# Maximizing Energy Efficiency for Multiple DF Relay System with QoS Constraint 

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#### Abstract

Multiple relays assisted transmission is an effective way to enhance the reliability of wireless communication network. However multiple relays joining in the transmission costs more energy and may reduce the energy efficiency of the system. In this paper, we investigate the energy efficiency performance of multiple decode-and-forward relays system under a quality of service constraint. In particular, we study a feedback-limited scenario where only the knowledge of average channel gain is available at the source. First, we define a tradeoff factor and based on it design a tradeoff mechanism between energy efficiency and outage probability. Second, we prove that the energy efficiency of the system is a piecewise strictly monotonic function of our tradeoff factor and has only one extreme value which is the global maximum. Third, by means of simulation we show that the numerical results perfectly match our theoretical analysis. In addition, we show that energy efficiency can achieve the extreme value under the loose QoS constraint.


Index Terms-energy efficiency, QoS constraint, decode-andforward, multiple relays, average CSI,

## I. Introduction

For future wireless communication networks one important design characteristic is reliability. Reliability is already important for data services such as video conferencing or voice-over-IP. However, with the advent of machine-to-machine type of applications, reliability together with latency will become even more important. Nevertheless, systems are also assumed to become more energy efficient as demonstrated by several projects on 'greening' radio communication sytems [1]. Therefore, reliability with quality-of-service ( QoS ) constraint and energy efficiency are the two points of major concern for future wireless communication networks.

In wireless fading channel environments, relaying is well known as an effective way to exploit the spatial diversity gain in order to enhance the transmission reliability [2][4]. Specifically, when multiple relays are available to assist data transmission, the performance of the transmission can be significantly improved [5], [6]. Unfortunately, more relay nodes joining in transmission also brings serious overhead on energy consumption to the system. And this directly affects the performance of energy efficiency. In order to save energy, [7]-[9] apply relay selection that only selects one or some best relays to help transmission. To apply relay selection the source needs to know the exact instantaneous channel state information (CSI), e.g., source-relay and relay-destination channel state information, before it selects and determines relays. However, this is a very overhead-prone system design.

In general, we cannot expect instantaneous feedback from the destination and all the relays to be available for the source.

In this paper, we assume perfect CSI of relay-destination channels are available at the destination while the source only has the knowledge of the average channel gains. We consider a decode-and-forward relaying (DF) protocol, hence without instantaneous CSI the source cannot select relays and does not know how many and which relays will decode the transmitted data correctly. We address the question how the system can still save energy. In detail, our contributions are three-fold:

1) Propose a simple 'protocol': to reduce the potential waste of energy by introducing a tradeoff factor, which is actually a tradeoff between outage probability and energy efficiency. The tradeoff parameter can be interpreted as an additional probability with which successfully decoding relays nevertheless discard the packet for the upcoming relaying phase, i.e., not participating in the relaying phase afterwards.
2) Analyze the tradeoff problem: we show that the energy efficiency is a piecewise strictly monotonic function of our tradeoff factor and has only one extreme value which is the global maximum. Based on this insight, we provide a scheme to maximize the energy efficiency under the QoS constraint.
3) Validation by simulation: we show that numerical results perfectly match our theoretical analysis. In addition, we show that energy efficiencies can achieve their extreme values conditionally on the different extent of the QoS requirement.
The rest of the paper is organized as follows. Section II introduces the multi-relay system model including scenario description, physical-layer assumption and energy efficiency model. Section III desribes the problem we are intrested in and presents our tradeoff problem in the form of the formula. In Section IV, we analyze how the tradeoff affects the energy efficiency and how to solve the tradeoff problem from a mathematical point of view. Section V validates our theoretical analysis and shows how our tradeoff mechanism works by means of simulation. Finally, we conclude the paper in Section VI.

## II. System Model

## A. Scenario Description

In this section, we first briefly describe the overview of our relaying scenario, and then introduce the physical-layer assumption of the system together with our previous theoretical work on outage probability. At last, the energy consumption and efficiency models are presented.

