Fast and Efficient Power and Rate Allocation for Multiuser OFDM Downlink

Chunhui Liu and Rudolf Mathar
Institute for Theoretical Information Technology,
RWTH Aachen University
D-52056 Aachen, Germany
Email: {liu,mathar}@ti.rwth-aachen.de

***Abstract*—Adaptive power and rate allocation for multiuser orthogonal frequency division multiplexing (OFDM) has been shown to improve the system performance significantly. In this paper, a group of resource allocation methods with low complexity for multiuser OFDM downlink is presented. The proposed methods initialize subcarrier assignments for users independently, which may result in conflicts on subcarriers. Following processes are designed to resolve such conflicts, where the output is further improved by simply sorting the subcarriers with conflicts according to appropriate criteria. Simulation results show that the best proposed method has less than 4% performance degradation with up to 95% complexity reduction compared to a newly suggested reference algorithm.**

I. Introduction

OFDM provides a low-complexity means of combating the effects of delay spread in high-speed wireless data transmission, where the transmission band is divided into orthogonal subcarriers. Depending on channel characteristics multiuser OFDM can allocate power and rate optimally on subcarriers in order to take advantage of channel diversity among users in different locations.

Optimal rate and power allocation for multiuser OFDM has been formulated in [1] and can be roughly classified into two groups according to different constraints: the rate-adaptive (RA) and margin-adaptive (MA) optimizations. Concerning the latter case, the algorithm proposed in [2] achieves near-optimal performance, but it is computationally intensive and difficult to implement for the case of large numbers of users. Heuristic approaches with low complexity are suggested in [3], [4], [5], [6], [7] at expense of some performance loss.

In this paper the proposed methods for multiuser resource allocation inherit the idea of initialization in [8]. Compared to the approach in [8] they have lower complexity and take more constraints into account while achieving comparable performance to the reference method in [3]. Furthermore, the efficiency of varying a subcarrier assignment is investigated and criteria for sorting subcarriers are developed to improve the system performance. These may be also adopted by other approaches, e.g., [4], [7]. Downlink in a single cell with one base station and multiple mobile users are considered.

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The remainder of this paper is organized as follows. Section II presents the system model and formulates the MA problem. In Section III, an efficient single-user water-filling algorithm is explained, which is further analyzed in Section IV to find a computationally inexpensive way to utilize it. Based on this analysis, new algorithms for multiuser resource allocation are designed. Numerical results are given in Section V. Finally, this paper is concluded.

II. System Model And Problem Formulation

We consider downlink transmission in a multiuser OFDM system with N subcarriers and K users. It is assumed that transmissions of different users are subject to independent frequency selective fading and that perfect channel state information (CSI) is available at the transmitter. In frequency domain an OFDM symbol received by user k can be written as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \boldsymbol{\omega}_k,$$

where the $N \times 1$ vectors $\mathbf{y}_k$ and $\mathbf{x}_k$ refer to the received and transmitted OFDM symbols, respectively. The noise vector $\boldsymbol{\omega}_k$ is complex Gaussian distributed with distribution $\boldsymbol{\omega}_k \sim \mathcal{CN}(\mathbf{0}, \sigma^2_{\omega_k}\mathbf{I_N})$. The diagonal matrix $\mathbf{H}_k$ is $diag(H_k[1], \dots, H_k[N])$, where $H_k[n]$ is the frequency response on the nth subcarrier of user k.

Let $G_k[n]$ denote the channel-to-noise ratio (CNR) on the nth subcarrier of user k defined as

$$G_k[n] = \frac{|H_k[n]|^2}{\Gamma_k \sigma^2_{\omega_k}},$$

where Γ_k refers to the signal-to-noise ratio (SNR) gap determined by the bit-error rate (BER) required by user k. The power-rate function can be expressed as

$$r_k[n] = \log_2(1 + P_k[n]G_k[n]), \tag{1}$$

where $P_k[n]$ and $r_k[n]$ denote the power and rate allocated on the nth subcarrier of user k, respectively.

