Power Optimization in Sensor Networks for Passive Radar Applications

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Abstract—**In the present work, we investigate the power allocation problem in distributed sensor networks that are used for passive radar applications. The signal emitted by a target is observed by the sensor nodes independently. Since these local observations are noisy and are thus unreliable, they are fused together as a single reliable observation at a remotely located fusion center in order to increase the overall system performance. The fusion center uses the best linear unbiased estimator in order to estimate the present target signal accurately. By using the proposed system architecture and fusion rule, we are able to optimize the power allocation analytically. Two different cases of power constraints are discussed and compared with each other. The main applications of the proposed results are issues concerning the sensor selection and energy efficiency in passive sensor networks.**

I. Introduction

In this publication, we consider a sensor network where each of K nodes individually and independently receives a signal from a jointly observed source. The type of the source and its signal are assumed to be abstract. For our purposes, we only need to know the quadratic mean of the radiated target signal. The particular information about the target signal at each node is sent to a fusion center, which combines the local observations into a single quantity in order to increase the system performance. This setup is illustrated in Figure 1 whose technical components will be specified in detail later. Both the sensing and the communication channels are subject to additive noise. Moreover, we assume that the sensor nodes (SN) have only limited sum-power available for communication and that each SN is in addition limited in its transmission power-range. A potential application of our approach is passive multiple-radar sensing, where an unknown target signal shall be estimated, detected or classified. Instead of using a complex single-radar system, this task is carried out by a network of cheap and energy-efficient SNs. To achieve comparable system performance, the fusion center combines a multitude of local observations into a single reliable quantity.

Quite naturally, the problem arises how to allocate a given sum-power to the SNs for transmitting the local observations to the fusion center. The problem of finding an optimum power allocation for a distributed radar system and a closed-form of the objective function is extremely hard, and it is even harder to determine optimal points under certain constraints. The main difficulty is associated with finding an explicit representation of the objective function as mentioned in [1]. In the present paper, the objective function is the average deviation between the estimated and the actual signal values. The proposed simple and abstract system architecture together with the contemplated objective function enable us to derive an analytical solution to the power allocation problem. Total and individual power constraints are considered. Both lead to explicit policies for the power allocation. These are the main contributions of the present work.

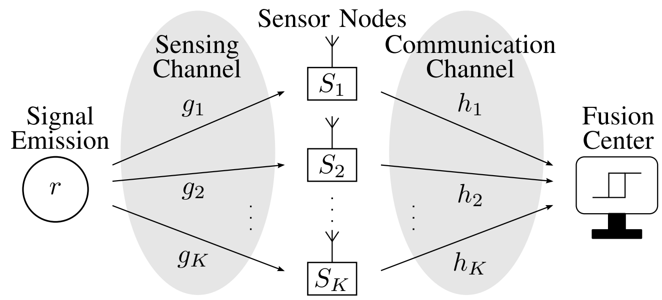

Fig. 1. Abstract representation of the distributed sensor network.

Distributed passive multiple-radar systems (DPMRS) have worthwhile applications nowadays. Physicists use this type of radars to detect or to determine specific characteristics of particles, for example, in the neutrino telescope ‘*IceCube Neutrino Observatory*’ at the Amundsen-Scott South Pole Station [2] in Antarctica, where a network with over 5000 nodes is implemented. They also use such radars for radio astronomy to study celestial objects, for instance in the ‘*Karl G. Jansky Very Large Array*’ of the National Radio Astronomy Observatory [3] in Socorro County, New Mexico. Many other applications of DPMRSs are military and some few are also for civil uses, but they are outside the scope of the present work. Because of the significance of DPMRSs it is important to investigate the power allocation problem in order to improve the radar accuracy while the power consumption of the whole network is kept constant. In the more recent past, some methods have been proposed to solve the power allocation problem. In particular, the authors in [4] investigated some game-theoretic approaches to solve the problem however without focusing on DPMRSs. The investigation of the power allocation only for localization is treated in [5] and [6]. An optimal solution for the power allocation problem is given in [7], where an active radar is considered instead of a passive radar system. In the present work, we treat the power allocation problem in general and find analytical solutions for DPMRSs.

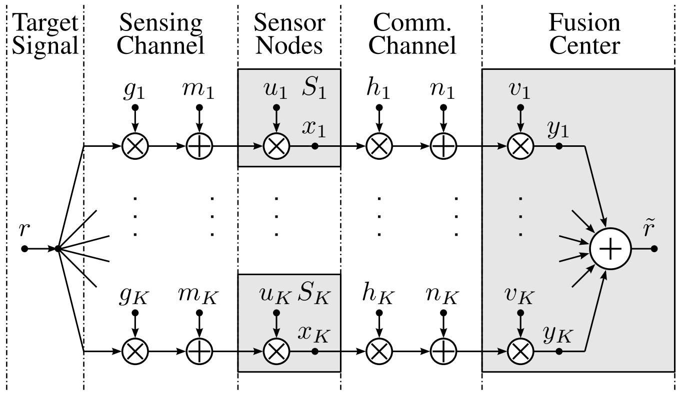

Fig. 2. System model of the distributed sensor network.

