Zadoff-Chu Sequence Based Time-Domain SNR Estimation for OFDM Systems

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***Abstract*—The optimization of wireless orthogonal frequency division multiplexing (OFDM) systems is based on the knowledge of the signal-to-noise ratio (SNR). Most of the existing methods for the SNR estimation operates in the frequency-domain and assumes perfect carrier frequency offset (CFO) synchronization. However, it has been shown that in real systems in the presence of the residual CFO this estimators are prone to performance degradation. This problem can be avoided by estimating the SNR in the time-domain. In this paper, a novel low-complexity time-domain SNR estimator based on time periodic Zadoff-Chu (ZC) sequence, named TDZCE, is proposed. When applied to the time periodic synchronization preamble, commonly used in packed based OFDM systems, the special property of a ZC sequence preserves the considered comb-type structure both in the frequency and time-domain. This allows for easy adaptation of previously proposed moment-based SNR estimator to be applied to the time-domain samples. The TDZCE outperforms the existing time-domain SNR estimators in frequency-flat channels in the presence of the CFO, and approaches the Cramer-Rao bound (CRB) as the number of periodic parts of the proposed preamble increases. Additionally, it offers considerable computational savings compared to the existing time-domain SNR estimators.**

***Index Terms*—OFDM, SNR estimation, Zadoff-Chu sequence**

I. Introduction

An important task in the design of future Orthogonal frequency division multiplexing (OFDM) system is to exploit frequency selective channels by adaptable transmission parameters (bandwidth, coding/data rate, power) to preserve power and bandwidth efficiency according to the channel conditions at the receiver. In order to achieve such improvements, an efficient and exact signal-to-noise ratio (SNR) estimation algorithm is requisite. The estimated SNR can be further exploited to improve the correction of carrier frequency offset (CFO), which arises due to the mismatch between the transmitter and receiver oscillators. The noise power estimate can be further utilized for channel estimation [1] and soft decoding [2].

The SNR estimators are well studied for single carrier transmissions [3] and most of these algorithms can be directly applied to OFDM systems in additive white Gaussian noise (AWGN) [4]. For packet based communications, the pilot symbols (preambles) used for synchronization and equalization can be utilized for the SNR estimation without the additional throughput reduction. In [5], we proposed a low-complexity periodic-sequence (PS) estimator based on the second-order moments of received samples in the frequency-domain, which utilizes the preamble structure proposed by Morelli and Mengali in [6]. The time periodic structure of this preamble corresponds to a comb-type structure in the frequency-domain wherein loaded subcarriers are separated by a certain number of nulled subcarriers, which are used to estimate the average noise power. Combined with the average signal plus noise power estimation obtained from the used (loaded) subcarriers, the average noise power estimate is utilized for the average SNR estimation.

However, as discussed in [7], in an imperfectly synchronized system, the CFO introduces the inter-carrier interference (ICI) causing the performance degradation of the frequency-domain SNR estimation methods. Therefore, either the CFO has to be compensated prior to the SNR estimation or SNR has to be estimated in the time-domain since the CFO impacts only the phase of the received time-domain samples [8].

The authors in [9] proposed the time-domain low-complexity SNR estimator (TLSE) based on the preamble containing two identical halves. However, the TLSE has been shown to be prone to performance degradation caused by the CFO. In [10], the authors introduced a maximum likelihood (ML) time-domain method for the joint estimation of the CFO, noise power and SNR, named JMLE, based on the time periodic preamble structure in [6]. However, the complexity of the JMLE increases with the number of periodic parts. Another approach is to design an SNR estimator robust to the CFO such that CFO acquisition can be applied afterwards, possibly by utilizing the previously estimated SNR. Accordingly, the authors in [8] proposed the time-domain preamble-based SNR estimator (TPSE) based on the time periodic structure [6] utilized by both the PS and JMLE estimators. The TPSE shows the good performance in the presence of the CFO at the cost of an increased complexity.