The MA resource allocation for the downlink of multiuser OFDM systems is equivalent to minimizing the total transmit power needed for all users under individual BER and data rate constraints of users.

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The MA optimization problem reads as

$$\min \sum_{k=1}^{K} \sum_{n=1}^{N} c_k[n] P_k[n] \quad (2)$$

s. t.

$$C1: \quad \sum_{n=1}^{N} c_k[n] r_k[n] \geq R_k, \quad \forall k \in \{1, \dots, K\}$$

$$C2: \quad 0 \leq r_k[n] \leq M, \ \forall k \in \{1, \dots, K\}, \ \forall n \in \{1, \dots, N\}$$

$$C3: \quad c[n] = \sum_{k=1}^{K} c_k[n] \leq 1, \quad \forall n \in \{1, \dots, N\},$$

where $c_k[n] \in \{0, 1\}$ refers to the assigned index for user k, $c_k[n] = 1$ if the nth subcarrier is assigned to user k, otherwise $c_k[n] = 0$. The minimal data rate required by user k is denoted by R_k in constraint $C1$. In constraint $C2$ the number of bits loaded on each subcarrier per OFDM symbol cannot be negative and larger than M, which refers to the maximal number of bits per modulation symbol. Constraint $C3$ illustrates that users are not allowed to share any subcarrier at a specific time. This leads to the non-convexity of (2), cf. [9].

Further, we define

$$P_k = \sum_{n=1}^{N} c_k[n] P_k[n]$$

to be the transmit power for user k. A subcarrier assignment for user k is denoted by vector $\mathbf{c}_k = (\, c_k[1], \dots, c_k[N]\,)$, where $c_k = \sum_{n=1}^{N} c_k[n]$ subcarriers are employed for user k.

III. Single-User Water-Filling

In this section, an efficient implementation for water-filling to solve the single-user MA problem is explained, where the user index k is suppressed for simplicity.

With perfect CSI, the optimal rate and power allocation is obtained by water-filling [3]

$$\begin{aligned} r[n] &= \log_2 (\lambda G[n]) \\ P[n] &= \lambda - \frac{1}{G[n]}, \end{aligned} \quad (3)$$

where λ is the water level determined by the rate constraint. Without constraint $C3$, convexity of the single-user MA problem ensures that the optimal solution is achieved at equality in $C1$, then

$$\lambda = 2^{\frac{R}{d}} \left(\prod_{n \in \mathcal{D}} \frac{1}{G[n]} \right)^{\frac{1}{d}}, \quad (4)$$

where set $\mathcal{D}$ contains d used subcarriers.

Algorithm 1 returns optimal solutions. Since high data rates are often demanded in practice, most of subcarriers are used. Therefore, $\mathcal{D}$ is initialized as containing all available subcarriers. Set $\mathcal{A}$ contains the subcarriers achieving the maximal allowed rate M. In the loop, we can only move the subcarriers from $\mathcal{D}$ to $\mathcal{A}$, provided there arise no negative rates on other subcarriers. Then $\mathcal{D}$ must be set to contain all subcarriers except the ones in $\mathcal{A}$ to investigate the possibility of using the before removed subcarriers.

Algorithm 1 Single-User Water-Filling (SUWF)