We start with a description of the underlying technical system in the next section. Subsequently, the power allocation problem is specified and analytically solved. The achieved results are then discussed and carefully compared with each other.

Mathematical Notations:

Throughout this paper, we denote the set of natural and real numbers by $\mathbb{N}$ and $\mathbb{R}$, respectively. Note that the set of natural numbers does not include the element zero. Moreover, $\mathbb{R}_+$ denotes the set of non-negative real numbers. Furthermore, we use the subset $\mathbb{F}_N \subseteq \mathbb{N}$, which is defined as $\mathbb{F}_N := \{1, \ldots, N\}$ for any given natural number N. We denote the absolute value of a real number z by $|z|$. The expected value of a random variable v is denoted by $\mathcal{E}[v]$. Moreover, the notation V^* stands for the value of an optimization variable V at the optimum point of the corresponding optimization problem. Finally, vectors and matrices are represented in bold typeface.

II. Overview and Technical System Description

At any instance of time, a network of $K \in \mathbb{N}$ independent and spatially distributed sensors, as shown in Figure 1, receives random observations. If a target signal r is present, then the received power at SN S_k is a part of the radiated power from the target source. Each received signal is weighted by the corresponding channel coefficient and is disturbed by additive noise. It is obvious that the sensing channel is wireless. All SNs continuously take samples from the disturbed received signal and amplify them without any additional data processing. The local measurements are then transmitted to a fusion center which is located in a remote location. The communication to the fusion center is performed by using distinct waveforms for each SN so as to distinguish the communication of different SNs. Each waveform has to be suitably chosen in order to suppress inter-user (inter-node) interference at the fusion center. Hence, the K received signals at the fusion center are uncorrelated and are assumed to be conditionally independent. Each received signal at the fusion center is also weighted by the corresponding channel coefficient and is disturbed by additive noise, as well. The communication channel between the SNs and the fusion center can either be wireless or wired. The disturbed received signals at the fusion center are weighted and combined together in order to obtain a single reliable observation $\tilde{r}$ of the actual target signal r.

Note that we disregard time delays within all transmissions and assume synchronized data communication.

In the following subsections, we mathematically describe the underlying system model that is depicted in Figure 2. The continuous-time system is modeled by its discrete-time equivalent, where the sampling rate of the corresponding signals is equal to the target observation rate, for the sake of simplicity.

A. Target signal

Often, the target source is not well known. Thus, we assume that we only know the quadratic mean $R := \mathcal{E}[r^2]$ with $0 < R < \infty$ of the real-valued target signal r. This knowledge is sufficient for further calculations. Furthermore, the target signal during each observation step is assumed to be stationary.

B. Sensing channel

Each propagation path of the sensing channel is described by a corresponding random channel coefficient g_k. For the investigation of the power allocation problem, the concrete realization of the channel coefficients is needed and hence can be used for postprocessing of the received signals at the SNs. We assume that the channel coefficients are real-valued and stationary during each target observation step. Thus, the expected value and the quadratic mean of each coefficient during each observation step can be assumed to be equal to their instantaneous values, i.e., $\mathcal{E}[g_k] = g_k$ and $\mathcal{E}[g_k^2] = g_k^2$. In practice, it is often difficult to measure or estimate these coefficients because the network is passive and is hence not able to sound the channel actively. Thus, the results of the present work are applicable for scenarios where the channel coefficients can somehow be estimated accurately during each observation process or they are nearly deterministic and thus can be measured before starting the radar task. This is the case, for example for the neutrino telescope where the SNs are installed deep in the icecap.

Furthermore, the channel coefficients are assumed to be uncorrelated and jointly independent. Note that the channel coefficients include the radar cross section, the influence of the antenna, the impact of the filters, as well as all additional attenuation of the target signal.

At the input of each SN, the disturbance is modeled by the real-valued additive white Gaussian noise (AWGN) m_k with zero mean and finite variance $M_k := \mathcal{E}[m_k^2]$ for all k. Note that the channel coefficient and the noise on the same propagation path are also uncorrelated and jointly independent.