In this paper we propose a novel low complexity time-domain SNR estimator for the frequency-flat channel. The proposed estimator, named TDZCE, is based on the comb-type time-domain Zadoff-Chu (ZC) sequence structure utilized for the preamble and shows the robust performance in the presence of the CFO. Due to the good correlation properties, ZC sequences have been extensively used in various parts of

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the LTE standard. Moreover, Beyme and Leung have recently proven in [11] that a ZC sequence and its DFT are time-scaled conjugates of each other, up to a constant factor. This leads us to the heuristic reasoning that, besides having the time periodic structure, the preamble also contains zero samples in the time-domain, i.e, the comb-type structure of nulled and loaded subcarriers in the frequency-domain corresponds to "nulled" and "loaded" samples in the time-domain. Since the CFO introduces only a phase offset to the time-domain received samples, applying the second order moment methods from the PS estimator [5] on nulled and loaded time-domain samples results in the SNR estimation irrespective of the CFO. Furthermore, the analysis of the statistical properties of the TDZCE can be utilized for improving the system procedures based on the SNR estimate.

The contribution of this paper is threefold: At first, we propose the time periodic preamble structure based on the ZC sequence; secondly, based on the given preamble we derive a low-complexity time-domain SNR estimator robust to the presence of the CFO; finally, we derive statistical properties of the proposed estimator.

The remainder of this paper is organized as follows. Section II introduces the system model and specifies the SNR estimation problem. In Section III, we propose the preamble structure, address the corresponding TDZCE estimator, and derive its statistical properties. Simulation results and computational complexity analysis are presented in Section IV. Finally, some concluding remarks are given in Section V.

II. System Model

The OFDM symbol contains the complex data symbols $C(n)$ on the nth subcarrier, for $n = 0, \dots, N-1$, each with the average power $E\{|C(n)|^2\} = \sigma_s^2$. The corresponding discrete time-domain transmitted signal after the N-point IDFT is given by

$$c(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} C(n) e^{j2\pi kn/N}, \; 0 \leq k \leq N-1, \tag{1}$$

where $E\{|c(k)|^2\} = \sigma_s^2$ is the signal power in the time-domain, which is due to the linearity of the IDFT transformation equal to the signal power in the frequency-domain, i.e., $\sigma_s^2 = \sigma_S^2$. At the receiver the received signal in the presence of the CFO can be expressed as

$$r(k) = c(k) e^{\frac{2\pi\varepsilon k}{N}} + w(k), \qquad k = 0, \dots, N-1, \tag{2}$$

where ε is the CFO normalized to the subcarrier spacing and $w(k)$ is sampled circularly symmetric complex AWGN with variance σ_w^2, i.e, $w(k) \sim SCN(0, \sigma_w^2)$. Assuming that the channel is constant during the frame, the average SNR is given by

$$\rho_{av} = \frac{E\{|y(k)|^2\}}{E\{|w(k)|^2\}} = \frac{\sigma_s^2}{\sigma_w^2}. \tag{3}$$

III. TDZCE Estimator

A. Preamble Structure

The preamble consisting of Q identical parts, each containing $N_p = N/Q$ samples is proposed as shown in Fig. 1a. The corresponding frequency-domain representation is shown in Fig. 1b. We further assume that Q divides N,

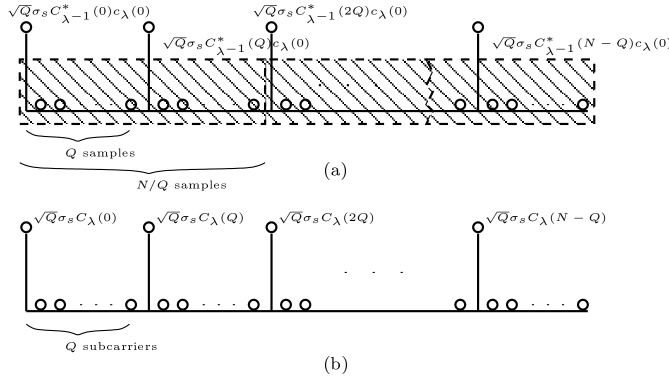

Fig. 1. Preamble structure in the (a) time and (b) frequency-domain.

so that the number of loaded subcarriers N_p is an integer. Starting from the 0th, each Qth subcarrier is modulated with a Zadoff-Chu (ZC) sequence symbol $C_\lambda(mQ)$ with $|C_\lambda(m)| = 1$ for $m = 0, 1, \dots, N_p - 1$. The remainder of $N_z = N - N_p = \frac{(Q-1)}{Q} N$ subcarriers is not used (nulled). To maintain the total energy level over all symbols within the preamble, the transmit power is scaled by the factor Q yielding a total transmit power of $Q\sigma_S^2$ at the loaded subcarriers.