initialization
  $\mathcal{D} \leftarrow \{1, \dots, N\}$
  $\mathcal{A} \leftarrow \emptyset$
  $P^M[n] = \frac{1}{G[n]}(2^M - 1), n \in \mathcal{D}$
repeat
  $\lambda \leftarrow$ (4)
  $P[n] \leftarrow$(3), $n \in \mathcal{D}$
  $\mathcal{S} \leftarrow \{n \in \mathcal{D} \mid P[n] \leq 0\}$
  $\mathcal{L} \leftarrow \{n \in \mathcal{D} \mid P[n] > P^M[n]\}$
  if $\mathcal{S} \neq \emptyset$ **then**
    $\mathcal{D} \leftarrow \mathcal{D} \setminus \mathcal{S}$
  else if $\mathcal{L} \neq \emptyset$ **then**
    $\mathcal{A} \leftarrow \mathcal{A} \cup \mathcal{L}$
    $\mathcal{D} \leftarrow \{1, \dots, N\} / \mathcal{A}$
    $R \leftarrow R - M \times |\mathcal{A}|$
  end if
until $\mathcal{S} = \emptyset$ and $\mathcal{L} = \emptyset$
$\mathcal{D} \leftarrow \mathcal{A} \cup \mathcal{D}$
output
  $\mathcal{D}, P \leftarrow \sum_{n \in \mathcal{D}} P[n]$

Compared to the conventional implementation [10], SUWF further consider the constraint $C2$ on the maximal rate over each subcarrier. To simply analyze its complexity, it is assumed that $d/2$ subcarriers on average are removed from $\mathcal{D}$ in each iteration. In such a case, N multiply operations for $P^M[n]$ and $2N$ multiply operations plus $\log_2(N)$ exponential operations for λ must be executed. Besides these, only simple operations, like compare and minus, are needed. Hence, the complexity of SUWF is $\mathcal{O}(N)$.

IV. Multiuser Power and Rate Allocation

By iteratively using SUWF, many algorithms have been developed for multiuser resource allocation, e.g., [4], [7], [8]. In this section a group of efficient methods, iteratively utilizing SUWF, is addressed.

A. Efficient Utilization of SUWF

An insight into SUWF can provide us an efficient way to utilize it. Varying a subcarrier assignment can be divided into two groups: one is to add a subcarrier to an assignment, the other is to remove a subcarrier from an assignment.

1) Adding a subcarrier to a subcarrier assignment: The optimal power allocation (3) can be employed to investigate the possibility of reducing transmit power by adding a subcarrier to a subcarrier assignment. Before changing $c_k[n]$ from zero to one, the reciprocal of its CNR is compared to the current water level first. If $\lambda_k > \frac{1}{G_k[n]}$, it is possible to reduce the transmit power, otherwise it is impossible. When $\lambda_k > \frac{1}{G_k[n]}$, the water level decreases while negative rates may probably appear and the maximal allowed rates over used subcarriers may still be exceeded. Hence, the full process of SUWF must be executed, for example, as shown in Fig. 1.

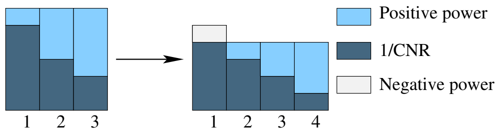

Fig. 1. An example of adding a subcarrier to a subcarrier assignment.

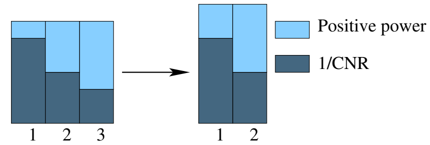

Fig. 2. An example of removing a subcarrier from a subcarrier assignment.

2) Removing a subcarrier from a subcarrier assignment: However, removing a subcarrier from a subcarrier assignment can only result in increase of water level and transmit power. It follows that comparison between a water level and the reciprocal of a CNR can be avoided and positive rates can be ensured over all assigned subcarriers, explained by Fig. 2. Therefore, Algorithm 1 can be simplified for such a case.

B. Heuristic Approaches for Multiuser Resource Allocation

When a large number of users are accommodated, a resource allocation scheme often becomes outdated after a short period of time, since resource allocation for all users needs to be updated even when the channel of only one user varies. In such a case complexity of resource allocation becomes crucial.

1) Initialization: We use the similar idea given in [8] to initialize the subcarrier assignment for each user independently, who utilizes SUWF greedily to choose useful subcarriers without considering others, as shown in Algorithm 2. The minimal number of subcarriers required by user k is N_k^{min}.