C. Sensor nodes

We model each SN by an amplify-and-forward unit, where the ratio of the output to the input signal is described by the non-negative real-valued amplification factor u_k. Thus, the output signal and the expected value of its instantaneous power are described by

$$x_k := (rg_k + m_k)u_k,\ k \in \mathbb{F}_K \tag{1}$$

and

$$X_k := \mathcal{E}[x_k^2] = (Rg_k^2 + M_k)u_k^2,\ k \in \mathbb{F}_K\ , \tag{2}$$

respectively. If the input signal is negligible in comparison to the output signal and if the nodes have smart power components with low-power dissipation loss, then the power consumption of each node is approximately equal to its output power X_k. In the present work, we assume that the linear output-power-range of each SN is individually limited by P_k and that the mean power consumption of all SNs together is limited by the sum-power constraint P_{tot}. Hence, the constraints

$$X_k \leq P_k \ \Leftrightarrow\ (Rg_k^2 + M_k)u_k^2 \leq P_k,\ k \in \mathbb{F}_K \tag{3}$$

and

$$\sum_{k=1}^{K} X_k \leq P_{\text{tot}} \ \Leftrightarrow\ \sum_{k=1}^{K}(Rg_k^2 + M_k)u_k^2 \leq P_{\text{tot}} \tag{4}$$

arise consequently.

Note that the sum-power constraint P_{tot} is a requirement to obtain an energy-efficient radar system.

D. Communication channel

Analogous to the sensing channel, each propagation path of the communication channel is described by a corresponding random channel coefficient h_k. But in contrast to the sensing channel, we assume that the concrete realization of the communication channel coefficients is measurable by using pilot sequences at each SN. Accordingly, the channel coefficients can be used for postprocessing of received signals at the fusion center. We assume that the channel coefficients are real-valued and stationary during each target observation step. Thus, the expected value and the quadratic mean of each channel coefficient can be assumed to be equal to their instantaneous values, i.e., $\mathcal{E}[h_k] = h_k$ and $\mathcal{E}[h_k^2] = h_k^2$. Furthermore, the channel coefficients are assumed to be uncorrelated and jointly independent. Note that the channel coefficients include the influence of the antenna, the impact of the filters, as well as all additional attenuation of the corresponding sensor signal.

At the input of the fusion center, the disturbance on each communication path is modeled by the real-valued AWGN n_k with zero mean and finite variance $N_k := \mathcal{E}[n_k^2]$ for all k. Note that the channel coefficient and the noise on the same propagation path are also uncorrelated and jointly independent.

E. Fusion center

The fusion center combines the different local observations into a single reliable one by applying a linear combiner. Thus, the received signals are weighted with the real-valued factors v_k and summed up to yield an estimate $\tilde{r}$ of the actual target signal r. In this way, we obtain

$$y_k := \big((rg_k + m_k)u_k h_k + n_k\big)v_k,\ k \in \mathbb{F}_K \tag{5}$$

and hence

$$\tilde{r} := \sum_{k=1}^{K} y_k = r\sum_{k=1}^{K} g_k u_k h_k v_k + \sum_{k=1}^{K}(m_k u_k h_k + n_k)v_k\ . \tag{6}$$

Note that the fusion center can separate the input streams because the communication channel is either wired or the data communication is performed by distinct waveforms for each SN.

In order to obtain a single reliable observation at the fusion center, the value $\tilde{r}$ should be a good estimate for the present target signal r. Thus, we optimize the amplification factors u_k and the weights v_k in order to minimize the average deviation between $\tilde{r}$ and the true r. This optimization procedure is elaborately explained in the next section.

F. Some remarks on the system model

All described assumptions are necessary to obtain a framework suitable for analyzing the power allocation problem, without studying detection, classification and estimation problems in specific systems and their settings.

The accurate estimation of the channel state is necessary for both the radar process and the power allocation. Sometimes it is not possible to estimate the transmission channels, consequently the channel coefficients g_k and h_k are unknown. In such cases, the radar usually fails to perform its task.

Since the channel coefficients g_k are in practice hard to estimate or to determine, the present work rather shows theoretical aspects of the power allocation than the practical realization and implementation. Hence, the presented results act as theoretical bounds and references for comparing real radar systems.

The introduced system architecture describes a baseband communication system without considering time, phase and frequency synchronization problems.

III. Power Optimization

In this section, we first introduce the power optimization problem and then present its analytical solution for the case where the transmission power of each SN is limited by the linear output-power-range limitation $P_k \in \mathbb{R}_+,\ k \in \mathbb{F}_K$. Afterwards, we present a suboptimal solution of the power allocation problem for the case where an additional sum-power constraint $P_{\text{tot}} \in \mathbb{R}_+$ for the expected power consumption of all SNs together is given.