We consider a simple relaying scenario with a source S , a destination D and a group of DF relays as schematically shown in Figure 1. The relay group has $J$ relays, all of them are randomly deployed in a certain area located between source and destination, and we assume the radius of the area is significantly smaller than the distance either from the source to the relay group or from the relay group to the destination. In this scenario the source has to transmit a packet of size $\rho$ to the destination with the help of relays. The entire system operates in a slotted fashion where time is divided into frames of length $T_{\mathrm{f}}$. The system spends two frames for each single transmission from source to destination, which are referred to as broadcasting frame and relaying frame. During a broadcasting frame, the source transmits the data packet to all relays. And afterwards, relays forward the packet to the destination during the relaying frame. The DF relays work in an opportunistic way, which means that only the relays who have decoded the packet successfully in the former broadcasting frame potentially forward the packet simultaneously to the destination during the relaying frame.


Fig. 1. Example of the considered multiple relay system scenario.

## B. Channel and Physical-layer Model

We assume that the channel states vary randomly due to Rayleigh block-fading. No instantaneous CSI but only the information of the average channel gain are available at the transmitters (both source and relays). All the transmitters apply fixed transmit powers denoted as $P_{\mathrm{S}}$ at source and $P_{\mathrm{R}}$ at each relay. The noise power is denoted as $\sigma^{2}$, and apart from it we do not assume the presence of any external source of interference. The successful reception of the packet at either the destination or the relays depends on the instantaneous channel quality between the respective transmitter and receiver. Denote the instantaneous channel gain during the broadcasting frame from the source to relay $j$ by $h_{\mathrm{S}, j}^{2}$. Then, the channel quality is given by the signal-to-noise-ratio (SNR) defined as

$$
\begin{equation*}
\gamma_{\mathrm{S}, j}=P_{\mathrm{S}} \cdot h_{\mathrm{S}, j}^{2} / \sigma^{2} \tag{1}
\end{equation*}
$$

Correspondingly, the channel gain and SNR from relay $j$ to the destination are given by $h_{j, \mathrm{D}}^{2}$ and $\gamma_{j, \mathrm{D}}=P_{\mathrm{S}} \cdot h_{j, \mathrm{D}}^{2} / \sigma^{2}$. Given an instantaneous SNR $\gamma$ during one frame (with $N$ symbols) at $\operatorname{most} N \cdot \log _{2}(1+\gamma)$ bits can be conveyed correctly. Hence, a currently transmitted packet of size $\rho$ is successfully received if the SNR of the link is above the threshold $\gamma^{*}=2^{\rho / N}-1$.
We denote the average channel gains by $\bar{h}_{\mathrm{S}, j}^{2}$ and $\bar{h}_{j, \mathrm{D}}^{2}$ which are actually the expected values of $h_{\mathrm{S}, j}^{2}$ and $h_{j, \mathrm{D}}^{2}$. Further, denote the average combined SNR of the channel from all the
relays to the destination by $\bar{\gamma}$ :

$$
\begin{equation*}
\bar{\gamma}=\sum_{j=1}^{J} P_{\mathrm{R}} \bar{h}_{j, \mathrm{D}}^{2} / \sigma^{2} \tag{2}
\end{equation*}
$$

Due to the fading, a varying amount of relays could decode the packet successfully during the broadcasting phase. We call this set the surviving relay set and denote it by $\Theta$. In the successive relaying frame, all relays in set $\Theta$ are able to forward the packet to the destination. Then, the SNR at the destination is simply the sum of the SNR of the individual links. Therefore, we have $\gamma_{\mathrm{D}}=\sum_{j \in \Theta} \gamma_{j, \mathrm{D}}$. Hence, the packet reception at the destination during the relaying phase depends on this joint SNR of all forwarding links.
In our previous work [10] we have shown that the expectation of the outage probability of a single two-frame transmission can be obtained by

$$
\begin{equation*}
\operatorname{Pr}^{\text {out }}=\sum_{n=0}^{J} \operatorname{Pr}_{2}(n) \operatorname{Pr}_{\mathrm{B}}\left(n ; J, \operatorname{Pr}_{1}\right) \tag{3}
\end{equation*}
$$

where $\operatorname{Pr}_{2}(n)$ is the outage probability of the relaying frame while $n$ relays being active:

$$
\operatorname{Pr}_{2}(n)=\left\{\begin{array}{l}
1-\sum_{j=0}^{n-1} \frac{1}{j!}\left(\frac{\gamma^{*}}{\beta}\right)^{j} e^{-\frac{\gamma^{*}}{\beta}} ; n>0  \tag{4}\\
1 ; \quad n=0
\end{array}\right.
$$