2) Conflict resolution: After the initialization, if $c[n] \leq 1$ for every subcarrier, $(P_1, \ldots, P_k)$ is the optimal power allocation. However, it happens often that $c[n] > 1$ on arbitrary subcarriers, which we refer to as the set $\mathcal{F}$ of *conflicting subcarriers*. Such conflicts are consecutively resolved by Algorithm 3 and Algorithm 4. Removing a conflicting subcarrier from a subcarrier assignment results in individual transmit power increase. A conflicting subcarrier is only assigned to the user with the largest individual power increment. However, conflicts may not be resolved by the procedure just explained, when $v \geq 1$ users intend to use a subcarrier with only N_k^{min} subcarriers assigned. We call them *tough users*. A conflicting subcarrier has to be assigned to the tough user if $v = 1$, included in set $\mathcal{Q}$ if $v = K$ or included in set $\mathcal{T}$ otherwise.

To solve one conflict in $\mathcal{T}$, we first select the non-tough user, who has the smallest power-to-rate ratio. Then we find the subcarrier used by this non-tough user, which can substitute this conflicting subcarrier at expense of the smallest increment of total transmit power in the first loop of Algorithm 4.

For a conflicting subcarrier n in $\mathcal{Q}$, there must exist $|\mathcal{I}| \geq c[n]-1$ remaining subcarriers, which are not used by any user. We finds the $c[n]-1$ remaining subcarriers to replace each conflicting subcarrier in $\mathcal{Q}$ for $c[n]-1$ tough users separately with the smallest increment of total transmit power. Obviously, the second loop in Algorithm 4 is rarely activated for large K.

3) Sorting conflicting subcarriers: We call the above group of algorithms RACS, because it is designed to **re-a**ssign **c**onflicting **s**ubcarriers. Apparently, conflicting subcarriers are re-assigned following a random order with respect to their CNRs in Algorithm 3. Sorting conflicting subcarriers in $\mathcal{F}$ after the initialization can result in better performance. It can be performed according to the following criterion.

The CNR variability of the nth subcarrier over users is

$$\text{VAR}[n] = \sum_{k=1}^{K} c_k[n] |g[n] - G_k[n]| \tag{5}$$

with the average CNR over users

$$g[n] = \frac{1}{c[n]} \sum_{k=1}^{K} c_k[n] G_k[n]. \tag{6}$$

Algorithm 2 Initialization

$\mathcal{K} \leftarrow \{1, \ldots, K\}$
$\mathbf{c}_k \leftarrow \mathbf{1},\ \forall k \in \mathcal{K}$
$(\mathbf{c}_k, P_k) \leftarrow$ SUWF $, \forall k \in \mathcal{K}$
$c[n] \leftarrow \sum_{k=1}^{K} c_k[n], \forall n \in \{1, \ldots, N\}$
$N_k^{min} \leftarrow \lceil \frac{R_k}{M} \rceil, \forall k \in \mathcal{K}$
$\mathcal{F} \leftarrow \{n \in \{1, \ldots, N\} \mid c[n] > 1\}$
$\mathcal{T} \leftarrow \emptyset$
$\mathcal{Q} \leftarrow \emptyset$

