Capacity and power control in spread spectrum macro-diversity radio networks revisited

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Outline

1. The macro-diversity model
2. Feasibility results compared
3. Discussion/outlook
Macro-diversity

- Macro-diversity [1]:
  - cellular structure is removed
  - each transmitter is jointly decoded by all receivers (RX "cooperation")
  - equivalently, ‘one cell’ with a distributed antenna array

- Macro-diversity can mitigate shadow fading[2] and increase capacity

- For $N$-transmitter, $K$-receiver system, $i$’s QoS given by:
  \[
  \frac{P_i h_{i,1}}{Y_{i,1} + \sigma_1} + \ldots + \frac{P_i h_{i,K}}{Y_{i,K} + \sigma_K}
  \]

  with $Y_{i,k} = \sum_{n \neq i} P_n h_{n,k}$
  $P_n$: power from transmitter $n$
  $h_{n,k}$: channel gain from transmitter $n$ to receiver $k$
Two fundamental questions

- Each terminal “aims” for certain level of QoS, $\alpha_i$
- With many terminals present, interference to a terminal grows with the power emitted by the others.
- Even without power limits, it is unclear that each terminal can achieve its desired QoS.
- Two fundamental questions:
  - Are the QoS targets feasible (achievable)?
    $\iff$ CRITICAL for admission control!
  - If yes, which power vector achieves the QoS targets?
Main result

Fact

The vector $\alpha$ of QoS targets is feasible, if for each transmitter $i$ at each receiver $k$,

$$
\sum_{n=1}^{N} \alpha_n g_{n,k} < 1
$$

where $g_{n,k} = h_{n,k}/\sum_k h_{n,k}$. The power vector that produces $\alpha$ can be found by successive approximations, starting from arbitrary power levels.

- Interpretation
  - Greatest weighted sum of $N - 1$ QoS targets must be $< 1$
  - The weights are relative channel gains.
  - At most $NK$ such simple sums need to be checked
Methodology: Fixed-point theory

- Power adjustment process $\Rightarrow$ a transformation $T$ that takes a power vector $p$ and “converts” it into a new one, $T(p)$.
- A limit of the process is a vector s.t. $p^* = T(p^*)$; that is, a “fixed-point” of $T$

**Fact**

*(Banach’s)* If $T: S \rightarrow S$ is a contraction in $S \subset \mathbb{R}^M$ (that is, $\exists r \in [0, 1)$ such that $\forall x, y \in S, \| T(x) - T(y) \| \leq r \| x - y \|$) then $T$ has a unique fixed-point, that can be found by successive approximation, irrespective of the starting point [3]

- We identify conditions under which the power-adjustment transformation is a contraction.
Methodology: key steps

- We replace
  - each $Y_{i,k}(P)$ with $\hat{Y}_i := \max_k \{ Y_{i,k} \}$ and
  - each $\sigma_k$ with $\hat{\sigma} := \max_k \{ \sigma_k \}$.

- Then, the power adjustment takes the simple form

$$
(h_i/\alpha_i)P_{i+1} = \hat{Y}_i(P^t) + \hat{\sigma}
$$

- We prove that $\hat{Y}_i := \max_k \{ Y_{i,k} \} \equiv \| Y_i(P) \|$ defines a “norm” on $P$. This allows us to invoke the “reverse” triangle inequality, which eventually leads to the result.
(Hanly, 1996 [1]) provides the condition

\[ \sum_{n=1}^{N} \alpha_n < K \]

Formula derived under certain simplifying assumptions:
- A TX contributes to own interference
- all TX’s can be “heard” by all RX’s
- non-overcrowding

Under certain practical situations condition is counter-intuitive:
- If there are 2 TX near each RX, it must be “better”, than if all TX’s congregate near same receiver
- In latter case, system should behave like a one-RX system
- But formula is insensitive to channel gains: cannot adapt!
Special symmetric scenario

- Our condition is most similar to original when $h_{i,k} \approx h_{i,m}$ for all $i, k, m$, in which case $g_{i,k} \approx 1/K$
- Example: TX along a road; the axis of the 2 symmetrically placed RX is perpendicular to road
- Under this symmetry (and with $\alpha_N \leq \alpha_n \forall n$ for convenience) our condition simplifies to

$$\sum_{n=1}^{N-1} \alpha_n < K$$

- Smallest $\alpha$ is left out of sum $\implies$ our condition is less conservative than original
Partial symmetry: one receiver “too far”

- If $K = 3$ and $h_{i,k} \approx h_{i,m}$ for all $i, k, m$, $g_{i,k} \approx 1/3$ and our condition becomes $\sum_{n=1}^{N-1} \alpha_n < 3$

- But suppose that $h_{i,1} \approx h_{i,2}$ but $h_{i,3} \approx 0$ (3rd receiver is “too far”), then $g_{i,3} \approx 0$ and $g_{i,1} \approx g_{i,2} \approx 1/2$