Write $n = mQ + q$, $m = 0, \dots, N_p - 1$, $q = 0, \dots, Q-1$. The transmitted signal on the nth subcarrier is given by

$$C(n) = C(mQ+q) = \begin{cases} \sqrt{Q} \cdot \sigma_s C_\lambda(mQ), & q = 0 \\ 0, & q = 1, \dots, Q-1 \end{cases}. \tag{4}$$

Due to the IDFT properties, the time-domain representation of the preamble with Q identical parts can be written as

$$c(k) = c(k + q\frac{N}{Q}), \; k = 0, \dots, \frac{N}{Q} - 1, \; q = 1, \dots, Q-1.$$

As shown in Appendix A, the time-domain representation of the preamble in (4) is given by

$$c(k) = c(mQ+q) = \begin{cases} \sqrt{Q}\sigma_s C^*_{\lambda^{-1}}(mQ) c_\lambda(0), & q = 0 \\ 0, & q = 1, \dots, Q-1 \end{cases}. \tag{5}$$

Both the frequency-domain and time-domain representations, given by (4) and (5), respectively, have the property that every Qth sample is "loaded", i.e., it is different from zero.

B. The Proposed SNR Estimation

Furthermore, according to (2), the kth received time sample can be written as

$$r(k) = r(mQ+q) = \begin{cases} r_p(m), & q = 0 \\ r_z(mQ+q), & q = 1, \dots, Q-1 \end{cases},$$

where

$$r_p(m) = c(mQ) e^{\frac{2\pi\varepsilon mQ}{N}} + w(mQ) \tag{6}$$

denotes the received time-domain samples containing the phase shifted signal and additional noise component and

$$r_z(mQ+q) = w(mQ+q) \tag{7}$$

represents the time-domain samples containing only noise.

The empirical second-order moment of the time-domain received samples containing the signal and noise component is given by

$$\hat{M}_{2,p} = \frac{1}{N_p} \sum_{m=0}^{N_p-1} |r_p(m)|^2 \tag{8}$$

with the expectation $\mathrm{E}\left\{\hat{M}_{2,p}\right\} = Q\sigma_s^2 + \sigma_w^2$.

Similarly, the empirical second moment of the received samples containing only the noise component

$$\hat{M}_{2,z} = \frac{1}{N_p(Q-1)} \sum_{m=0}^{N_p-1} \sum_{q=1}^{Q-1} |r_z(mQ+q)|^2 \tag{9}$$

has expectation $\mathrm{E}\left\{\hat{M}_{2,z}\right\} = \sigma_w^2$.

In summary, the average SNR ρ_{av} can be estimated by forming

$$\begin{aligned}\hat{\rho}_{av} &= \frac{1}{Q}\frac{\hat{M}_{2,p}-\hat{M}_{2,z}}{\hat{M}_{2,z}} = \frac{1}{Q}\left(\frac{\hat{M}_{2,p}}{\hat{M}_{2,z}} - 1\right) \\ &= \frac{1}{Q}\Big((Q-1)\frac{\sum_{m=0}^{N_p-1}|r_p(m)|^2}{\sum_{m=0}^{N_p-1}\sum_{q=1}^{Q-1}|r_z(mQ+q)|^2} - 1\Big),\end{aligned} \tag{10}$$

where, by the strong law of large numbers, $\hat{M}_{2,p}$ and $\hat{M}_{2,z}$ are strongly consistent unbiased estimators of $QS+W$ and average noise power W, respectively.

C. Statistical Properties

To derive the statistical properties of the average SNR $\hat{\rho}_{av}$ in (10), it can be shown that $2N_p\frac{\hat{M}_{2,p}}{\sigma_w^2}$ and $2N_z\frac{\hat{M}_{2,z}}{\sigma_w^2}$ are noncentral chi-squared and central chi-squared random variables, respectively.