$\operatorname{Pr}_{2}(n)$ is actually a cumulative distribution function of the gamma distribution with the scale parameter $\beta=2 \bar{\gamma} / J$. The number of forwarding relays $n$ is a binomial distributed random variable. In Equation (3), the $\operatorname{Pr}_{\mathrm{B}}\left(n ; J, \operatorname{Pr}_{1}\right)$ is the probability density function of $n$ :

$$
\begin{equation*}
\operatorname{Pr}_{\mathrm{B}}\left(n ; J, \operatorname{Pr}_{1}\right)=\binom{J}{n}\left(1-\operatorname{Pr}_{1}\right)^{n}\left(\operatorname{Pr}_{1}\right)^{J-n} \tag{5}
\end{equation*}
$$

where $\operatorname{Pr}_{1}$ is the outage probability of the source-relay $(j)$ link:

$$
\begin{equation*}
\operatorname{Pr}_{1}=1-\exp \left(-\gamma^{*} \sigma_{0}^{2} / 2 \bar{h}_{\mathrm{S}, j}^{2} P_{\mathrm{S}}\right) \tag{6}
\end{equation*}
$$

Both the gamma distribution and the binomial distribution are approximations based on the topology simplification, where we assume that the distance among the relays is fairly small compared to the distance to the source or to the destination. We have shown in [10] that both two distributions are indeed good approximations even if the relays are a bit more distributed.

## C. Energy Consumption Model

Regarding the consumption of power, this paper adopts a linear power consumption model which was introduced in [11]. Besides, we also consider the idling power [12] when the transmitter is inactive.
In general, we divide the energy consumption of the system over a broadcasting and a relaying frame into two parts:

- Variable power consumption: which is consumed by RF amplifiers at relays when the relays have packets to send. The per packet (two frames) variable power consumption can be obtained by: $n \cdot P_{\mathrm{R}} \cdot T_{\mathrm{f}}$. It is a random variable, since the number of forwarding relays $n$ in one single
packet transmission (two frames) is modeled as a random variable due to the channel fading.
- Constant power consumption: we consider two types of constant power consumption. One is the fundamental power consumption which is independent of transmission power and includes signal processing, battery backup, cite cooling and so on. We denote by $P_{\mathrm{S}}^{\circ}, P_{\mathrm{D}}^{\circ}$ and $P_{\mathrm{R}}^{\circ}$ the per second fundamental power consumption at the source, the destination and at every relay node. For the other one, we also treat the transmission power consumption of the source (with power $P_{\mathrm{S}}$ ) as the constant power consumption, because the source is certain to transmit a packet once every two frames with a fixed transmission power.
Hence, $E_{\text {packet }}(n)$, the total consumed energy for transmitting a packet in two frames, can be given as:

$$
\begin{equation*}
E_{\text {packet }}(n)=\left(P_{\mathrm{S}}^{\circ}+P_{\mathrm{D}}^{\circ}+J \cdot P_{\mathrm{R}}^{\circ}\right) \cdot 2 T_{\mathrm{f}}+\left(P_{\mathrm{S}}+P_{\mathrm{R}} \cdot n\right) \cdot T_{\mathrm{f}} \tag{7}
\end{equation*}
$$

Let $P^{\circ}=P_{\mathrm{S}}+2 P_{\mathrm{S}}^{\circ}+2 P_{\mathrm{D}}^{\circ}+2 J \cdot P_{\mathrm{R}}^{\circ}$, hence $P^{\circ}$ is the constant power consumption per two adjacent frames. Therefore $E_{\text {packet }}(n)$ can be further expressed as:

$$
\begin{equation*}
E_{\text {packet }}(n)=\left(P^{\circ}+P_{\mathrm{R}} \cdot n\right) \cdot T_{\mathrm{f}} . \tag{8}
\end{equation*}
$$

