_{k=0}^{K-1}p[n+kM]w[n+kM+\frac{\rho-1}{2}M]\right)^2\right\}\\
&=\sum_{k=0}^{K-1}p^2[n+kM]E\left\{w^2[n+kM+\frac{\rho-1}{2}M]\right\}\\
&=N_0\sum_{k=0}^{K-1}p^2[n+kM].
\end{aligned}\tag{22}$$

By substituting Equation 22 into Equation 21, we get,

$$E\left\{\hat{W}_\rho[i]\right\}=\frac{1}{M}\sum_{n=0}^{M-1}N_0\sum_{k=0}^{K-1}p^2[n+kM]=N_0.\tag{23}$$

C. BER derivation for both systems

From the two previous subsectoins, the power of the noise that influences QAM and OQAM demodulators are derived, both are N_0. And it is clear that OQAM can be treated as a special case QAM because it only separates a QAM symbol into two symbols. This means that for I-ary rectangular QAM or OQAM symbol, the BER expression is the same under the AWGN channel for both OFDM and FBMC, which is

$$\mathrm{Pb}\approx\frac{\sqrt{I}-1}{\sqrt{I}\log_2\sqrt{I}}\mathrm{erfc}\left[\sqrt{\frac{3\log_2 I}{2\,(I-1)}\cdot\frac{E_b}{N_0}}\right],\ \text{see [7]}.\tag{24}$$

IV. Systems under Rayleigh Channel

In this section, the two systems under Rayleigh channel are studied. We use $h[n]$ to denote the impulse response of the Channel.

A. OFDM

In this case, the received signal is $r[n]=h[n]*x[n]+w[n]$, where $*$ meas convolution. At the receiver side for OFDM, a frequency domain equalizer will be applied to mitigate the distortion introduced by the channel, denoted as $C[i]$. The equalizer will satisfy the constraint which is

$$C[i]=\frac{1}{H[i]},\tag{25}$$

where $H[i]$ is the frequency response of the channel. Therefore, the signal entering the QAM demodulator is

$$\begin{aligned}
\hat{X}[i]&=C[i]\cdot\mathcal{F}\{r[n]\}=C[i]\cdot\mathcal{F}\{h[n]*x[n]+w[n]\}\\
&=C[i]\mathcal{F}\{h[n]*x[n]\}+C[i]\mathcal{F}\{w[n]\}\\
&=C[i]H[i]\mathcal{F}\{\mathcal{F}^{-1}\{X[i]\}\}+C[i]\mathcal{F}\{w[n]\}\\
&=X[i]+C[i]W[i]=X[i]+\hat{W}[i].
\end{aligned}\tag{26}$$

Thus, the channel attenuated noise power is

$$\begin{aligned}
E\left\{\hat{W}^2[i]\right\}&=E\left\{(C[i]W[i])^2\right\}\\
&=E\left\{C^2[i]\right\}\cdot E\left\{W^2[i]\right\}\\
&=N_0E\left\{C^2[i]\right\}.
\end{aligned}\tag{27}$$

As a result, the average SNR per bit is

$$\gamma=\frac{E_b}{N_0E\left\{C^2[i]\right\}}=\frac{E_b}{N_0}E\left\{H^2[i]\right\}=\frac{E_b}{N_0}\Omega.\tag{28}$$

B. FBMC

For the PPN based FBMC system, a time domain equalizer will be applied for the compensation of channel fading. By denoting the impulse response of the equalizer as $c[n]$ with property of

$$c[n]*h[n]=\delta[n],\tag{29}$$

where $\delta[n]$ is the Dirac delta function, we can achieve the received signal as $r_n=h[n]*y[n]+w[n]$. Thus, by passing it through the equalizer, the mitigated sequence $\hat{r}[n]$ is

$$\begin{aligned}
\hat{r}[n]&=c[n]*r[n]=c[n]*(h[n]*y[n]+w[n])\\
&=y[n]+\underbrace{c[n]*w[n]}_{v[n]}.
\end{aligned}\tag{30}$$