Let $r_z(mQ+q)$, for $m = 0, 1, \ldots, N_p - 1$ and $q = 1, \ldots, Q-1$, denote independent circular symmetric complex Gaussian random variables with zero means and identical variances σ_w^2, i.e., $r_z(mQ+q) \sim SCN(0, \sigma_w^2)$. Then, $Z = \frac{2}{\sigma_w^2}\sum_{m=0}^{N_p-1}\sum_{q=1}^{Q-1}|r_z(mQ+q)|^2 = 2N_z\frac{\hat{M}_{2,z}}{\sigma_w^2}$ has a *central chi-squared* distribution, $\chi^2_{\nu_z}$, with $\nu_z = 2N_z$ degrees of freedom. Therefore, $Z \sim \chi^2_{\nu_z}$, with the pdf given by

$$f_Z(z) = \frac{1}{2^{\frac{\nu_z}{2}}\Gamma\left(\frac{\nu_z}{2}\right)} z^{\frac{\nu_z}{2}-1} e^{-\frac{z}{2}}, \quad z \geq 0, \tag{11}$$

where $\Gamma(u)$ is the Gamma function [12].

Let $r_p(m)$, for $m = 0, 1, \ldots, N_p - 1$, denote independent circular symmetric complex Gaussian random variables with expected values $\sqrt{Q}\sigma_s$ and identical variances σ_w^2, i.e., $r_p(m) \sim SCN(\sqrt{Q}\sigma_s, \sigma_w^2)$. Then, $P = \frac{2}{\sigma_w^2}\sum_{m=0}^{N_p-1}|r_p(m)|^2 = 2N_p\frac{\hat{M}_{2,p}}{\sigma_w^2}$ has a noncentral chi-squared distribution, $\chi^2_{\nu_p,\lambda}$, with $\nu_p = 2N_p$ degrees of freedom and noncentrality parameter $\lambda = 2Q\frac{\sigma_S^2}{\sigma_w^2} = 2Q\rho_{av}N_p$. Therefore, $P \sim \chi^2_{\nu_p,\lambda}$, with the pdf given by

$$f_P(p) = \frac{1}{2}\left(\frac{p}{\lambda}\right)^{\frac{\nu_p-2}{4}} e^{\frac{p+\lambda}{2}} I_{\frac{\nu_p}{2}-1}(\sqrt{p\lambda}), \quad p \geq 0, \tag{12}$$

where $I_a(u)$ is the modified Bessel function of the first kind and order a [12].

Furthermore, given $Z = 2N_z\frac{\hat{M}_{2,z}}{\sigma_w^2}$ as a central chi-squared random variable with $\nu_z = 2N_z$ degrees of freedom and $P = 2N_p\frac{\hat{M}_{2,p}}{\sigma_w^2}$ as a noncentral chi-squared random variable with $\nu_p = 2N_p$ degrees of freedom and noncentrality parameter $\lambda = 2Q\rho_{av}N_p$, the random variable $V = \frac{\hat{M}_{2,p}}{\hat{M}_{2,z}}$ is noncentral F-distributed. From the empirical second order moments $\hat{M}_{2,p}$ and $\hat{M}_{2,z}$ given in (8) and (9), respectively, it can be noticed that the different sets of data samples are used for the estimation. Therefore, $\hat{M}_{2,p}$ and $\hat{M}_{2,z}$ can be considered as independent random variables, thus giving random variables P and Z also as independent, respectively. According to [13], having P and Z stochastically independent, a noncentral F-distributed random variable V is obtained when dividing a noncentral chi-squared random variable $P \sim \chi^2_{\nu_p,\lambda}$ by a central chi-squared random variable $Z \sim \chi^2_{\nu_z}$,

$$V = \frac{P/\nu_p}{Z/\nu_z}. \tag{13}$$

Replacing P, Z, ν_p, and ν_z in (13) directly yields

$$V = \frac{\left(\frac{2N_p\hat{M}_{2,p}}{\sigma_w^2}\right)/(2N_p)}{\left(\frac{2N_z\hat{M}_{2,z}}{\sigma_w^2}\right)/(2N_z)} = \frac{\hat{M}_{2,p}}{\hat{M}_{2,z}}. \tag{14}$$

