

## Exercise 10 in Advanced Methods of Cryptography

Prof. Dr. Rudolf Mathar, Henning Maier, Markus Rothe

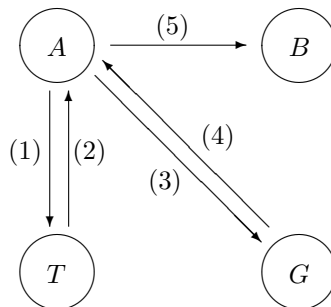
2015-01-16

**Problem 32.** (*Kerberos with ticket granting server*) We introduce a ticket granting server for the simplified Kerberos protocol.

To establish secure *unilateral* authentication from  $A$  (Alice) to  $B$  (Bob) with a trusted authority server  $T$  (Trent) and a ticket granting server  $G$  (Grant), we use the following parameters:

- $k_{AT}$  is a shared key between  $A$  and  $T$
- $k_{AG}$  is a session key for secure communication between  $A$  and  $G$
- $TGT$  is a ticket granting ticket to authenticate  $A$  to  $G$
- $k_{TG}$  is a shared key between  $T$  and  $G$
- $a_{AG}$  is an authenticator between  $A$  and  $G$
- $k_{AB}$  is a session key for secure communication between  $A$  and  $B$
- $k_{BG}$  is a shared key between  $G$  and  $B$
- $ST$  is a service ticket to authenticate  $A$  to  $B$
- $a_{AB}$  is an authenticator between  $A$  and  $B$
- Time stamps  $t_i$  and validity periods  $l_i$ , for  $i = 1, 2, \dots$

The sequence of messages to be exchanged by the protocol is provided in the figure below. Formulate<sup>1</sup> the corresponding protocol and describe it with the parameters as given above.



<sup>1</sup>Feel free to use textbooks, www, etc.

**Problem 33.** (*zero-knowledge factorization*) James Bond (JB) wants to prove to the British secret service (MI5) that he knows the factorization of a composite number  $n$  without revealing the factors. These factors are two distinct primes  $p$  and  $q$  fulfilling the congruences  $p, q \equiv 3 \pmod{4}$ . JB suggests the following protocol:

- (i) The MI5 chooses an arbitrary quadratic residue  $y$  modulo  $n$ , and sends  $y$  to JB.
- (ii) JB computes the square root  $x$  of  $y$ , and sends  $x$  to the MI5.
- (iii) The MI5 checks whether  $x^2 \equiv y \pmod{n}$ .

These steps are repeated 20 times. If JB can compute the square roots modulo  $n$  in all 20 attempts, the MI5 believes him.

- a) Show that the MI5 can factor  $n$  with very high probability.
- b) Does this protocol satisfy the requirements of a zero-knowledge protocol?
- c) Is a third party able to derive useful information about the factorization of  $n$  by intercepting the communication between JB and the MI5?

**Problem 34.** (*Feige-Fiat-Shamir-signature*) Zero-knowledge-protocols can also be used to construct signature schemes. Construct a signature scheme from the Feige-Fiat-Shamir identification protocol by replacing the challenge  $(b_1, \dots, b_k)$  with a hash value  $h(m, x)$ . Specify the signing and the verification algorithm.