A Better Decision Rule for OFDM with Subcarrier Index Modulation

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***Abstract*—Orthogonal frequency division multiplexing with subcarrier index modulation (OFDM-SIM) is a promising technique which has been proposed and developed recently. Unlike the conventional OFDM system, the indices of the subcarriers in this new system are also utilized for transmitting the information bits. The On-Off Keying (OOK) is employed to control the subcarrier status. This innovative structure leads to a better trade-off between the system performance and energy efficiency. Meanwhile, the peak to average power ratio (PAPR) is decreased with this enhanced system. Therefore, finding the status of the carrier correctly is critical. In this paper, a new decision rule for determining the subcarrier's status is proposed and evaluated. And the analytical bit error rate (BER) expression of the OFDM-SIM system with this new decision rule is derived either.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) can be seen as the most important multicarrier transmission technique in the field of modern communication. The applications of this technique are various, such as the asymmetric digital subscriber line (ADSL), the wireless local area network (WLAN) and the long term evolution (LTE) in today's 4G standard. However, there are also some disadvantages of the traditional OFDM system that limited the performance. One critical drawback of it is the high peak to average power ratio (PAPR) which induces unnecessary distortion to the amplified signal due to the limited linear range of a power amplifier. Many efforts have been spent on reducing the PAPR and one modification of the original OFDM that introduces the subcarrier index modulation (OFDM-SIM) is proved to be effective.

The essential idea of OFDM-SIM is inspired by the spatial modulation (SM) (see [1], [2], [3]), which employs the antenna indices of an multiple-input multiple-output (MIMO) system as an additional dimension to convey the information. Obviously, one can extend this extra dimension from space domain to frequency domain for an OFDM system. The status of the subcarriers are set to either active or inactive. Therefore, it can be exploited to transmit data in an On-Off Keying (OOK) fashion. Afterwards, the modulated constellation symbols, for example, the multilevel quadrature amplitude modulation (M-QAM) symbols, can be carried by these active subcarriers.

Regarding the utilization of the subcarriers indices, a normal scheme can be found in [4]. For a system with N subcarriers, a block of data stream for one transmission has a length of $N + \frac{N}{2}\log_2(M)$ bits, and it can be divided into two parts, see Fig. 1. The first subblock with N bits controls the status of the subcarriers and we denote it as B_{OOK}. The second subblock is referd to as B_{QAM} and it contains $N/2$ M-QAM symbols that will be transmitted by the active carriers. Before the start of each transmission, the majority bit type of B_{OOK}

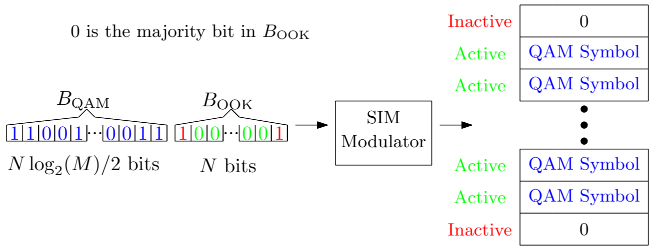


Fig. 1. The system model of OFDM-SIM [5]

has to be determined, i.e., to find out which kind of bit ('1' or '0') presents more in the subblock. Then, the majority bit type will be labelled as 'active' and the B_{QAM} symbols will be transmitted by the corresponding 'active' subcarriers. At the same time, the type of majority bit will be sent by the remaining active subcarriers. The disadvantages of this scheme are obvious. First, a threshold is required at the receiver side to detect the OOK bits. Further, it is relatively difficult to find the suitable threshold when the signal to noise ratio (SNR) is low as this threshold must be below the lowest power of the M-QAM symbol and above the power of the Gaussian noise. The misjudgement on the status of a subcarrier will not only lead to an incorrect demodulation of the OOK bits but also change the order of the subsequent QAM symbols. Second, the decision step for the majority bit type in the transmitter side introduces additional complexity to the system. Besides, this extra information may be corrupted during the transmission. Once the majority bit type is faulty determined at the receiver side, the whole B_{OOK} will be decoded incorrectly. Hence, in order to overcome these drawbacks, an enhanced OFDM-SIM system is proposed in [5], which utilizes the subcarrier pairs to convey the OOK bits. And we only consider the enhanced model in this paper.