The F-distributed random variable V in (13) has a pdf given by

$$\begin{aligned}f_V(v) = e^{-\frac{\lambda}{2}} \sum_{k=0}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^k}{k!} \frac{\left(\frac{\nu_p}{\nu_z}\right)^{\frac{1}{2}\nu_p+k}}{B\left(\frac{\nu_p+2k}{2}, \frac{\nu_z}{2}\right)} v^{\frac{\nu_p}{2}+k-1} \\ \cdot \left(1 + \frac{\nu_p}{\nu_z}v\right)^{-\frac{1}{2}(\nu_p+\nu_z)-k},\end{aligned} \tag{15}$$

where $B(u,v)$ is the Beta function, which can be represented by the Gamma functions [12].

The mean and the variance of the F-distributed random variable V, defined in (13), are given by

$$\mathrm{E}(V) = \frac{\nu_z(\nu_p+\lambda)}{\nu_p(\nu_z-2)}, \quad \nu_z > 2, \tag{16}$$

and

$$\mathrm{Var}(V) = 2\left(\frac{\nu_z}{\nu_p}\right)^2 \frac{(\nu_p+\lambda)^2 + (\nu_z-2)(\nu_p+2\lambda)}{(\nu_z-2)^2(\nu_z-4)}, \quad \nu_z > 4, \tag{17}$$

respectively [13]. Replacing ν_p and ν_z in (16) and (17) yields

$$\begin{aligned}\mathrm{E}(V) &= \frac{2N_z \cdot 2N_p(1+Q\rho_{av})}{2N_p \cdot 2(N_z-1)} \\ &= \frac{1+Q\rho_{av}}{(1-\frac{1}{N_z})}, \quad N_z > 1,\end{aligned} \tag{18}$$

and

$$\begin{aligned}\mathrm{Var}(V) &= 2\left(\frac{N_z}{N_p}\right)^2 \frac{4N_p^2(1+Q\rho_{av})^2 + 4N_p(N_z-1)(1+2Q\rho_{av})}{4(N_z-1)^2 \cdot 2(N_z-2)}, \\ &= \frac{1}{N_z}\frac{(1+Q\rho_{av})^2 + \frac{1}{N_p}(N_z-1)(1+2Q\rho_{av})}{(1-\frac{1}{N_z})^2(1-\frac{2}{N_z})}, \quad N_z > 2,\end{aligned} \tag{19}$$

respectively. Comparing (10) with (14), the estimated average SNR $\hat{\rho}_{av}$ can be obtained by transforming the random variable V

$$\hat{\rho}_{av} = \frac{1}{Q}\left(\left(\frac{\hat{M}_{2,p}}{\hat{M}_{2,z}}\right) - 1\right) = \frac{1}{Q}(V-1). \tag{20}$$

Its expectation $\mathrm{E}(\hat{\rho}_{av})$ is given by

$$\begin{aligned}\mathrm{E}(\hat{\rho}_{av}) &= \frac{1}{Q}\left(\mathrm{E}\left(\frac{\hat{M}_{2,p}}{\hat{M}_{2,z}}\right) - 1\right) = \frac{1}{Q}(\mathrm{E}(V) - 1) \\ &= \frac{1}{Q}\left(Q\rho_{av} + \frac{1+Q\rho_{av}}{N_z-1}\right) = \rho_{av} + \frac{\frac{1}{Q}+\rho_{av}}{N_z-1} \\ &\approx \rho_{av},\end{aligned} \tag{21}$$