As we define $c[n]*w[n]=v[n]$, then $\hat{r}[n]$ has the same structure as equation (14) in Section III, which means the

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same derivations are obtained here. Using equation (20), the received signal of the OQAM demodulator are,

$$\hat{X}_\rho[i] = X_\rho[i] + \mathcal{F}\{\hat{v}_\rho[n]\} = X_\rho[i] + \hat{V}_\rho[i]. \tag{31}$$

Now, we consider $\hat{V}_{\rho,i}$, which is

$$\begin{aligned}\hat{V}_\rho[i] &= \mathcal{F}\{\hat{v}_\rho[n]\}\\ &= \mathcal{F}\left\{\sum_{k=0}^{K-1} p[n+kM]v[n+kM+\frac{\rho-1}{2}M]\right\}\\ &= \frac{1}{\sqrt{M}}\sum_{n=0}^{M-1}\sum_{k=0}^{K-1} p[n+kM]\cdot\\ &\qquad v[n+kM+\frac{\rho-1}{2}M]e^{-j2\pi ni/M}.\end{aligned} \tag{32}$$

Similar as equation (22), we get

$$\begin{aligned}E\left\{\hat{V}_\rho[i]^2\right\} &= E\left\{v^2[n+kM+\frac{\rho-1}{2}M]\right\}\\ &= E\left\{v^2[\lambda]\right\} = E\left\{(c[\lambda]*w[\lambda])^2\right\}\\ &= E\left\{\left(\sum_\eta w[\eta]c[\lambda-\eta]\right)^2\right\}\\ &= \sum_\eta E\left\{(w[\eta]c[\lambda-\eta])^2\right\}\\ &= \sum_\eta E\left\{w^2[\eta]\right\}E\left\{c^2[\lambda-\eta]\right\}\\ &= N_0\sum_\eta E\left\{c^2[\lambda-\eta]\right\}\\ &= N_0 E\left\{c^2[\lambda]\right\}.\end{aligned} \tag{33}$$

It is easy to prove that $E\left\{c^2[\lambda]\right\} = E\left\{C^2[i]\right\}$ when $C[i] = \mathcal{F}\left\{c[\lambda]\right\}$. Thus, the average SNR per bit for FBMC is the same as OFDM like equation (28) shows.

C. BER derivation for both systems

The BER expression for both systems can be achieved base on the previous conclusion. As FBMC and OFDM have the same average SNR per bit for the demodulator, the BER expression for them are the same. For I-ary rectangular QAM or OQAM modulation over Rayleigh channel, the BER is [8],

$$\begin{aligned}P_b(e) = &\frac{2}{\log_2 I}\left(\frac{\sqrt{I}-1}{\sqrt{I}}\right)\left(1-\sqrt{\frac{1.5\gamma_s}{I-1+1.5\gamma_s}}\right)\\ &-\left(\frac{\sqrt{I}-1}{\sqrt{I}}\right)^2\left[1-\sqrt{\frac{1.5\gamma_s}{M-1+1.5\gamma_s}}\right.\\ &\left.\left(\frac{4}{\pi}\tan^-1\sqrt{\frac{I-1+1.5\gamma_s}{1.5\gamma_s}}\right)\right]\end{aligned}, \tag{34}$$

