Homework 8 in Optimization in Engineering

Prof. Dr. Anke Schmeink, Michael Reyer, Alper Tokel

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Exercise 1. (Geometric Programming) Let $p(\boldsymbol{x})$ and $q(\boldsymbol{x})$ be posynomials defined as

$$p(\boldsymbol{x}) = x_1^2 x_2 + \frac{1}{x_1 x_2} \quad \text{and} \quad q(\boldsymbol{x}) = \frac{x_1^2}{x_2} + \frac{x_2^2}{x_1},$$

and the monomial $r(\boldsymbol{x})$ is defined as

$$r(\boldsymbol{x}) = 2x_1 x_2.$$

Express the following problems as geometric programming problems in $\mathbb{R}^2_{>0}$ and find the optimal solutions by means of cvx.

(a) Minimize $\max\{p(\boldsymbol{x}), q(\boldsymbol{x})\}$.

(b) Minimize $\frac{p(\boldsymbol{x})}{r(\boldsymbol{x})-q(\boldsymbol{x})}$ subject to $r(\boldsymbol{x}) > q(\boldsymbol{x})$.

Remark: To solve geometric programming problems in monomial and posynomial form in cvx, the **cvx_begin gp** command must be used.

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Exercise 2. (Optimal transmitter power allocation) Consider a wireless network as discussed in the lecture with m users/transmitters and n receivers. The signal-to-interference ratio of user i is

$$SIR_i = \frac{h_{ii}p_i}{\sum_{j \neq i} h_{ij}p_j + \sigma_i^2}$$

where p_i is the transmit power of user i, h_{ij} is the channel gain from user j to the home receiver of user i, and σ_i^2 is the noise power of receiver i. In this exercise we consider a network with 4 users and 4 base stations. The channel gain matrix $\boldsymbol{H}$ is given as

$$\boldsymbol{H} = (h_{ij}) = \frac{1}{100}\begin{bmatrix} 37 & 2 & 1 & 6 \\ 10 & 30 & 3 & 6 \\ 1 & 14 & 354 & 3 \\ 10 & 8 & 6 & 171 \end{bmatrix}$$

It is assumed that $\sigma_i^2 = 1$ for all users.

Formulate the following optimization problem as a geometric programming problem and solve it using cvx.

$$\begin{aligned} &\text{maximize} \ \min_{1 \leq i \leq 4} \text{SIR}_i \\ &\text{subject to} \ 0 \leq p_i \leq 30, \quad i = 1, \ldots, 4. \end{aligned}$$

Remark: To solve geometric programming problems in monomial and posynomial form in cvx, the **cvx_begin gp** command must be used.