- Thus our condition leads to $\sum_{n=1}^{N-1} \alpha_n < 2$

- Our condition automatically “adapts”, whereas original remains at $\sum_{n=1}^{N} \alpha_n < 3$

- Original can over-estimate capacity if applied when some RX’s are “out of range” (because under this situation — of practical interest — some assumptions underlying the original are not satisfied)
Symmetric 3TX, 2RX scenario

- 3 TX “equidistant” from 2 RX
- Original $\rightarrow$ darker pyramid
- Ours ADDs grayish triangle
- If a 3rd RX cannot “hear” TX’s, original overestimates region to outer pyramid
Asymmetric 3TX, 2RX scenario: our region

- 3 TX, 2 RX
- relative gains to RX-1: 2/3, 1/3, 1/2
Asymmetric 3TX, 2RX: ours vs original

- 3 TX, 2 RX with relative gains to RX-1: 2/3, 1/3, 1/2
- original yields region (up to yellow volume) that neither contains nor is contained by ours
Recapitulation

- With macro-diversity receivers “cooperate” in decoding each TX
- Scheme can mitigate shadow fading and increase capacity
- Original feasibility formula may overestimate capacity under certain practical situations (e.g. a given TX is in a range of only a few RX)
- On the foundation of Banach’ fixed-point theory, a new formula has been derived that,
  - is only slightly more complex than original,
  - adjusts itself — through a dependence on relative channel gains – to non-uniform geographical distributions of TX
  - leads to a practical admission-control algorithm (see paper)
- Analysis has been extended to other practical schemes, and to a generalised multi-receiver radio network
Generalised multi-receiver radio network

- Analysis extended to a generalised radio network
- \( i \)'s QoS requirement given by

\[
\mathcal{D}_i \left( \frac{P_i h_{i,1}}{\mathcal{Y}_{i,1}(P) + \sigma_1}, \ldots, \frac{P_i h_{i,K}}{\mathcal{Y}_{i,K}(P) + \sigma_K} \right) \geq \alpha_i
\]

- \( \mathcal{D}_i \) and \( \mathcal{Y}_{i,k} \) are general functions obeying certain simple properties (monotonicity, homogeneity, etc)

- For macro-diversity
  - \( \mathcal{Y}_{i,k}(P) = \sum_{n \neq i} P_n h_{n,k} \)
  - \( \mathcal{D}_i(x) = \mathcal{D}(x) = x_1 + \cdots + x_K \) (same function works for all \( i \))

- Feasibility results obtained for multiple-connection reception and all other scenarios of (Yates, 1995) ([4])

- See IEEE-WCNC, 5-8 April 2009, Budapest
Questions?
**Norms I**

Let $V$ be a vector space (see [5, pp. 11-12] for definition).

**Definition**
A function $f: V \to \mathbb{R}$ is called a *semi-norm* on $V$, if it satisfies:

1. $f(v) \geq 0$ for all $v \in V$ (non-negativity)
2. $f(\lambda v) = |\lambda| \cdot f(v)$ for all $v \in V$ and all $\lambda \in \mathbb{R}$ (homogeneity)
3. $f(v + w) \leq f(v) + f(w)$ for all $v, w \in V$ (*triangle ineq.*)

**Definition**
If $f$ also satisfies $f(v) = 0 \iff v = \theta$ (where $\theta$ is the zero element of $V$), then $f$ is called a *norm* and $f(v)$ is denoted as $\|v\|$.
The Hölder norm with parameter $p \geq 1$ ("$p$-norm") is denoted as $\| \cdot \|_p$ and defined for $x \in \mathbb{R}^N$ as $\|x\|_p = (|x_1|^p + \cdots + |x_N|^p)^{\frac{1}{p}}$.

With $p = 2$, the Hölder norm becomes the familiar Euclidean norm. Also, $\lim_{p \to \infty} \|x\|_p = \max(|x_1|, \cdots, |x_N|)$, thus:

For $x \in \mathbb{R}^N$, the infinity or "max" norm is defined by $\|x\|_\infty := \max(|x_1|, \cdots, |x_N|)$. 
Banach fixed-point theorem

**Definition**

A map $T$ from a normed space $(V, \| \cdot \|)$ into itself is a **contraction** if there exists $r \in [0, 1)$ such that for all $x, y \in V$, 

$$\| T(x) - T(y) \| \leq r \| x - y \|$$

**Theorem**

*(Banach’ Contraction Mapping Principle)* If $T$ is a contraction mapping on $V$ there is a unique $x^* \in V$ such that $x^* = T(x^*)$. Moreover, $x^*$ can be obtained by successive approximation, starting from an arbitrary initial $x_0 \in V$. [3]
For Further Reading I


For Further Reading II