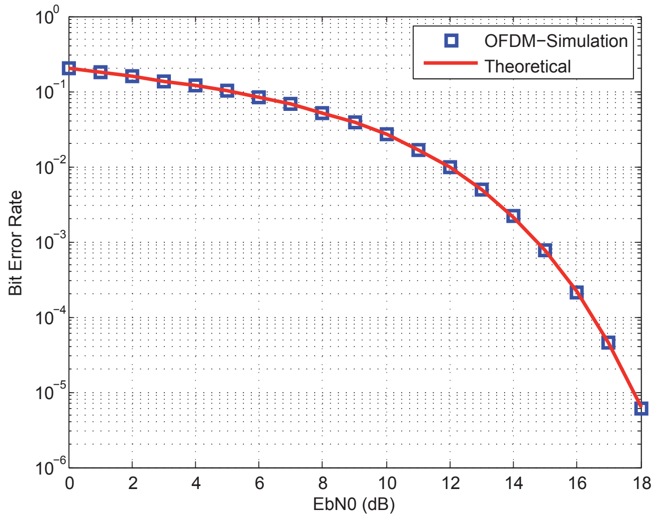
where $\gamma_s = \gamma\log_2 I$ denotes the average SNR per symbol.

V. Simulation and Evaluation

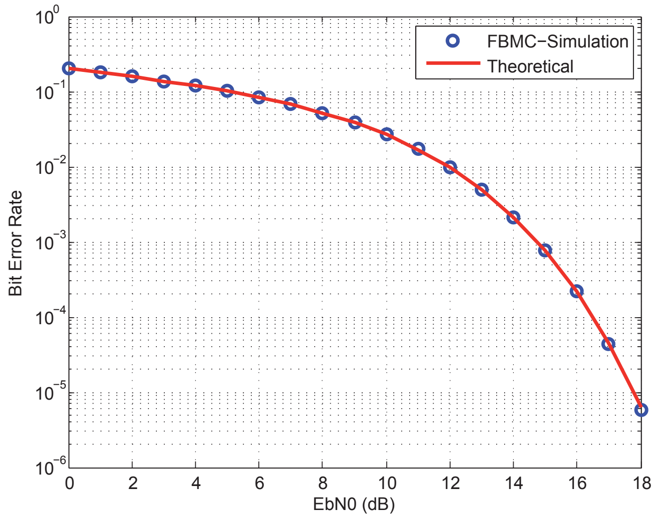
In this section, the theoretical and simulated BER will be compared under both AWGN and Rayleigh channels for FBMC and OFDM. The carrier size M of them are 64. And the 64-QAM (or OQAM) is applied in the simulation. For the OFDM system, the length of CP is 16. And in the FBMC system, the overlapping factor $K = 4$ is used. The Rayleigh multipath channel applied here is a 5-tap channel. We assume that the perfect channel state information is available at the receiver.

A. Under AWGN Channel

For the AWGN channel, the CP part of OFDM system can be neglected as there is no ISI. Therefore, the CP are not taken into the simulation.



(a) OFDM under the AWGN



(b) FBMC under the AWGN

Fig. 8. BER under the AWGN channel

Figure 8. shows the simulation results of OFDM and FBMC in AWGN channel and compared with the derived BER expression. Both two simulations match the analytical expression perfectly. This proves that their performances in the AWGN channel is exactly the same and validates our expression too.

B. Under Rayleigh Channel

Firstly, the FBMC system with different time domain equalizers are simulated. The taps' length of these equalizers varies from 6 to 1200, as shown in Figure 9.

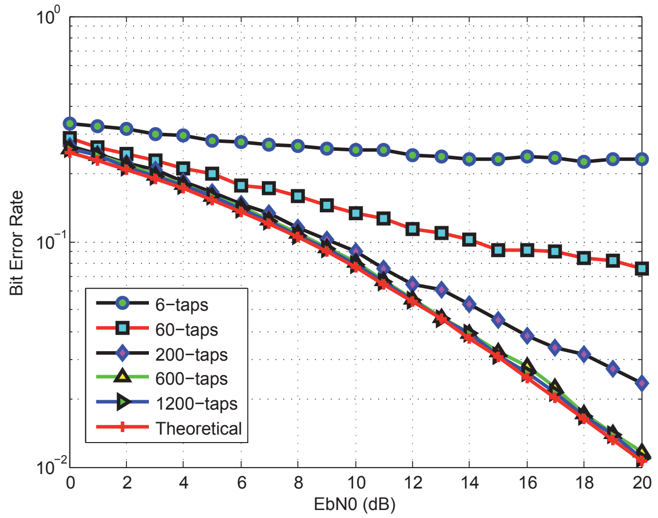


Fig. 9. FBMC with 200-tap equalizer in the Rayleigh Channel

From the figure above, we can tell that in order to reach the theoretical BER performance for FBMC system, a very long time domain equalizer is necessary. However, this will introduce extra time delay to the system with growth in the computational complexity.