We now summarize notations that we will use hereinafter and are needed for the description of each observation process:

- r and R: target signal and its quadratic mean;
- $\tilde{r}$: the estimate of r;
- g_k and h_k: real-valued channel coefficients;
- m_k, n_k, M_k and N_k: real-valued zero-mean AWGN and their variances;
- u_k and v_k: amplification factors and weights;
- y_k: input signals of the combiner;
- P_k: linear output-power-range limitation per SN;
- P_{tot}: sum-power constraint.

A. The optimization problem

As mentioned in the last section, the value $\tilde{r}$ should be a good estimate for the present target signal r. In particular, we aim at finding estimators $\tilde{r}$ of minimum mean squared error in the class of unbiased estimators for each r.

The estimate $\tilde{r}$ is unbiased simultaneously for each r if $\mathcal{E}[\tilde{r}-r]=0$, i.e., from equation (6) we obtain the identity

$$\sum_{k=1}^{K} g_k u_k h_k v_k = 1 \ . \tag{7}$$

This identity is our first constraint in what follows. Note that the mean of the second term in (6) vanishes since the noise is zero-mean. Furthermore, we do not consider the impact of both random variables g_k and h_k as well as their estimates in our calculations because the coherence time of both channels is assumed to be much longer than the target observation time.

The objective is to minimize the mean squared error $\mathcal{E}[(\tilde{r}-r)^2]$. By using equation (6) and the identity (7) we may write the objective function as

$$V := \mathcal{E}\big[(\tilde{r}-r)^2\big] = \sum_{k=1}^{K} \big(M_k u_k^2 h_k^2 + N_k\big) v_k^2 \ . \tag{8}$$

Note that (8) is only valid if m_k and n_k are white and jointly independent.

As mentioned in the last section, each SN has a linear output-power-range limitation and the expected overall power consumption is also limited. Hence, the objective function is also subject to (3) and (4), which are our second and last constraints, respectively.

In summary, the optimization problem is to minimize the mean squared error in (8) with respect to u_k and v_k, subject to constraints (3), (4) and (7). Note that the optimization problem is a *signomial program*, which is a generalization of *geometric programming*, and is thus non-convex in general, see [8].

B. No sum-power limitation

In order to solve the optimization problem, we use the method of Lagrangian multipliers and obtain the Lagrange function as

$$L_1(u_k, v_k, \eta, \lambda_k; \varrho_k) := \sum_{k=1}^{K} \big(M_k u_k^2 h_k^2 + N_k\big) v_k^2 + \Big(1 - \sum_{k=1}^{K} g_k u_k h_k v_k\Big)\eta + \sum_{k=1}^{K} \big(P_k - \varrho_k - (R g_k^2 + M_k) u_k^2\big)\lambda_k , \tag{9}$$

where η and λ_k are Lagrange multipliers while ϱ_k are slack variables.

At any stationary point of L_1, all first partial derivatives must vanish, i.e.,

$$\frac{\partial L_1}{\partial v_l} = 2\big(M_l u_l^2 h_l^2 + N_l\big) v_l - \eta g_l u_l h_l = 0 \, , \ l \in \mathbb{F}_K \ , \tag{10}$$

$$\frac{\partial L_1}{\partial \eta} = 1 - \sum_{k=1}^{K} g_k u_k h_k v_k = 0 \tag{11}$$

and

$$\frac{\partial L_1}{\partial \lambda_l} = P_l - \varrho_l - (R g_l^2 + M_l) u_l^2 = 0 \, , \ l \in \mathbb{F}_K \ . \tag{12}$$

Note that the first partial derivative with respect to u_l, $l \in \mathbb{F}_K$, is not needed because the optimal point lies on the boundary of the feasible set as we will see later.

From (10), we deduce the equation

$$v_l = \frac{\eta}{2} \frac{g_l u_l h_l}{M_l u_l^2 h_l^2 + N_l} \ . \tag{13}$$

By incorporating (13) into (11), it yields the relationship

$$\frac{\eta}{2} = \left(\sum_{k=1}^{K} \frac{g_k^2 u_k^2 h_k^2}{M_k u_k^2 h_k^2 + N_k} \right)^{-1} \ . \tag{14}$$

In turn, we replace $\frac{\eta}{2}$ in (13) with (14) and obtain

$$v_l = \frac{g_l u_l h_l}{M_l u_l^2 h_l^2 + N_l} \left(\sum_{k=1}^{K} \frac{g_k^2 u_k^2 h_k^2}{M_k u_k^2 h_k^2 + N_k} \right)^{-1} \ . \tag{15}$$