The remainder of this paper is organized as follows. In Section II, the enhanced OFDM-SIM system is introduced in details. A better decision rule for this system is presented and evaluated in Section III. The analytical expression of the bit error rate (BER) with this new decision rule is derived in Section IV. Finally, we conclude this paper in Section V.

II. Enhanced OFDM-SIM

The enhanced OFDM-SIM is a kind of modification of the original OFDM-SIM. The essential concept of it is quite simple. Instead of mapping the bits in B_{OOK} to each subcarrier one by one, we can use two consecutive carriers to encode one bit. Thus, the status order of a subcarrier pair conveys the information from OOK. As a result of using carrier pairs, the length of the subblock B_{OOK} reduces to $N/2$ when the total

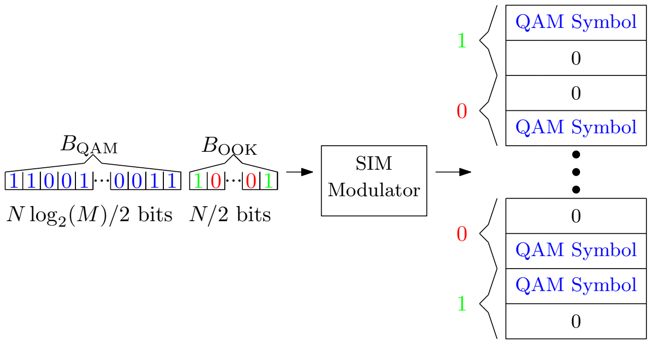
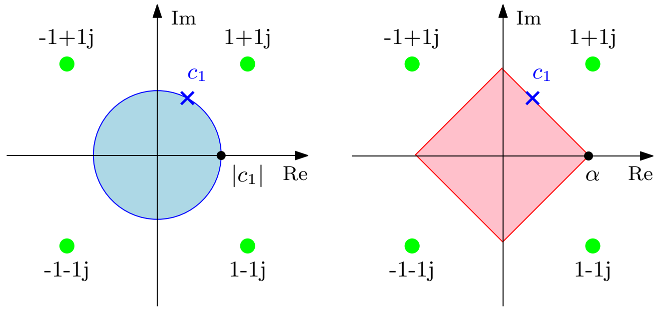


Fig. 2. The system model of enhanced OFDM-SIM [5]



(a) The old decision rule (b) The new decision rule

Fig. 3. The received symbol under different decision rules

number of the subcarriers is N. One possible scheme to encode the OOK bits with carrier pairs is illustrated in Fig. 2. When value 1 is encountered in the OOK series, the first subcarrier in the pair is set to active while the second subcarrier is kept to silent. On the contrary, if the bit is 0, the first carrier in the pair remains to inactive and the second one is invoked for data transmission. Clearly, only $N/2$ carriers are set as active to transmit the M-QAM symbols.

The benefits of this new scheme are obvious, the step to decide the majority bit type is neglected; the threshold for judging the status of the carriers at the receiver side is no longer necessary as we can compare the carrier pair instead; and the order of the QAM symbols is not influenced by the misjudgement of the carriers status as each pair must contain one QAM symbol. In fact, the misjudgement can only lead to an error within the pair. The only disadvantages of this enhanced OFDM-SIM lie in the spectral efficiency. For the conventional OFDM-SIM system, the average bits conveyed by each subcarrier is expressed as

$$\Psi^{\text{Original}}_{\text{OFDM-SIM}} = \frac{\log_2(M)}{2} + 1, \tag{1}$$

while the spectral efficiency of the enhanced OFDM-SIM is measured by

$$\Psi^{\text{Enhanced}}_{\text{OFDM-SIM}} = \frac{\log_2(M)}{2} + \frac{1}{2}. \tag{2}$$

However, compared to the advantages the enhanced OFDM-SIM have, this price is worth paying.

III. A Better Decision Rule

As mentioned in the previous section, at the receiver side of the system, the status of each subcarrier inside of the carrier pair must be distinguished so that one can obtain the B_{OOK} bits. Then, the B_{QAM} sequence can be achieved by demodulating the QAM symbols at the corresponding active subcarriers. Hence, finding a suitable and feasible decision rule to detect the active subcarrier correctly becomes critical.