Secondly, the OFDM system is simulated and evaluated. Figure 10 and Figure 11 show the BER performance of this system in the situation with and without CP correspondingly.

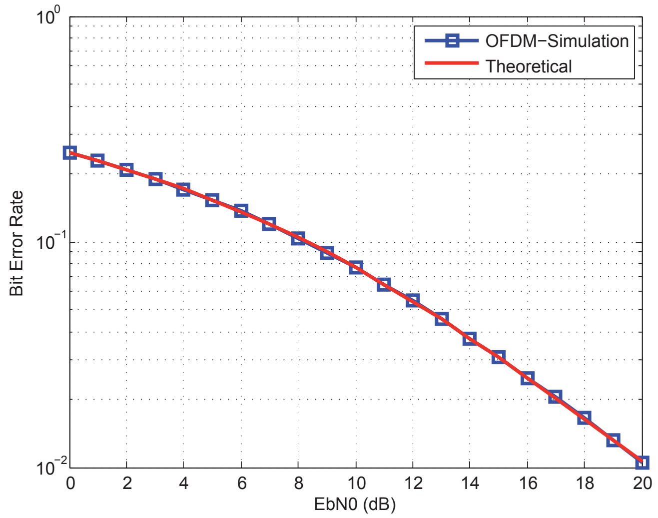


Fig. 10. FBMC with 200-tap equalizer in the Rayleigh Channel

Unlike in the AWGN channel where the CP does not really matter at all, in fading channels CP plays a vital role to remove the ISI. As we can see from it, when the length of CP is longer than the channel impulse response, namely it can eliminate the ISI perfectly, the analytical BER and the simulated one are matching ideally. Thus, the OFDM system without the cyclic prefix has a very poor performance. However, the implementation of CP will decrease the bandwidth efficiency.

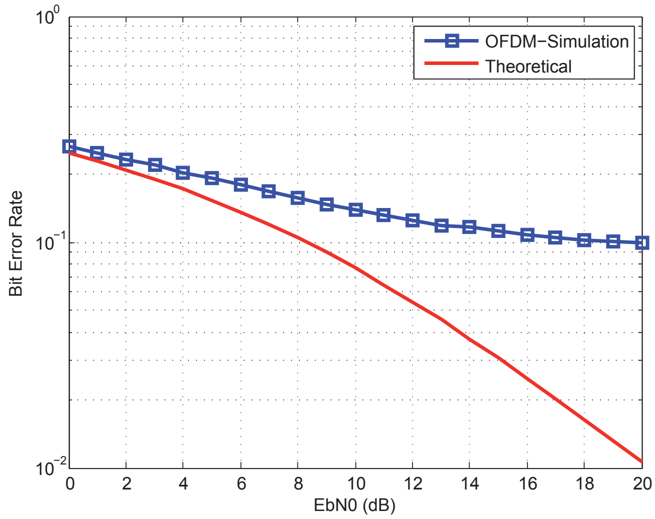


Fig. 11. OFDM without CP in the Rayleigh channel

VI. Conclusion

In this paper, the analytical BER expressions for OFDM and PPN-FBMC systems under AWGN and Rayleigh channels are derived and validated. We prove that the BER performance of the two systems are the same when the perfect recover conditions are satisfied. In order to achieve this, OFDM requires the cyclic prefix to eliminate the inter symbol interference, while FBMC needs a long time domain equalizer.

Hence, the further research can concentrate on the new equalization technique for PPN-FBMC which reduces the complexity for mitigating the signal.

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