Algorithm 3 Conflict Resolution

for each $\check{n} \in \mathcal{F}$ **do**
  $\mathcal{U} \leftarrow \{k \in \{1, \ldots, K\} \mid c_k[\check{n}] = 1\}$
  $c_k \leftarrow \sum_{n=1}^{N} c_k[n],\ \forall k \in \mathcal{U}$
  $\mathcal{V} \leftarrow \left\{k \in \mathcal{U} \mid c_k = N_k^{min}\right\}$
  $v \leftarrow |\mathcal{V}|$
  if $v = 0$ **then**
    $c_k[\check{n}] \leftarrow 0, \forall k \in \mathcal{U}$
    $\check{P}_k \leftarrow$ SUWF$, \forall k \in \mathcal{U}$
    $k_m \leftarrow \text{argmax}_{k \in \mathcal{U}} \check{P}_k - P_k$
    $c_{k_m}[\check{n}] \leftarrow 1$
    $P_k \leftarrow \check{P}_k, \forall k \in \mathcal{U} \setminus \{k_m\}$
  else if $v = K$ **then**
    $\mathcal{Q} \leftarrow \mathcal{Q} \cup \{\check{n}\}$
  else
    $c_k[\check{n}] \leftarrow 0, \forall k \in \mathcal{U} \setminus \mathcal{V}$
    $P_k \leftarrow$ SUWF$,\ \forall k \in \mathcal{U} \setminus \mathcal{V}$
    if $v \neq 1$ **then**
      $\mathcal{T} \leftarrow \mathcal{T} \cup \{\check{n}\}$
    end if
  end if
end for

Algorithm 4 Tough User Resolution

for each $\tilde{n} \in \mathcal{T}$ **do**
  $\mathcal{U} \leftarrow \{k \in \mathcal{K} \mid c_k[\tilde{n}] = 1\}$
  for each $k \in \mathcal{U}$ **do**
    $\mathcal{Z} \leftarrow \{k \in \mathcal{K} \setminus \mathcal{U} \mid c_k = N_k^{min}\}$
    if $|\mathcal{U}| + |\mathcal{Z}| < K$ **then**
      $\mathcal{B} \leftarrow \{\tilde{k} \in \mathcal{K} \setminus (\mathcal{U} \cup \mathcal{Y}) \mid \exists\ n\ c_{\tilde{k}}[n] = 1 \wedge c_k[n] = 0\}$
      $k' \leftarrow \mathrm{argmin}_{k \in \mathcal{B}} P_k / R_k$
      $\mathcal{S} \leftarrow \{n \in \{1, \dots, N\} \mid c_{k'}[n] = 1 \wedge c_k[n] = 0\}$
      for each $n \in \mathcal{S}$ **do**
        $(c_k[n], c_{k'}[n], c_k[\tilde{n}]) \leftarrow (1, 0, 0)$
        $\tilde{P}_{k',n} \leftarrow$ SUWF
        $\tilde{P}_{k,n} \leftarrow$ SUWF
        $(c_k[n], c_{k'}[n]) \leftarrow (0, 1)$
        $\Delta P_{k,n} = \tilde{P}_{k,n} + \tilde{P}_{k',n} - P_{k,n} - P_{k',n}$
      end for
      $n_m \leftarrow \mathrm{argmin}_{n \in \mathcal{S}} \Delta P_{k,n}$
      $\Delta P_k \leftarrow \tilde{P}_{k,n_m} - P_{k,n_m}$
      $(c_k[n_m], c_{k'}[n_m]) \leftarrow (1, 0)$
      $(P_{k'}, P_k) \leftarrow (P_{k',n_m}, P_{k,n_m})$
    else
      $\mathcal{Q} \leftarrow \mathcal{Q} \cup \{\tilde{n}\}$
      break
    end if
  end for
  $k_m \leftarrow \mathrm{argmax}_{k \in \mathcal{U}} \Delta P_k$
  $c_{k_m}[\tilde{n}] \leftarrow 1$
  $P_{k_m} \leftarrow$ SUWF
end for
for each $\hat{n} \in \mathcal{Q}$ **do**
  $\mathcal{U} \leftarrow \{k \in \mathcal{K} \mid c_k[\tilde{n}] = 1\}$
  repeat
    $\mathcal{U} \leftarrow \mathcal{U} \setminus \{k\}$
    $\mathcal{I} \leftarrow \{n \in \{1, \dots, N\} \mid c[n] = 0\}$
    for $n \in \mathcal{I}$ **do**
      $(c_k[n], c_k[\hat{n}]) \leftarrow (1, 0)$
      $\hat{P}_{k,n} \leftarrow$ SUWF
      $c_k[n] \leftarrow 0$
    end for
    $n_m \leftarrow \mathrm{argmin}_{n \in \mathcal{I}} \hat{P}_{k,n} - P_{k,n}$
    $(P_k, c_k[n_m]) \leftarrow (\hat{P}_{k,n_m}, 1)$
  until $|\mathcal{U}| = 1$
end for