Next, we define the energy efficiency of the system as the ratio per one single transmission between the average correct received data and the average consumed energy:

$$
\begin{equation*}
\Phi=\bar{R} / \bar{E} . \tag{9}
\end{equation*}
$$

As the packet size was denoted as $\rho$, the average correct received data can be derived based on the packet size and outage probability as:

$$
\begin{equation*}
\bar{R}=\left(1-\operatorname{Pr}^{\text {out }}\right) \cdot \rho . \tag{10}
\end{equation*}
$$

Based on the energy consumption model, the average energy consumption of one packet transmission (spanning two frames) $\bar{E}$ can be obtained by:

$$
\begin{equation*}
\bar{E}=\mathscr{E}\left[E_{\text {packet }}(n)\right]=\left(P^{\circ}+\mathscr{E}[n] \cdot P_{\mathrm{R}}\right) \cdot T_{\mathrm{f}} \tag{11}
\end{equation*}
$$

where $\mathscr{E}[\cdot]$ is the expectation of a random variable. Hence the energy efficiency can be further expressed as:

$$
\begin{equation*}
\Phi=\frac{\bar{R}}{\bar{E}}=\frac{\left(1-\operatorname{Pr}^{\text {out }}\right) \cdot \rho}{\left(P^{\circ}+\mathscr{E}[n] \cdot P_{\mathrm{R}}\right) \cdot T_{\mathrm{f}}} \tag{12}
\end{equation*}
$$

## III. Tradeoff Problem Statement

Based on the system model above, this paper mainly focuses on the energy efficiency of the system with certain QoS requirements for the transmission. We treat the outage probability as the quality-of-service indicator. Therefore, the motivation of our work is to maximize the energy efficiency of the system under the constraint of achieving a certain outage probability threshold. Recall that for this task there is no instantaneous channel state information available at the source or relays which would allow to do a precise power control. On the other hand, considering the fixed outage probabiliy requirement, it is possible that the number of forwarding
relays is higher than what the system really needs to meet the outage requirement. Hence the system may waste energy by the excessive use of forwarding relays.
In order to reduce this waste of energy, we propose a simple 'protocol' solution by introducing a factor $\omega$. The factor $\omega$ might reduce the number of forwarding relays by moderating the relationship between the following two probabilities of the relaying: Probability $\operatorname{Pr}_{1}$ of fail decoding in the broadcasting frame and the probability $\operatorname{Pr}_{\mathrm{X}}$ of keeping silence in relaying phase. Unlike the previous works [7], [10], [13] which treat the two probabilities as the same, we propose to increase the probability of a relay keeping silent during the relaying frame by the factor $\omega$.

$$
\begin{equation*}
\operatorname{Pr}_{\mathrm{X}}(\omega)=\omega \cdot \operatorname{Pr}_{1} ; \quad \omega \in\left[1,1 / \operatorname{Pr}_{1}\right) \tag{13}
\end{equation*}
$$

Based on [10], the number of forwarding relays $n$ can be modeled as the binomial distributed variable as $n \sim$ $B\left(J, \operatorname{Pr}_{\mathrm{x}}\right)$. Thus, the expected value of $n$ in equation (11) and equation (12) can be obtained by:

$$
\begin{equation*}
\mathscr{E}[n]=\left[1-\operatorname{Pr}_{\mathrm{X}}(\omega)\right] \cdot J . \tag{15}
\end{equation*}
$$

And as a result, the outage probability of the transmission changes from equation (3) to:

$$
\begin{equation*}
\operatorname{Pr}^{\text {out }}(\omega)=\sum_{n=0}^{J} \operatorname{Pr}_{2}(n) \operatorname{Pr}_{\mathrm{B}}\left(n ; J, \operatorname{Pr}_{\mathrm{X}}(\omega)\right) \tag{16}
\end{equation*}
$$

Denote the QoS requirement of outage probability as $\mathrm{Pr}_{\mathrm{QoS}}$. Hence, our tradeoff problem statement can be given as:

$$
\begin{align*}
& \max \Phi(\omega) \\
& \text { s.t. } \operatorname{Pr}^{\text {out }}(\omega) \leq \operatorname{Pr}_{\mathrm{QoS}} ; \quad \omega \in\left[1,1 / \operatorname{Pr}_{1}\right) \tag{17}
\end{align*}
$$

## IV. Tradeoff Analysis

In this section we analyze how the tradeoff factor $\omega$ affects the system's energy efficiency and how to solve the tradeoff problem.