Apparently, the subcarrier pairs are independent to each other. Thus, the analysis of one pair can be generalized to the whole system. In this paper, we only consider the presence of the AWGN channel. As illustrated in Fig. 3, a 4-QAM symbol is transmitted through the AWGN channel and the received symbol at the corresponding active carrier is represented by the blue cross 'X'. Let us denote this symbol as $c_1 = x_1 + jy_1$ and the received symbol on the inactive carrier is represented by $c_0 = x_0 + jy_0$ which is just a Gaussian noise.

Intuitively, a possible decision rule for detecting the active and inactive carriers is to compare the power or the amplitude of the two received symbols. Thus, in order to make a correct decision, the amplitude of the symbol on the active carrier must be higher than the amplitude of the symbol on the inactive carrier, i.e., $|c_1| > |c_0|$. As we can seen from Fig. 3(a), the light blue area inside the solid circle represents the feasible range of the symbol on the inactive carrier which can fulfil the requirement of a correct detection.

In this work, we propose a new scheme to identify the active and inactive subecarriers within a carrier pair. Instead of comparing the power of the received symbols over these two subcarriers directly, the absolute value of the real and imaginary parts for the complex QAM symbols are utilized for the detection. In this decision rule, a successful detection requires that the summation of the absolute values for the QAM symbol's real and imaginary parts over the active subcarrier is bigger than the summation over the inactive subcarrier, i.e., $|x_1| + |y_1| > |x_0| + |y_0|$. An example with 4-QAM of this new decision rule is illustrated in Fig. 3(b). The point of the received symbol on the inactive subcarrier must be inside of the pink square so that the correct decision can be made.

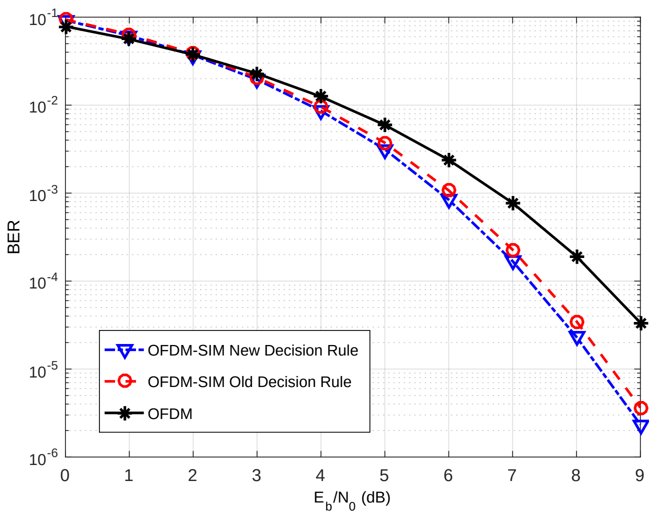


Fig. 4. The comparison of two decision rules

In order to compare the performance of these two decision rules and evaluate the OFDM-SIM system, a simulation is conducted with the carrier size $N = 16$ and the QAM constellation size $M = 4$. Meanwhile, the traditional OFDM system is also included in the simulation to provide a performance baseline. The result is plotted in Fig. 4. Based on this figure, we can draw two conclusions. First, under either new or old decision rules, the OFDM-SIM system outperforms the

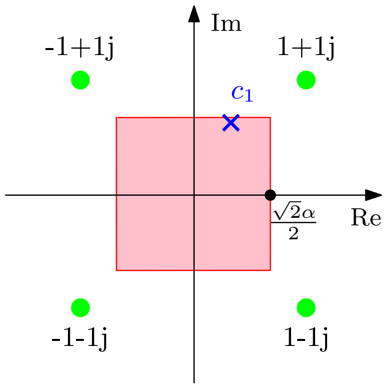


Fig. 5. The new decision rule with rotated feasible region

traditional OFDM from the point where E_b/N_0 equals to 2dB. And there is only a slight difference when E_b/N_0 is less than 2dB. Second, the OFDM-SIM with the new decision rule has a better performance when compared to the one with the old decision rule, especially in the higher E_b/N_0 range. Although this improvement is not significant, it is still worth introducing the new decision to the OFDM-SIM system as the complexity does not increase. And the BER performance will be analysed in the following section.