Consider the extreme case that only one user can use the nth subcarrier and all others have too low CNRs to use it. Improperly assigning this subcarrier may hardly happen while VAR$[n]$ is large. On the contrary, when VAR$[n]$ is small, users have similar CNRs on the nth subcarrier, which may be assigned to a wrong user with higher probability. Hence, the conflicting subcarriers with larger variabilities are supposed to be re-assigned earlier. The revised RACS, where conflicting subcarriers in $\mathcal{F}$ are re-assigned following a descending order of their variabilities by Algorithm 3 instead of the power variabilities in [8], is called **o**rdered RACS (ORACS).

The variability in (5) may be improved by balancing users' different attenuations and noise powers. Alternatively, instead of CNR used in (5) and (6), a normalized CNR over subcarriers of each user is employed and written as

$$\overline{G_k[n]} = G_k[n] / \Sigma_{n=1}^{N} G_k[n]. \tag{7}$$

RACS is further revised as **n**ormalized ORACS (NORACS).

C. Complexity Analysis

To briefly analyze the complexity, we consider the worst case that all N subcarriers are conflicting, which implies that there may exist almost no frequency selective fading. In the initialization SUWF must be executed K times for all users, so the complexity of this step is $\Omega(KN)$ due to the linear complexity of SUWF. SUWF must be called at most KN times to process all conflicts in Algorithm 3. If it is assumed that $\hat{N}$ subcarriers are used by each user on average after the initialization, then the complexity of this step must be lower than $\Omega(KN\hat{N})$. Conflict resolution in the first loop in Algorithm 4 occurs not often, its complexity is $\Omega(K\hat{N}^2)$. The complexity of the second loop in Algorithm 4 is $\Omega(N^2\hat{N})$ while it is called very rarely. From the abovementioned analysis the complexity of the suggested methods is bound by $\Omega(KN + KN\hat{N} + K\hat{N}^2 + N^2\hat{N})$.

V. Numerical Results

In this section, numerical results are obtained to compare the performance and complexity of RACS, ORACS and NORACS with the successive user integration algorithm (SUSI) newly suggested in [3]. SUSI has better performance than most of other heuristic methods, e.g., [4], [5], [6], [7].

The frequency selective channels of different users are independent with each other and each of them is modeled as consisting of 16 independently Rayleigh distributed multipaths with an exponentially decaying profile. The maximal expected CNR on each subcarrier is set to be 5 dB, which fades with the distance from the transmitter to the receiver. We consider a multiuser OFDM system with 64 subcarriers and 2 to 12 users for simulations, which can serve three types of users, as shown in Table I. The Rate of a data user is exponentially distributed with a maximal rate of 32 bits per OFDM symbol. The maximal sum rate of the system, 384 bits per OFDM symbol, can be possibly achieved when 12 users are served.

Since the proposed methods only provide suboptimal solutions to the MA problem, performance degradation cannot be avoided. To evaluate such degradation and quantify the complexity reduction, SUSI is used as benchmark. Fig. 3

TABLE I
Users in the simulation system

User type	Proportion	Rate(bits/OFDM symbol)	SNR gap (dB)
Video user	10%	32	7.5
Audio user	40%	8	8.8
Data user	50%	8 (mean)	9.5