Based on the definition of $\omega$, it is obvious that the bigger $\omega$ is, the lower the average forwarding number of relays is and therefore, the lower the received SNR at the destination is. Hence, the outage probability of the system is a strictly increasing function of $\omega$. Thus, in our tradeoff the minimum value of outage probability $\operatorname{Pr}_{\text {min }}$ can be achieved when $\omega=1$. In other words, $\operatorname{Pr}_{\min }=\operatorname{Pr}^{\text {out }}(\omega=1)$. For a target outage probability requirement $\operatorname{Pr}_{\mathrm{QoS}}$ of the served transmission, there are two different cases that need to be distinguished before doing tradeoff:

- $\operatorname{Pr}_{\mathrm{QoS}}<\operatorname{Pr}_{\text {min }}$, the system can't support the transmission without any tradeoff operation for saving energy.
- $\operatorname{Pr}_{\mathrm{QoS}} \geq \operatorname{Pr}_{\text {min }}$, the system has a probability for tradeoff to promote energy efficiency.
Regarding the first case, we cannot save any energy from a system that already cannot support the transmission with the maximum power consumption. Therefore, we are going to focus on the second case to maximize the energy efficiency under the outage probability requirement $\mathrm{Pr}_{\mathrm{QoS}}$. First, consider the following proposition:

$$
\begin{equation*}
\frac{\partial \operatorname{Pr}^{\text {out }}}{\partial \operatorname{Pr}_{\mathrm{X}}}=\sum_{n=0}^{J}\left\{\operatorname{Pr}_{2}(n)\binom{J}{n} \operatorname{Pr}_{\mathrm{X}}^{J-n}\left(1-\operatorname{Pr}_{\mathrm{X}}\right)^{n}\left[1+\left(\frac{J-n}{\operatorname{Pr}_{\mathrm{X}}}-\frac{n}{1-\operatorname{Pr}_{\mathrm{X}}}\right)\left(\frac{P^{\circ}}{J P_{\mathrm{R}}}+1-\operatorname{Pr}_{\mathrm{X}}\right)\right]\right\} \tag{14}
\end{equation*}
$$

Proposition 1. Let $\mathrm{g}_{1}: \mathbb{R} \rightarrow \mathbb{R}, y \rightarrow \mathrm{~g}_{1}(y)$, with $\mathrm{g}_{1}(y) \geq 0$ $\forall y$. Let $\mathrm{g}_{2}: \mathbb{R}^{2} \rightarrow \mathbb{R},(x, y) \rightarrow \mathrm{g}_{2}(\mathrm{x}, \mathrm{y})$ be monotonically decreasing in $x$. If $\mathrm{f}: \mathbb{R}^{2} \rightarrow \mathbb{R},(x, z) \rightarrow \mathrm{f}(x, z)$ is monotonically decreasing in $z$ and strictly increasing in $x$, then $\mathrm{f}\left(x, \mathrm{~g}_{1}(y) \mathrm{g}_{2}(x, y)\right)$ is also strictly decreasing in $x$.

## Proof:

$\because \mathrm{g}_{2}(x, y)$ is monotonically decreasing in $x$,
$\therefore \forall x_{1}<x_{2}, \mathrm{~g}_{2}\left(x_{1}, y\right) \geq g_{2}\left(x_{2}, y\right)$.
$\because \forall y, \mathrm{~g}_{1}(y) \geq 0$,
$\therefore \mathrm{g}_{1}(y) \mathrm{g}_{2}\left(x_{1}, y\right) \geq \mathrm{g}_{1}(y) g_{2}\left(x_{2}, y\right)$.
$\because \mathrm{f}(x, z)$ is monotonically decreasing in $z$ and strictly increasing in $x$,