Multiplying (10) with v_l, summing up the outcome for all l and using equation (11), we deduce

$$V = \frac{\eta}{2} = \left(\sum_{k=1}^{K} \frac{g_k^2 u_k^2 h_k^2}{M_k u_k^2 h_k^2 + N_k} \right)^{-1} \tag{16}$$

for the objective function which consequently becomes only a function of u_l. According to equation (12), we calculate the factors

$$u_l^2 = \frac{P_l - \varrho_l}{R g_l^2 + M_l} \, , \ l \in \mathbb{F}_K \, , \tag{17}$$

where ϱ_l is in the range $0 \leq \varrho_l \leq P_l$. After replacing u_k^2 in (16) with (17), we obtain

$$V = \left(\sum_{l=1}^{K} \frac{g_l^2 h_l^2}{M_l h_l^2 + N_l \frac{R g_l^2 + M_l}{P_l - \varrho_l}} \right)^{-1} \, , \tag{18}$$

which is strictly increasing with respect to ϱ_l and strictly decreasing with respect to K. Thus, it follows that

$$\varrho_k^* = 0 \, , \ k \in \mathbb{F}_K \, , \tag{19}$$

$$u_k^* = \sqrt{\frac{P_k}{R g_k^2 + M_k}} \, , \ k \in \mathbb{F}_K \, , \tag{20}$$

and hence

$$V^* = \left(\sum_{l=1}^{K} \frac{g_l^2 h_l^2 P_l}{M_l h_l^2 P_l + N_l (R g_l^2 + M_l)} \right)^{-1} \ . \tag{21}$$

By incorporating (20) into (15), we infer

$$v_k^* = \frac{V^* g_k h_k \sqrt{P_k} \sqrt{R g_k^2 + M_k}}{M_k h_k^2 P_k + N_k (R g_k^2 + M_k)} \, , \ k \in \mathbb{F}_K \ . \tag{22}$$

Since $\varrho_k^* = 0$, it follows that the optimal point lies on the boundary of the feasible set, especially on a corner, where the first derivatives of the objective with respect to u_k do not vanish in general.

The equations (20), (21) and (22) are the optimal solutions of the power allocation problem and hence are the first main contributions of the present work.

C. Interpretation of the solution

The solution of the power allocation problem has the following interpretation: All K SNs are active and their output power is equal to their linear output-power-range limitation P_k. By using the amplification factors from (20) and the weights from (22), the single observation $\tilde{r}$ is an estimator of minimum mean squared error in the class of unbiased estimators for the target signal r. Hence, we obtain the estimate

$$\tilde{r} = r + \sum_{k=1}^{K} (m_k u_k^* h_k + n_k) v_k^* \tag{23}$$

from (6). The above equation shows that $\tilde{r}$ is equal to r with some additional noise. Hence, $\tilde{r} - r$ is a zero-mean Gaussian random variable with a variance of V^*, see (8) and (21).

D. Optimization subject to all constraints

In the current subsection, we consider the optimization problem from Subsection III-A subject to all constraints, which yields the Lagrange function

$$L_2(u_k, v_k, \eta, \lambda_k, \tau; \varrho_k, \xi) := L_1(u_k, v_k, \eta, \lambda_k; \varrho_k) + \left(P_{\text{tot}} - \xi - \sum_{k=1}^{K} (R g_k^2 + M_k) u_k^2 \right) \tau \tag{24}$$

with additional Lagrange multiplier τ and the slack variable ξ. The first partial derivatives of (24) with respect to v_l, η and λ_l are identical to those which are given in (10), (11) and (12), respectively. Thus, we also obtain the same results for v_l and V as given in (15) and (16), respectively. Consequently, both the sum-power constraint and the output-power-range limitations remain unused, thus far.

For the sake of simplicity, we define new quantities as

$$w_k := (R g_k^2 + M_k) u_k^2 \;\Rightarrow\; w_k \in \mathbb{R}_+ \;, \tag{25}$$

$$\alpha_k := \sqrt{\frac{g_k^2}{M_k}} \;\Rightarrow\; \alpha_k \in \mathbb{R}_+ \;, \tag{26}$$

and

$$\beta_k := \sqrt{\frac{N_k (R g_k^2 + M_k)}{M_k h_k^2}} \;\Rightarrow\; \beta_k \in \mathbb{R}_+ \;. \tag{27}$$

Before we continue the investigation of the optimization problem, we highlight three important properties of the resulting objective function. First, the objective function in (16) is strictly decreasing with respect to w_k, which can easily be seen if the objective is written as

$$V = \left(\sum_{k=1}^{K} \frac{\alpha_k^2}{1 + \beta_k^2 / w_k} \right)^{-1} \;. \tag{28}$$