IV. Analytical BER derivation

As described in the preceding part, the received complex symbol of the inactive carrier must be located inside of the pink square, see Fig.3(b). And this complex symbol is indeed an additive complex Gaussian noise, i.e., $c_0 \sim \mathcal{CN}(0, N_0)$, where N_0 is the noise spectral density. Therefore, the probability of a successful detection can be expressed as

$$
\begin{aligned}
&P\left(d=1|c_1=x_1+jy_1\right)\\
&=\int_{-|x_1|-|y_1|}^{|x_1|+|y_1|}\int_{-|x_1|-|y_1|+|y_0|}^{|x_1|+|y_1|-|y_0|}\frac{1}{\pi N_0}e^{-\frac{x_0^2+y_0^2}{N_0}}\mathrm{d}x_0\mathrm{d}y_0,
\end{aligned}
\tag{3}
$$

where '$d=1$' represents that the carriers are correctly distinguished. Equation (3) actually gives us a conditional probability, which means that if c_1 is received at the active subcarrier, then the probability of detecting that this is the active subcarrier in the carrier pair is as stated there.

It is clear that the probability density function (PDF) of the Gaussian noise c_0 is unchanged as long as its amplitude $|c_0|$ remains same. Further, we know that the feasible region is a square centred in the origin. Therefore, this region of integration can be rotated by $45°$ without interfering the computational result, as shown in Fig. 5. By denoting $\alpha = |x_1|+|y_1|$, the original integral can be modified as

$$
\begin{aligned}
&P\left(d=1|c_1=x_1+jy_1\right)\\
&=\frac{1}{\pi N_0}\int_{-\frac{\sqrt{2}}{2}\alpha}^{\frac{\sqrt{2}}{2}\alpha}\int_{-\frac{\sqrt{2}}{2}\alpha}^{\frac{\sqrt{2}}{2}\alpha}e^{-\frac{x_0^2+y_0^2}{N_0}}\mathrm{d}x_0\mathrm{d}y_0\\
&=\frac{4}{\pi N_0}\int_0^{\frac{\sqrt{2}}{2}\alpha}\int_0^{\frac{\sqrt{2}}{2}\alpha}e^{-\frac{x_0^2+y_0^2}{N_0}}\mathrm{d}x_0\mathrm{d}y_0\\
&=\frac{4}{\pi N_0}\int_0^{\frac{\sqrt{2}}{2}\alpha}e^{-\frac{x_0^2}{N_0}}\mathrm{d}x_0\int_0^{\frac{\sqrt{2}}{2}\alpha}e^{-\frac{y_0^2}{N_0}}\mathrm{d}y_0\\
&=\left[\mathrm{erf}\left(\frac{\alpha}{\sqrt{2N_0}}\right)\right]^2=\mathrm{erf}^2\left(\frac{|x_1|+|y_1|}{\sqrt{2N_0}}\right).
\end{aligned}
\tag{4}
$$

Now, we will obtain the probability that the value c_1 is presented at the active subcarrier. For a M-QAM constellation, we suppose that each constellation point appears with the same probability $1/M$ on the active subcarrier at the transmitter side. After passing this symbol through the AWGN channel, it will be distorted and the probability density of each constellation point becomes to a Gaussian function centred at this point. Thus, the probability density of receiving the symbol at the active subcarrier with value c_1 becomes a weighted combination of these M points' probability densities. By denoting μ_m as the mth point in the constellation, the weighted combination is calculated as

$$
\begin{aligned}
&P\left(c_1=x_1+jy_1\right)\\
&=\frac{1}{M}\sum_{m=1}^{M}\frac{1}{\pi N_0}e^{-\frac{(x_1-\Re\{\mu_m\})^2+(y_1-\Im\{\mu_m\})^2}{N_0}},
\end{aligned}
\tag{5}
$$

where $\Re\{\cdot\}$ and $\Im\{\cdot\}$ represent the real and imaginary part of a complex symbol, respectively. In order to shorten the notation, we let $R_m=\Re\{\mu_m\}$ and $I_m=\Im\{\mu_m\}$.