$$
\begin{aligned}
& \therefore \mathrm{f}\left(x_{1}, \mathrm{~g}_{1}(y) \mathrm{g}_{2}\left(x_{1}, y\right)\right) \leq \mathrm{f}\left(x_{1}, \mathrm{~g}_{1}(y) g_{2}\left(x_{2}, y\right)\right) \\
& \quad<\mathrm{f}\left(x_{2}, \mathrm{~g}_{1}(y) g_{2}\left(x_{2}, y\right)\right) \\
& \therefore \mathrm{f}\left(x, \mathrm{~g}_{1}(y) \mathrm{g}_{2}(x, y)\right) \text { is strictly increasing in } \mathrm{x} .
\end{aligned}
$$

As we mentioned above, $\operatorname{Pr}^{\text {out }}(\omega)$ is a strictly increasing function of $\omega$. So based on Equation (12) and Equation (15), we observe that both the numerator and denominator of Equation (12) are monotonic decreasing functions of $\omega$. However, they might differ by their slope. With the derivative of $-\operatorname{Pr}_{1} P_{R} J$ the denominator decreases in a linear way while the derivative of the numerator can be obtained as follows. The derivative of the numerator is equal to $-\frac{\partial \mathrm{Pr}^{\text {out }}}{\partial \omega}$. And $\frac{\partial \mathrm{Pr}^{\text {out }}}{\partial \omega}$ can be further derived as

$$
\begin{equation*}
\frac{\partial \operatorname{Pr}^{\text {out }}}{\partial \omega}=\frac{\partial \operatorname{Pr}^{\text {out }}}{\partial \operatorname{Pr}_{\mathrm{X}}} \frac{\partial \operatorname{Pr}_{\mathrm{X}}}{\partial \omega}=\frac{\partial \operatorname{Pr}^{\text {out }}}{\partial \operatorname{Pr}_{\mathrm{X}}} \operatorname{Pr}_{1} \tag{18}
\end{equation*}
$$

where $\frac{\partial \mathrm{Pr}^{\text {out }}}{\partial \mathrm{Pr}_{\mathrm{X}}}$ can be obtained by equation (14).
Based on the comparison on the derivatives between the numerator and the denominator of the energy efficiency $\Phi$, we have the preliminary conclusions:

- if $\frac{\partial \mathrm{Pr}^{\text {out }}}{\partial \omega_{\text {a }}}>\operatorname{Pr}_{1} P_{\mathrm{R}} J$, then $\Phi$ is strictly decreasing.
- if $\frac{\partial \mathrm{Pr}^{\text {out }}}{\partial \omega_{\text {out }}}<\operatorname{Pr}_{1} P_{\mathrm{R}} J$, then $\Phi$ is strictly increasing.
- if $\frac{\partial \mathrm{Pr}^{\text {out }}}{\partial \omega}=\operatorname{Pr}_{1} P_{\mathrm{R}} J$, then $\Phi$ achieves an extreme value by one point of $\omega$ or a stationary value over a continuous interval of $\omega$.
Denote for short that $\ell\left(\operatorname{Pr}_{\mathrm{X}}, n\right)=1+$ $\left(\frac{J-n}{\operatorname{Pr}_{\mathrm{X}}}-\frac{n}{1-\operatorname{Pr} \mathrm{X}}\right)\left(\frac{P^{\circ}}{J P_{\mathrm{R}}}+1-\operatorname{Pr}_{\mathrm{X}}\right)$, then we can simplify equation (14) into:

$$
\begin{equation*}
\frac{\partial \operatorname{Pr}^{\text {out }}}{\partial \operatorname{Pr}_{\mathrm{X}}}=\underset{n \sim B\left(J, \operatorname{Pr}_{\mathrm{X}}\right)}{\mathscr{E}}\left[\operatorname{Pr}_{2}(n) \ell\left(\operatorname{Pr}_{\mathrm{X}}, \mathrm{n}\right)\right] \tag{19}
\end{equation*}
$$

where $\underset{n \sim B\left(J, \operatorname{Prx}_{\mathrm{X}}\right)}{\mathscr{E}}[\mathrm{X}(\mathrm{n})]$ is the expectation of $\mathrm{X}(\mathrm{n})$ while the random variable $n$ is binomially distributed. Similarly, $\operatorname{Pr}^{\text {out }}$ can be expressed as $\underset{n \sim B\left(J, \operatorname{Prx}_{\mathrm{X}}\right)}{\mathscr{E}}\left[\operatorname{Pr}_{2}(n)\right]$.