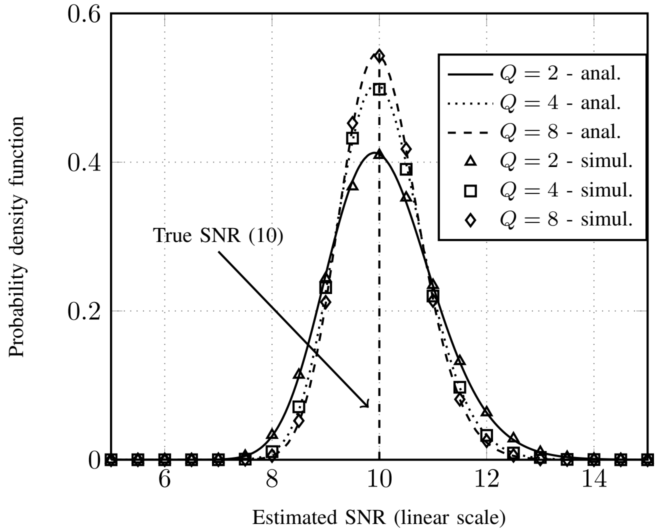

Fig. 2. Pdf of SNR estimate for true SNR = 10 dB, $N = 256$, and the number of periodic parts $Q = 2, 4$, and 8.

for $N >> 1$, which is a regular case for real OFDM systems. Similarly, using the transformation (20), the mean square error (MSE) of the estimated average SNR is given by

$$\begin{aligned} \text{MSE}(\hat{\rho}_{av}) &= \text{E}\left((\hat{\rho}_{av} - \rho_{av})^2\right) \approx \text{Var}(\hat{\rho}_{av}) \\ &= \frac{1}{Q^2}\text{Var}(V). \end{aligned} \tag{22}$$

Inserting (19) into (22), for $N_z = \frac{Q-1}{Q}N$ and $N_z = \frac{N}{Q}$ yields

$$\text{MSE}(\hat{\rho}_{av}) \approx \frac{(1+Q\rho_{av})^2 + (Q-1-\frac{Q}{N})(1+2Q\rho_{av})}{NQ(Q-1)(1-\frac{Q}{(Q-1)N})^2(1-\frac{2Q}{(Q-1)N})}.$$

By using the transformation of the F-distributed random variable V in (20), the pdf of the estimated average SNR $\hat{\rho}_{av}$ can be derived as

$$\begin{aligned} f_{\hat{\rho}_{av}}(\hat{\rho}_{av}) &= Q \cdot f_V(1+Q\hat{\rho}_{av}) \\ &= Q \cdot e^{-\frac{\lambda_n}{2}} \sum_{k=0}^{\infty} \frac{(\frac{\lambda_n}{2})^k}{k!} \frac{\left(\frac{\nu_p}{\nu_z}\right)^{\frac{1}{2}\nu_p+k}}{B\left(\frac{\nu_p+2k}{2}, \frac{\nu_z}{2}\right)} \\ &\cdot (1+Q\hat{\rho}_{av})^{\frac{\nu_p}{2}+k-1}\left(1+\frac{\nu_p}{\nu_z}(1+Q\hat{\rho}_{av})\right)^{-\frac{1}{2}(\nu_p+\nu_z)-k}. \end{aligned} \tag{23}$$

Replacing λ, ν_p, and ν_z in (23) and having $N_z = \frac{Q-1}{Q}N$, yields

$$\begin{aligned} f_{\hat{\rho}_{av}}(\rho_{av}) &= \frac{e^{-N\rho_{av}} \cdot (Q-1)^{N(1-\frac{1}{Q})}}{Q^{N-1}} \cdot \frac{(1+Q\rho_{av})^{\frac{N}{Q}-1}}{(1+\rho_{av})^N} \\ &\cdot \sum_{k=0}^{\infty} \frac{\rho_{av}^k \cdot \left(\frac{N}{Q}\right)^k}{k! \cdot B\left(k+\frac{N}{Q}, \frac{Q-1}{Q}N\right)} \left(\frac{1+Q\rho_{av}}{1+\rho_{av}}\right)^k. \end{aligned} \tag{24}$$

Figure 2 shows the pdf of the estimated average SNR, given in (24) for true SNR = 10 dB. It can be seen that the simulation results agree with the derived analytical model. Moreover, increasing the number of periodic parts Q improves the accuracy of SNR estimation, i.e., the pdf curve becomes more concentrated around the true SNR value for the higher Q values.

TABLE I
THE COMPLEXITY COMPARISON OF CONSIDERED SNR ESTIMATORS.