Consequently, the probability for correctly determining the active subcarrier is obtained by combining equation (4) with (5) and integrating it over the complex plane, which is

$$
\begin{aligned}
P_1&=P(d=1)\\
&=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}P\left(d=1|c_1=x_1+jy_1\right)\\
&\qquad\qquad P\left(c_1=x_1+jy_1\right)\mathrm{d}x_1\mathrm{d}y_1\\
&=\frac{1}{\pi MN_0}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{erf}^2\left(\frac{|x_1|+|y_1|}{\sqrt{2N_0}}\right)\\
&\qquad\sum_{m=1}^{M}e^{-\frac{(x_1-R_m)^2+(y_1-I_m)^2}{N_0}}\mathrm{d}x_1\mathrm{d}y_1\\
&=\frac{1}{\pi MN_0}\sum_{m=1}^{M}e^{-\frac{\mu_m^2}{N_0}}\int_0^{\infty}\int_0^{\infty}\mathrm{erf}^2\left(\frac{x_1+y_1}{\sqrt{2N_0}}\right)e^{-\frac{x_1^2+y_1^2}{N_0}}\\
&\qquad\left(e^{\frac{2x_1R_m}{N_0}}+e^{\frac{-2x_1R_m}{N_0}}\right)\left(e^{\frac{2y_1I_m}{N_0}}+e^{\frac{-2y_1I_m}{N_0}}\right)\mathrm{d}x_1\mathrm{d}y_1.
\end{aligned}
\tag{6}
$$

In order to simplify equation (6), we apply a substitution to the integration. Let $x_1=\frac{p-q}{2}$ and $y_1=\frac{p+q}{2}$, then $p=x_1+y_1$ and $q=x_1-y_1$. The determinant of the Jacobian matrix is calculated as

$$
\det J(p,q)=\begin{vmatrix}\frac{\partial x_1}{\partial p} & \frac{\partial x_1}{\partial q}\\ \frac{\partial y_1}{\partial p} & \frac{\partial y_1}{\partial q}\end{vmatrix}=\begin{vmatrix}\frac{1}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{vmatrix}=\frac{1}{2}.
\tag{7}
$$

Therefore, we can rewrite equation (6) as

$$
\begin{aligned}
P_1&=\frac{1}{2\pi MN_0}\sum_{m=1}^{M}e^{-\frac{\mu_m^2}{N_0}}\int_{-\infty}^{\infty}\int_0^{\infty}\mathrm{erf}^2\left(\frac{p}{\sqrt{2N_0}}\right)e^{-\frac{p^2+q^2}{2N_0}}\\
&\left(e^{\frac{R_m+I_m}{N_0}p}e^{\frac{-R_m+I_m}{N_0}q}+e^{\frac{R_m-I_m}{N_0}p}e^{\frac{-R_m-I_m}{N_0}q}+\right.\\
&\left.e^{\frac{-R_m+I_m}{N_0}p}e^{\frac{R_m+I_m}{N_0}q}+e^{\frac{-R_m-I_m}{N_0}p}e^{\frac{R_m-I_m}{N_0}q}\right)\mathrm{d}p\mathrm{d}q.
\end{aligned}
\tag{8}
$$

The equation above provides us the probability for successfully distinguishing the active and inactive subcarrier in a carrier

$$\widehat{P_1} = \frac{1}{3} + \frac{1}{3\sqrt{3\pi}} e^{\frac{-2l^2}{N_0}} \sum_{n=0}^{\infty} \frac{1}{3^n} \sum_{k=0}^{n} \frac{1}{\Gamma\left(\frac{3}{2}+k\right)\Gamma\left(\frac{3}{2}+n-k\right)} \sum_{v=0}^{\infty} \frac{\Gamma\left(n+\frac{3}{2}+v\right)}{\Gamma(2v+1)} \left(\frac{8l^2}{3N_0}\right)^v \tag{15}$$