First, as a probability, $\operatorname{Pr}_{2}(n)$ is non-negative. Second, according to the earlier analysis, $\underset{n \sim B\left(J, \operatorname{Pr}_{\mathrm{X}}\right)}{\mathscr{E}}\left[\operatorname{Pr}_{2}(n)\right]$ is a strictly increasing function of $\operatorname{Pr}_{\mathrm{X}}$. For a given $\operatorname{Pr}_{\mathrm{X}}$, $\underset{n \sim B\left(J, \operatorname{Pr}_{\mathrm{x}}\right)}{\mathscr{E}}\left[\operatorname{Pr}_{2}(n)\right]$ has a fixed value, which can be treated as special monotonically decreasing function of $\operatorname{Pr}_{2}(\mathrm{n})$. Third,
$\frac{\partial \ell}{\partial \mathrm{Pr}_{\mathrm{X}}}=-(J-n)-\left(\frac{P^{\circ}}{J P_{\mathrm{R}}}+1\right) \frac{J-n}{\left(\operatorname{Pr}_{\mathrm{X}}\right)^{2}}-\frac{n P^{\circ}}{J P_{\mathrm{R}}\left(1-\mathrm{Pr}_{\mathrm{X}}\right)^{2}} \leq 0$ since the forwarding relay number $n$ satisfies that: $0 \leq n \leq J$. Hence $\ell\left(\operatorname{Pr}_{\mathrm{X}}, n\right)$ is a monotonically decreasing function of $\operatorname{Pr}_{\mathrm{X}}$. Therefore, based on the proposition above, $\frac{\partial \mathrm{Pr}^{\text {out }}}{\partial \mathrm{Pr}_{\mathrm{X}}}$ is a strictly increasing function of $\operatorname{Pr}_{\mathrm{X}}$. Consider that the relationship between $\operatorname{Pr}_{X}$ and $\omega$ in equation (18), we can further figure out a very important analysis result that $\frac{\partial \mathrm{Pr}^{\text {out }}}{\partial \omega}$ is also strictly increasing in $\omega$. This means that if equation $\frac{\partial \mathrm{Pr}^{\text {out }}}{\partial \omega}=\operatorname{Pr}_{1} P_{\mathrm{R}} J$ has a solution $\omega^{\circ}$, then the solution $\omega^{\circ}$ is unique. Combined with our former analysis, we have:

- if $\omega>\omega^{\circ}$, then $\Phi$ is strictly decreasing.
- if $\omega<\omega^{\circ}$, then $\Phi$ is strictly increasing.
- if $\omega=\omega^{\circ}$, then $\Phi$ achieves the unique extreme value: the global maximum.
Take into account that our tradeoff is subject to the target statistical outage probability, therefore, if $\operatorname{Pr}^{\text {out }}\left(\omega^{\circ}\right) \leq \operatorname{Pr}_{\mathrm{QoS}}$, the extreme point $\omega^{\circ}$ is the solution of the tradeoff. However if $\operatorname{Pr}^{\text {out }}\left(\omega^{\circ}\right)>\operatorname{Pr}_{\text {QoS }}$, the solution of the tradeoff is not the extreme point anymore but the boundary solution $\omega^{\bullet}$ which can be obtained by the QoS edge equation:

$$
\begin{equation*}
\operatorname{Pr}^{\text {out }}\left(\omega^{\bullet}\right)=\operatorname{Pr}_{\mathrm{QoS}} ; \omega^{\bullet} \in\left[1, \omega^{\circ}\right] \tag{20}
\end{equation*}
$$

And $\omega^{\bullet}$ is easy to find since the energy efficiency is a strictly increasing function of $\omega$ during the interval $\left[1, \omega^{\circ}\right]$.