Estimator	TLSE [9]	TPSE [8]	JMLE [10]	TDZCE
FLOPs	$5N$	$7.5N + 4$	$4N(Q+2) + 12Q$	$4N + 2$

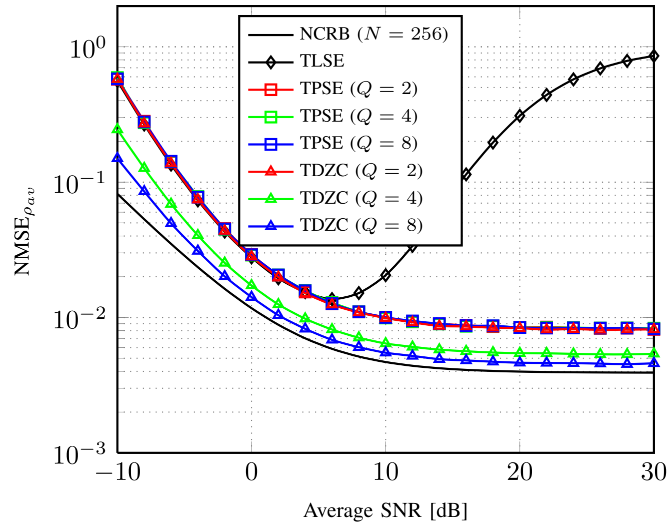

Fig. 3. NMSE of the estimated average SNR in a frequency-flat channel, $N = 256$, $\varepsilon = 5 \cdot 10^{-2}$.

IV. SIMULATION RESULTS

The performance of the TDZCE is evaluated and compared with the performance of the JMLE, TLSE, and TPSE using the Monte-Carlo simulations. OFDM system parameters consider $N = 256$ subcarriers and cyclic prefix length of 32 samples while the channel is assumed to be flat fading. Without loss of generality, we set $\varepsilon = 5 \cdot 10^{-2}$. The evaluation of the accuracy of $\hat{\rho}_{av}$ performance is done in the terms of the normalized MSE (NMSE), which is defined as $\text{NMSE}_{\rho_{av}} = \text{E}\left\{(\hat{\rho}_{av} - \rho_{av})^2\right\}/\rho_{av}^2$.

The performance of the TDZCE is evaluated for different number of the preamble's periodic parts, i.e., $Q = 2, 4$ and 8. Fig. 3 shows the NMSE_{av} as a function of the SNR for the considered estimators. In order to assess the absolute performances of the estimators, they are compared with the Normalized Cramer-Rao bound (NCRB), which is the lower bound for the normalized variance of any unbiased estimator, see [14]. The TLSE performs poorly in the presence of the CFO experiencing the increasing NMSE_{av} in the high SNR region as a consequence of the estimation bias. Furthermore, the TPSE performs closely to the proposed TDZCE and its accuracy is independent of the number of the periodic parts Q. By increasing Q, i.e., by introducing more nulled samples in the preamble, the accuracy of the TDZCE approaches the NCRB. The reason for this is that more samples are used for the average noise power estimation (9) while loaded samples are getting higher power due to the scaling by Q, thus resulting in more accurate estimate in (8). The accuracy of the JMLE estimator is not shown in Fig. 3 due to the clarity because it performs identically to the PS (TDZCE), as indicated in [10].

To compare the computational complexity of TDZCE, JMLE, TLSE, and TPSE we analyze floating point operations per second (FLOPs) required to perform one SNR estimate. Implementing the TLSE, as investigated in [10], requires $5N$ FLOPs and depends on the number of periodic parts Q. Table I shows that the TPSE and JMLE are more complex, requiring $7.5N+4$ [8] and $4N(Q+2)+12Q$ [10] FLOPs, respectively. Moreover, the JMLE complexity depends on the number of periodic parts Q. As shown in (10), the TDZCE requires $2N$ real additions and $2N+2$ real multiplications, which results in $4N+2$ FLOPs and makes the TDZCE less computationally complex compared to the other estimators. Furthermore, the TDZCE depends only on the number of subcarriers N and provides computational savings of approximately 80% (for $M = 4$), 45%, and 20% when used instead of the JMLE, TPSE, and TLSE, respectively.