Second, the objective function is twice differentiable with respect to w_k because its first and second derivatives exist. Third, the objective function is a convex function with respect to w_k which can be shown by calculating the Hessian $\mathbf{H} := (\frac{\partial^2 V}{\partial w_k \partial w_l})_{k,l}$ that leads to a positive-definite matrix, i.e.,

$$\mathbf{z}'\mathbf{H}\mathbf{z} = 2V^2 \sum_{k=1}^{K} \frac{\alpha_k^2 \beta_k^2 z_k^2}{(w_k + \beta_k^2)^3} + 2V^3 \left(\sum_{k=1}^{K} \frac{\alpha_k^2 \beta_k^2 z_k}{(w_k + \beta_k^2)^2} \right)^2 \geq 0 \;, \quad \forall \mathbf{z} := (z_1, z_2, \ldots, z_K)' \in \mathbb{R}^K \;. \tag{29}$$

Due to the monotonicity of the objective function, the optimal point of (24) must lie on the boundary of its feasible set which consequently leads to $\xi = 0$ and in turn it concludes in

$$\frac{\partial L_2}{\partial \tau} = P_{\text{tot}} - \sum_{k=1}^{K} w_k = 0 \;. \tag{30}$$

By incorporating w_l from (25) into (12), we obtain the equations

$$\frac{\partial L_2}{\partial \lambda_l} = P_l - \varrho_l - w_l = 0 \,, \; l \in \mathbb{F}_K \,, \tag{31}$$

which are also convex functions with respect to w_l. Since (28) and (30) are also convex with respect to w_k, we thus can formulate a new convex minimization problem. If we assume for the new problem that only some of the amplification factors u_k are positive at the optimum point, and define the subset $\mathbb{K} \subseteq \mathbb{F}_K$ for all SNs for which $u_k > 0 \Leftrightarrow w_k > 0 \, \forall k \in \mathbb{K}$ holds, then the corresponding Lagrange function is described by

$$\tilde{L}_2(w_k, \lambda_k, \tau; \varrho_k) := \left(\sum_{k \in \mathbb{K}} \frac{\alpha_k^2 w_k}{w_k + \beta_k^2} \right)^{-1} + \sum_{k \in \mathbb{K}} (P_k - \varrho_k - w_k) \lambda_k + \left(P_{\text{tot}} - \sum_{k \in \mathbb{K}} w_k \right) \tau \;. \tag{32}$$

In order to solve the new optimization problem, we have to consider three different cases depending on P_{tot} as well as the number $\tilde{K}$, $1 \leq \tilde{K} \leq K$, of nodes which are elements of the set $\mathbb{K}$, i.e., $\tilde{K} := |\mathbb{K}|$.

1) The case of $\sum_{k \in \mathbb{F}_K} P_k < P_{tot}$:

In this case, the sum-power constraint does not have any effect on the optimization problem, because the feasible set is only limited by the individual output-power-range constraints. Hence, the solution of the power allocation problem is equal to the results from Subsection III-B, i.e., (20), (21) and (22). The only difference is that a part of the sum-power remains unallocated and cannot be used.

2) The case of $P_{tot} \leq \sum_{k \in \mathbb{F}_K} P_k$ with $\tilde{K} = 1$:

In this case, where the optimum point is unique and would lie on a corner of the feasible set, we re-index the SNs such that the inequality chain

$$\frac{\min\{P_{\text{tot}}, P_k\} + \beta_k^2}{\alpha_k^2 \min\{P_{\text{tot}}, P_k\}} < \frac{\min\{P_{\text{tot}}, P_{k+1}\} + \beta_{k+1}^2}{\alpha_{k+1}^2 \min\{P_{\text{tot}}, P_{k+1}\}} \,, k \in \mathbb{F}_{K-1} \,, \tag{33}$$

holds. Then, only the first SN will minimize the objective and hence the problem in (32) has the simple solution

$$V^* = \frac{\min\{P_{\text{tot}}, P_1\} + \beta_1^2}{\alpha_1^2 \min\{P_{\text{tot}}, P_1\}} = \frac{M_1 h_1^2 \min\{P_{\text{tot}}, P_1\} + N_1(Rg_1^2 + M_1)}{g_1^2 h_1^2 \min\{P_{\text{tot}}, P_1\}} \ , \quad (34)$$

$$u_1^* = \sqrt{\frac{\min\{P_{\text{tot}}, P_1\}}{Rg_1^2 + M_1}} \ , \text{ and } v_1^* = \frac{1}{g_1 h_1}\sqrt{\frac{Rg_1^2 + M_1}{\min\{P_{\text{tot}}, P_1\}}} \ , \quad (35)$$

which follows from the monotonicity of the objective function. If $\tilde{k}$ is not unique or the optimal point is not on a corner of the feasible set, then the number of elements in $\mathbb{K}$ must be greater than one and hence we have to consider the next case.