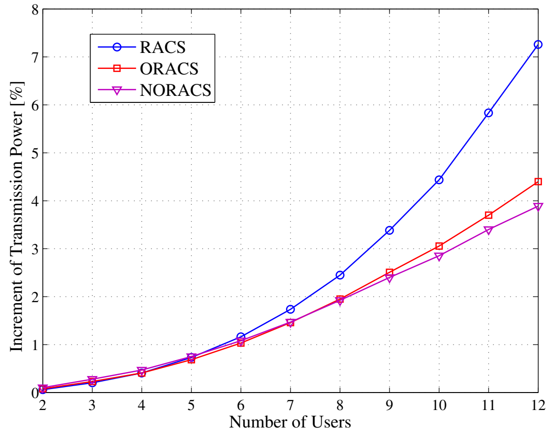

Fig. 3. Increment of total transmit power by using RACS, ORACS and NORACS compared to SUSI in percent

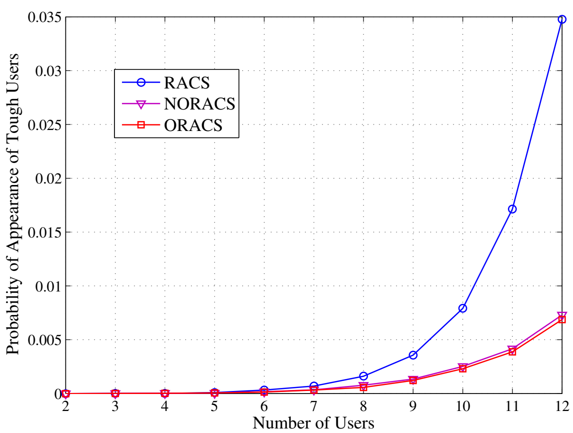

Fig. 4. Probability of appearance of tough users

shows the increment of total transmit power by using the proposed methods compared to SUSI. By simply sorting conflicting subcarriers, the performance of RACS is improved by around 2% with ORACS and by around 3% with NORACS, when the number of users in the system is large.

Adding a subcarrier to an assignment happens mostly in the second loop of Algorithm 3, when tough users appear. Fig. 4 gives the probability of the appearance of tough users in our algorithms, who almost does not appear when not more than six users are in the simulation system. The simple sorting can reduce the probability significantly while a large number of users are in the simulation system, which means that the complexity may be further reduced.

Fig. 5 shows that the three proposed algorithms almost have the same complexity. Compared to SUSI, they reduce up to 98% times of adding a subcarrier to an assignment and up to 92% times of removing a subcarrier from an assignment while many users are served in the simulation system. The reduction of times of adding a subcarrier to an assignment dominates due

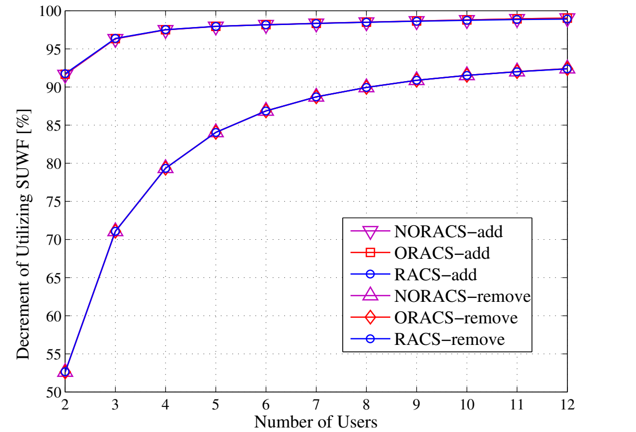

Fig. 5. Decrement of times of adding and removing subcarriers while using RACS, ORACS and NORACS compared to SUSI in percent

to the property of utilizing SUWF in Section IV.

VI. Conclusion

Efficient algorithms for multiuser resource allocation allow mobile networks to promptly adapt to fast-varying environments. In this paper, a group of low-complexity methods has been proposed to provide suboptimal power and rate allocation for the downlink of multiuser OFDM systems. Solving the problem by three steps enables the algorithms to utilize SUWF efficiently, where the simple sorting of conflicting subcarriers has been applied and can effectively improve the performance of resource allocation and reduce the computational complexity. Simulations have shown their low complexity and comparable performance compared to the reference algorithm.

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