V. Conclusion

We have proposed a novel low-complexity time-domain SNR estimator based on the property that ZC sequence and its DFT are the time-scaled conjugates of each other, up to a constant factor. The comb-type time-domain structure of the proposed preamble allows us to utilize our previously proposed moment based SNR estimator, which results into its robustness to the CFO when applied to the time-domain samples. Moreover, we have derived statistical properties of the TDZCE, which can be further utilized to improve the performance of the CFO acquisition, channel estimation, or soft-decoding procedures. Simulation results show that the accuracy of the TDZCE approaches the NCRB irrespective of the CFO value and outperforms the considered time-domain SNR estimators. Moreover, the TDZCE offers non-negligible computational savings compared to other methods available in the literature.

Appendix

The λth root ZC sequence of even-numbered length N is defined as [15]

$$C_\lambda(n) = e^{-\frac{j\pi\lambda n^2}{N}}, \quad n = 0, \ldots, N-1. \tag{25}$$

To construct the comb-type structure of the preamble given in (4), every Qth subcarrier is loaded with the appropriate member of the ZC sequence given in (25), where $C_\lambda(mQ) = e^{-\frac{j\pi\lambda m^2 Q^2}{N}}$ for $m = 0, \ldots, N_p - 1$. The IDFT of $C(n)$ can be written as

$$\begin{aligned} c(k) &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} C_\lambda(n) e^{\frac{j2\pi kn}{N}} = \frac{\sqrt{Q}\sigma_s}{\sqrt{N}} \sum_{m=0}^{N_p-1} C_\lambda(mQ) e^{\frac{j2\pi kmQ}{N}} \\ &= \sqrt{Q}\sigma_s c_\lambda(k), \quad k = 0, \ldots, N-1, \end{aligned} \tag{26}$$

where $c_\lambda(k)$ denote the IDFT of the ZC sequence given by

$$c_\lambda(k) = \frac{1}{\sqrt{N}} \sum_{m=0}^{N_p-1} C_\lambda(mQ) e^{\frac{j2\pi kmQ}{N}}, \quad k = 0, \ldots, N-1. \tag{27}$$

By replacing (25) in (27), it yields

$$\begin{aligned} c_\lambda(k) &= \frac{1}{\sqrt{N}} \sum_{m=0}^{N_p-1} e^{-\frac{j\pi\lambda m^2 Q^2}{N}} e^{\frac{j2\pi kmQ}{N}} \\ &= e^{\frac{j\pi\lambda^{-1}k^2}{N}} \frac{1}{\sqrt{N}} \sum_{m=0}^{N_p-1} e^{-\frac{j\pi\lambda\left(mQ-\lambda^{-1}k\right)^2}{N}} \\ &= \frac{C^*_{\lambda^{-1}}(k)}{\sqrt{N}} \sum_{m=0}^{N_p-1} C_\lambda(mQ - \lambda^{-1}k), \quad k = 0, \ldots, N-1, \end{aligned} \tag{28}$$

where $*$ presents the complex conjugation and λ^{-1} is the modular multiplicative inverse of λ, i.e., an integer with the property $\lambda \cdot \lambda^{-1} = 1 \mod N$.

By following the approach given in [11], [16] and having the property that every Qth member of the sequence is different from zero, it is shown that

$$C_\lambda(mQ - \lambda^{-1}k) = \begin{cases} C_\lambda(mQ), & k = mQ \\ 0, & k \neq mQ \end{cases}. \tag{29}$$

Therefore, by applying (29) to (28), we obtain

$$\begin{aligned} c_\lambda(k) &= \begin{cases} C^*_{\lambda^{-1}}(k) \frac{1}{\sqrt{N}} \sum_{m=0}^{N_p-1} C_\lambda(mQ), & k = mQ \\ 0, & k \neq mQ \end{cases} \\ &= \begin{cases} C^*_{\lambda^{-1}}(k) c_\lambda(0), & k = mQ \\ 0, & k \neq mQ \end{cases}. \end{aligned} \tag{30}$$

Finally, by substituting (30) in (26), the time-domain representation of preamble is given by

$$c(k) = \begin{cases} \sqrt{Q}\sigma_s C^*_{\lambda^{-1}}(k) c_\lambda(0), & k = mQ \\ 0, & k \neq mQ \end{cases}. \tag{31}$$

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