3) The case of $P_{tot} \leq \sum_{k\in\mathbb{F}_K} P_k$ with $\tilde{K} > 1$:

In this case, the amount of the given sum-power is on the one hand possibly inadequate to supply all SNs. Hence, the sum-power can only be allocated to some of the SNs while all others remain inactive. On the other hand, some of the SNs can attain the limit of their individual output-power-range and are thus saturated. Therefore, we have to separate all active SNs in two groups. The first group contains all active SNs, which are saturated, and is denoted by the subset $\mathbb{K}_{\text{Sat}}$. The second group contains all other active SNs, which operate within their linear output-power-range, and is denoted by the subset $\mathbb{K}_{\text{Lin}}$. Note that both subsets are disjoint and their union is the subset of all active SNs, i.e., $\mathbb{K} = \mathbb{K}_{\text{Sat}} \cup \mathbb{K}_{\text{Lin}}$ and $\mathbb{K}_{\text{Sat}} \cap \mathbb{K}_{\text{Lin}} = \emptyset$ with $\tilde{K}_{\text{Sat}} := |\mathbb{K}_{\text{Sat}}|$, $\tilde{K}_{\text{Lin}} := |\mathbb{K}_{\text{Lin}}|$ and $\tilde{K} := |\mathbb{K}| = \tilde{K}_{\text{Sat}} + \tilde{K}_{\text{Lin}}$. Since all SNs from the subset $\mathbb{K}_{\text{Lin}}$ are active and not saturated, the first derivative of (32) with respect to w_l, $l \in \mathbb{K}_{\text{Lin}}$, must vanish. It gives

$$\frac{\partial \tilde{L}_2}{\partial w_l} = -\frac{V^2 \alpha_l^2 \beta_l^2}{(w_l + \beta_l^2)^2} - \lambda_l - \tau = 0\,, \ l \in \mathbb{K}_{\text{Lin}} \ . \quad (36)$$

By using (30), (31) and (36), the optimization problem (32) can be solved. Since this type of optimization problems are hard to solve, iterative methods or numerical tools are usually used to find an optimum solution. Thus, we propose a different method to find only a suboptimal solution for the power allocation problem including individual output-power-range limitations as well as a sum-power constraint. The idea is first to select some of the SNs with the highest signal-to-noise ratio (SNR) and perform the optimization results from Subsection III-B subsequently.

In order to select the right SNs, we first consider equation (5) and determine the SNR of each received signal at the fusion center by using the results from (20) and (22). It follows that the SNR is equal to

$$\gamma_k := \frac{\mathcal{E}[(rg_k u_k h_k v_k)^2]}{\mathcal{E}[(m_k u_k h_k + n_k)^2 v_k^2]} = \frac{Rg_k^2 h_k^2 P_k}{M_k h_k^2 P_k + N_k(Rg_k^2 + M_k)} \quad (37)$$

for all $k \in \mathbb{F}_K$. Second, we assume that the nodes are re-indexed such that the inequality chain

$$\gamma_k \geq \gamma_{k+1}, \ k \in \mathbb{F}_{K-1} \ , \quad (38)$$

holds. In this manner, the first SNs are the best sensors in the sense that their SNRs are the largest ones. Thus, it is wise to allocate the given sum-power only to the first $\tilde{K}$ SNs.

Because of the results from Subsection III-B, the first $\tilde{K}-1$ SNs attain an output power equal to their linear output-power-range limitation P_k and thus are members of the subset $\mathbb{K}_{\text{Sat}}$. The last SN $S_{\tilde{K}}$ attains the remaining power P_{remain} and thus belongs to either $\mathbb{K}_{\text{Sat}}$ or $\mathbb{K}_{\text{Lin}}$ depending on the difference $P_{\tilde{K}} - P_{\text{remain}}$. We infer

$$P_{\text{remain}} := P_{\text{tot}} - \sum_{k=1}^{\tilde{K}-1} P_k \ , \quad (39)$$

where $\tilde{K}$ is the smallest integer number for which the inequality

$$P_{\text{tot}} \leq \sum_{k=1}^{\tilde{K}} P_k \quad (40)$$

holds. Now we can use the results from Subsection III-B with a minor change in the indices of the nodes. The first $\tilde{K}-1$ nodes are indexed as given in (38). The remaining nodes are then re-indexed again such that the inequalities $P_{\tilde{K}} \geq P_{\text{remain}}$ and

$$\frac{Rg_{\tilde{K}}^2 h_{\tilde{K}}^2 P_{\text{remain}}}{M_{\tilde{K}} h_{\tilde{K}}^2 P_{\text{remain}} + N_{\tilde{K}}(Rg_{\tilde{K}}^2 + M_{\tilde{K}})} > \frac{Rg_k^2 h_k^2 P_{\text{remain}}}{M_k h_k^2 P_{\text{remain}} + N_k(Rg_k^2 + M_k)} \quad (41)$$