So far, we have analyzed the way to solve the tradeoff problem as shown in flow chart in Figure 2.


Fig. 2. The flow chart of the process of solving the tradeoff problem

## V. NUMERICAL RESULTS AND DISCUSSION

In order to validate the above analysis of the solution of the tradeoff problem, this section shows by simulation how the tradeoff factor $\omega$ affects the energy efficiency and the outage probability. In the simulation, we randomly deploy 9 relays in a cirlular area with radius $R=10 \mathrm{~m}$ while the distance of broadcasting and relaying links are both set to 200 m . We assume that the center frequency is 2 GHz and the frame length is set to $T_{\mathrm{f}}=20 \mathrm{~ms}$. Besides, we set $P_{\mathrm{S}}=P_{\mathrm{R}}=25$ $\mathrm{dBm}, P_{\mathrm{S}}^{\circ}=P_{\mathrm{D}}^{\circ}=P_{\mathrm{R}}^{\circ}=22 \mathrm{dBm}$ and $\sigma^{2}=-100 \mathrm{dBm}$. For calculating the path-loss, we utilize the well-known COST231 model.
Figure 3 presents the curves of energy efficiency (top) and outage probability (bottom) versus the factor $\omega$ for three cases with different packet sizes $\rho$. The top graph shows that every
energy efficiency curve has an extreme value over $\omega$, and that each curve strictly increases at first and then decreases after the extreme value. This perfectly matches our theoretical analysis. In the bottom figure the outage probability curves also prove our analysis that they are strictly increasing in $\omega$. Second, during the simulation we consider 2 different QoS constraints on the outage probability: the red threshold line $\operatorname{Pr}_{\mathrm{QoS}}=10^{-4}$ and the green threshold line $\operatorname{Pr}_{\mathrm{QoS}}=10^{-1}$. The threshold lines intersect three outage probability curves. For each case, in order to satisfy a QoS constraint, the effective interval of $\omega$ should be from 1 to the intersection of the outage probability curve of the case and the threshold line of the constraint. For a big packet size case $\rho=200$ bits with the strict QoS constraint $\operatorname{Pr}_{\mathrm{QoS}}=10^{-4}$, there is no intersection between the probability curve and the threshold line. This means that with such a big packet size the system cannot achieve the QoS constraint at all no matter how the value $\omega$ is set. Apart from this, there are 5 intersections in the bottom part, and we marked the corresponding energy efficiency value of the intersections by additional arrows.


Fig. 3. The energy efficiency and the outage probability on tradeoff factor

Consider the effective interval of $\omega$, we can find that not all the extreme values of energy efficiency can be achieved by varying $\omega$ in the relative effective interval. In case $\rho=100$ bits, energy efficiency achieves its maximum when $\omega=6$ which is out of the effective interval $[1,3.7]$ under the strict QoS constraint (red line and arrows) but in the effective interval $[1,6.3]$ with the loose QoS constraint (green line and arrows). In fact, for all the cases the maximum of energy efficiency can only be achieved when the QoS constraint is not strict, otherwise the tradeoff only has boundary solutions(intersections). This actually matches our analysis in last section and the flow chart of Figure 2.

## VI. Conclusion

Multiple-relay assisted transmission is an effective way to enhance the reliability in wireless systems. The downside
to this approach is the enhanced energy consumption which may even reduce the energy efficiency if too many relays participate in the transmission. In this paper, we have considered a scenario where the nodes (source and relays) do not have instantaneous CSI. We then focused on maximizing the energy efficiency for a two-hop transmission with an outage probability constraint.
The main contribution of our work is a tradeoff mechanism between outage probability and energy efficiency which is achieved by introducing an additional tradeoff factor. The work could be a solution for a sensor network and the tradeoff factor might actually determine the sleeping cycle of the nodes. The tradeoff factor allows relays to discard a packet even if they received it successfully. We can prove analytically that in such a set-up the energy efficiency is a piecewise strictly monotonic function of the tradeoff factor and has only one extreme value which is the global maximum. Besides, we analyzed the tradeoff problem and provided a scheme to efficiently calculate the optimal solution. By means of simulation, we finally showed that the numerical results perfectly match our theoretical analysis.

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