pair. Apparently, the probability of the incorrect detection is given by

$$P_0 = P(d=0) = 1 - P_1. \tag{9}$$

As the detection probabilities P_1 and P_0 are already achieved, now the analytical BER performance of the system can be acquired. The bit stream is composed of two subblocks B_{OOK} and B_{QAM}. Therefore, the BER of each subblock is able to be achieved individually. For a carrier pair, the total number of the transmitted bits is equal to $\log_2(M)+1$ in which 1 is the bit carried by the OOK modulation. Therefore, for an erroneous detection, the BER of B_{OOK} is calculated by

$$E_{\mathrm{OOK}} = P_0 \frac{1}{\log_2(M)+1}. \tag{10}$$

For the subblock B_{QAM}, there are two parts that constitute its BER. The first one is under the condition of the incorrect detection. That is to say the QAM demodulator will map the noise received by the inactive subcarrier to an M-QAM constellation point. A reasonable hypothesis is that most of the erroneous detection appear at the four constellation points that are closest to the origin, especially in the higher SNR range with higher modulation order M. For a Gray coded QAM, the average bit error among these four points is 1. Thus, the BER in this case is achieved as the first term of equation (11). Now the second part which has the carrier correctly distinguished must be analysed. The M-QAM symbol occupies $\log_2(M)$ bits out of the total bits. And the Gray mapping results in the BER performance of the QAM symbols being very similar. This is because that most misjudgements occur at the adjacent constellation points. It yields a result that the BER of QAM symbol is almost independent of the status of the subcarrier. Therefore, the BER expression for M-QAM P_e (see [6]) can be applied and the BER for the subblock B_{QAM} is

$$E_{\mathrm{QAM}} = P_0 \frac{1}{\log_2(M)+1} + P_1 P_e \frac{\log_2(M)}{\log_2(M)+1}. \tag{11}$$

Clearly, the final BER expression of the enhanced OFDM-SIM system is the summation of the two subblocks' BER, which is

$$E_{\mathrm{OFDM-SIM}} = E_{\mathrm{OOK}} + E_{\mathrm{QAM}}. \tag{12}$$

As 4-QAM is used in the previous simulation, we will analyse it particularly. It is easy to know that 4-QAM has the feature which is $|R_m| = |I_m| = l$. By substituting this value to equation (8), the correct detection probability for the 4-QAM $\widehat{P_1}$ is expressed as

$$\widehat{P_1} = \frac{1}{2\pi N_0} e^{-\frac{2l^2}{N_0}} \int_{-\infty}^{\infty}\int_{0}^{\infty} \mathrm{erf}^2\left(\frac{p}{\sqrt{2N_0}}\right) e^{-\frac{p^2+q^2}{2N_0}} \left(e^{\frac{2l}{N_0}p} + e^{\frac{-2l}{N_0}q} + e^{\frac{2l}{N_0}q} + e^{\frac{-2l}{N_0}p}\right) \mathrm{d}p\mathrm{d}q. \tag{13}$$

In order to compute this integration, the square of the error function must be expanded, which is

$$\mathrm{erf}^2(x) = e^{-2x^2} x^2 \sum_{n=0}^{\infty}\sum_{k=0}^{n} \frac{x^{2n}}{\Gamma\left(\frac{3}{2}+k\right)\Gamma\left(\frac{3}{2}+n-k\right)}, \tag{14}$$

where $\Gamma(\cdot)$ is the gamma function. By combining equation (13) with equation (14) and referring to some helpful equations of integrals (see [7], [8], [9]), the final expression of $\widehat{P_1}$ can be computed by equation (15). And with this equation, the BER for 4-QAM is achievable.

V. Conclusion

In this paper, an enhanced OFDM system with index subcarrier modulation is introduced and studied. Generally, the system outperforms the traditional OFDM system with a lower bit error rate. Meanwhile, a new decision rule for this system is proposed. This decision rule is compared with the conventional one by simulation. And the result proves that the new decision has a better performance. Furthermore, a closed form BER expression for the OFDM-SIM with the new decision rule is derived.

For the future work, first, the analytical BER expression may be reduced by some handy approximations as the current one is rather complex. This is likely to result in a numerical problem for the simulation. Second, some other decision rules can be studied and tested.

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