hold for all $k \in \mathbb{F}_K \backslash \mathbb{K}_{\text{Sat}}$. Then, the linear output-power-range limitation $P_{\tilde{K}}$ of the last node can artificially be replaced by P_{remain} in order to use the same results as given in (20), (21) and (22). Overall, we obtain the equations

$$u_k^* = \sqrt{\frac{P_k}{Rg_k^2 + M_k}} \ , \ k \in \mathbb{K}\,, \quad (42)$$

$$V^* := \left(\sum_{l=1}^{\tilde{K}} \frac{g_l^2 h_l^2 P_l}{M_l h_l^2 P_l + N_l(Rg_l^2 + M_l)}\right)^{-1} \quad (43)$$

and

$$v_k^* = \frac{V^* g_k h_k \sqrt{P_k}\sqrt{Rg_k^2 + M_k}}{M_k h_k^2 P_k + N_k(Rg_k^2 + M_k)} \ , \ k \in \mathbb{K}\,, \quad (44)$$

for the current case.

Note that the point $(u_1^*, \ldots, u_{\tilde{K}}^*, v_1^*, \ldots, v_{\tilde{K}}^*)$, with u_k^* and v_k^* from (42) and (44), and $P_{\tilde{K}} = P_{\text{remain}}$, is only a feasible point of the objective function since all constraints (3), (4) and (7) are simultaneously fulfilled. But, it is not the global minimum of the objective function and is hence suboptimal, because not all equations in (36) are simultaneously fulfilled in general.

Note that by using the above results, the corresponding fusion rule is simplified by discarding the influence of inactive SNs from the fusion rule. The fusion rule (6) becomes

$$\tilde{r} = \sum_{k\in\mathbb{K}} y_k \ . \quad (45)$$

The equations (34) and (35) as well as (42), (43) and (44) are (suboptimal) solutions of the power allocation problem and hence are the last main contributions of the present work.

E. Discussion of the solution

In contrast to the first case, in which the sum-power constraint is not considered, only some of the SNs are active. The amount of the given sum-power is inadequate to supply all SNs coincident. Hence, the sum-power can only be allocated to the SNs, which are included in the subset $\mathbb{K}$, while all others remain inactive, since their information reliability is too poor to be considered for data fusion.

IV. Conclusion

The main contribution of the present work is a solution to the power allocation problem for increasing the system performance while the power consumption of the whole sensor network is kept constant. We have introduced a simplified system model and a linear fusion rule, which enable us to solve the power allocation problem analytically. Two different cases of power constraints have been investigated. For a limitation of transmission power per sensor node as well as for an additional sum-power limitation, we have obtained optimal and suboptimal solutions analytically. We have seen that the power allocation problem is harder to solve if an additional sum-power constraint is given. Hence, we have proposed a new method to achieve a suboptimal solution of the power allocation problem including all constraints. The proposed results enable us to calculate the optimal or nearly optimal power allocation fast and accurately in distributed passive multiple-radar systems. This is essential for sensor networks with a large number of nodes in order to avoid complex numerical computations.

References

[1] P. K. Varshney, *Distributed Detection and Data Fusion*. New York: Springer, 1997.

[2] (2010, Dec.) Icecube neutrino observatory. University of Wisconsin-Madison and National Science Foundation. Amundsen-Scott South Pole Station, Antarctica. [Online]. Available: http://icecube.wisc.edu/

[3] (2012, Mar.) The very large array. The National Radio Astronomy Observatory. Socorro County, New Mexico, USA. The National Radio Astronomy Observatory is a facility of the National Science Foundation operated under cooperative agreement by Associated Universities, Inc. [Online]. Available: http://www.vla.nrao.edu/

[4] F. Meshkati, H. Poor, and S. Schwartz, "Energy-efficient resource allocation in wireless networks," *Signal Processing Magazine, IEEE*, vol. 24, no. 3, pp. 58–68, 2007.

[5] H. Godrich, A. Petropulu, and H. Poor, "Power allocation strategies for target localization in distributed multiple-radar architectures," *Signal Processing, IEEE Transactions on*, vol. 59, no. 7, pp. 3226–3240, 2011.

[6] Y. Shen, W. Dai, and M. Win, "Optimal power allocation for active and passive localization," in *Global Communications Conference (GLOBECOM), 2012 IEEE*, 2012, pp. 3713–3718.

[7] G. Alirezaei and R. Mathar, "Optimum power allocation for sensor networks that perform object classification," in *2013 Australasian Telecommunication Networks and Applications Conference (ATNAC 2013)*, Christchurch, New Zealand, Nov. 2013.

[8] S. Boyd and L. Vandenberghe, *Convex Optimization*. California: Cambridge University Press, 2004.