Homework 5 in Optimization in Engineering

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Exercise 1. (minimum points) Let $f\colon \mathcal{C} \to \mathbb{R}$ denote a convex function defined on the convex set $\mathcal{C}$. A (global) minimum of f is an $\boldsymbol{x}^* \in \mathcal{C}$ with $f(\boldsymbol{x}^*) \leq f(\boldsymbol{x})$ for all $\boldsymbol{x} \in \mathcal{C}$. Show that the set $\mathcal{S}$ of all minimum points of f is convex.

Exercise 2. (monotone mappings) A function $\psi\colon \mathbb{R}^n \to \mathbb{R}^n$ is called *monotone* if for all $\boldsymbol{x}$, $\boldsymbol{y} \in \mathbb{R}^n$,

$$(\psi(\boldsymbol{x}) - \psi(\boldsymbol{y}))^T(\boldsymbol{x} - \boldsymbol{y}) \geq 0.$$

Suppose $f\colon \mathbb{R}^n \to \mathbb{R}$ is a differentiable convex function. Show that its gradient ∇f is monotone.

Exercise 3. (running average) Suppose $f : \mathbb{R} \to \mathbb{R}$ is a differentiable convex function. Show that its *running average* F, defined as

$$F(x) = \frac{1}{x}\int_0^x f(t)\,\mathrm{d}t, \quad x > 0,$$

is convex.

Hint: For a twice differentiable function F defined on a real interval, convexity is equivalent to $F''(x) \geq 0$ for all x. This is the one-dimensional formulation of the second order condition for convexity.