Homework 9 in Optimization in Engineering

Prof. Dr. Anke Schmeink, Michael Reyer, Alper Tokel

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Exercise 1. (One-dimensional trust region problem) Consider the one-dimensional, real-valued trust region problem.

$$\begin{array}{rl} \text{minimize} & ax^2 + 2bx \\ \text{subject to} & x^2 \leq 1. \end{array}$$

(a) Determine all pairs (a, b) for which the problem is non-convex.

In the following the problem shall be non-convex.

(b) Calculate the dual function $L_D(\lambda)$

(c) Give the optimal parameter λ^* which maximizes L_D and the corresponding value d^*.

(d) Show that the optimal value of the primal problem p^* equals d^*.

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Exercise 2. (Geometric interpretation of duality) Consider the optimization problem

$$\begin{array}{ll} \text{minimize} & f_0(x) \\ \text{subject to} & f_1(x) \leq 0 \end{array}$$

with $f_0, f_1 : \mathbb{R} \to \mathbb{R}$, the sets

$$\begin{aligned} \mathcal{G} &= \{(u,t) \mid \exists\, x \in \mathcal{D},\ f_0(x) = t,\ f_1(x) = u\} \text{ and} \\ \mathcal{A} &= \{(u,t) \mid \exists\, x \in \mathcal{D},\ f_0(x) \leq t,\ f_1(x) \leq u\} \end{aligned}$$

and the following realizations of the optimization problem.

(a) $f_0(x) = x,\ f_1(x) = x^2 - 1$

(b) $f_0(x) = x,\ f_1(x) = x^2$

(c) $f_0(x) = x,\ f_1(x) = |x|$

(d) $f_0(x) = x,\ f_1(x) = \Gamma(x)$, with

$$\Gamma(x) = \begin{cases} -x-2 & , x < -1 \\ x & , -1 \leq x \leq 1 \\ -x+2 & , x > 1 \end{cases}$$

(e) $f_0(x) = x^3,\ f_1(x) = -x + 1$

(f) $f_0(x) = x^3,\ f_1(x) = -x + 1$ and additional constraint $x \geq 0$

What is the geometric interpretation of the given sets?

For all those realizations:

- formulate the dual problem,
- solve both the primal and the dual problem and
- answer the following three questions:
  - Is the problem convex?
  - Is Slater's constraint qualification satisfied?
  - Does strong duality hold?