Generalised link-layer adaptation with higher-layer criteria for energy-constrained and energy-sufficient data terminals

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Executive Overview

- **Link-layer** parameters (modulation, packet size, coding, etc) should be (adaptively) **optimised**
- Typical approach: choose modulation to **maximise spectral efficiency** (bps/Hertz) with bit error rate (BER) constraint
- For packet communication, **higher-layer criteria** are better
- We find the **link-layer configuration** for maximal “goodput”
- Limited and unlimited energy supplies studied separately

- **the key**: a tangent line from (0,0) to the scaled packet-success rate function (PSRF) graph (PSR = 1 minus PER)
- the **steeper** the tangent (greater slope) the **better** the configuration
- true whenever **PSRF** is an “S-curve”
Idealised packetised communication system

- TX makes $L$-bit packets including $C$ error-detection bits ($L - C$ information bits)
- Each packet transmitted symbol by symbol (e.g., M-QAM)
- $W$-bandwidth flat-fading channel adds white noise
- Received packet goes through ideal error detector (CRC)
- RX sends positive or negative acknowledgement (ACK/NACK) over idealised feedback channel
- TX re-sends packet until it gets the corresponding ACK
Link configuration criteria

- Link-layer configuration: (adaptively) choose modulation, bits per symbol, packet length, code length, power

- Possible optimisation criteria:
  - **Spectral efficiency**: maximise bits/second/Hertz with bit error rate constraint (Webb, 1995 [1]); (Chung & Goldsmith, 2001 [2])
  - **“Goodput”**: maximise total information bits transferred over a period of interest, e.g., bits per second, or bits per Joule (Goldsmith, Goodman, et al., 2006 [3]); present work
  - network utility maximisation (NUM): maximise an index of network performance (e.g., sum of each link performance) with average power constraint (O’Neill & Goldsmith, 2008 [4])
Goodput-optimal link configuration

- (Goldsmith, Goodman, et al., 2006 [3]) proposes it for
  - single communication link
  - M-QAM modulation
  - error-detecting codes (CRC)

- performance index: (net) throughput (goodput), given by

\[ T = \frac{L - C}{L} b R_s f(b, \gamma_s, L) \]  

- \( L, C \) : packet length, CRC length in bits
- \( b, R_s \) bits per symbol, symbol rate
- \( \gamma_s \) : per symbol signal-to-noise ratio.
- \( f(b, \gamma_s, L) = [1 - P_b(\gamma_s, b)]^{L/b} \) packet-success rate (1 - PER)
- \( P_b(\gamma_s, b) \) symbol-error probability

- Basic idea: choose parameters that maximise \( T \)
Issues with analysis in reference

- [3]'s algebraic approach requires PSRF in explicit formula
- Such formulae valid only under strong assumptions, and/or major simplifications, and for very specific systems
- Expressions barely tractable. Approximation for M-QAM:

\[
T = \frac{L - C}{L} b R_s \left[ 1 - 4(1 - 2^{-b/2}) Q \left( \sqrt{\frac{p}{N_0 R_s} \left( \frac{3}{2^b - 1} \right)} \right) \right]^{L/b}
\]

with \( Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp(-\frac{1}{2} t^2) dt \) \( \Leftarrow \) NO explicit solution!

- Certain technical steps seem controversial:
  - all parameters are treated as continuous (even bits/symbol)
  - derivatives are taken with respect to them
- Solutions are hard to interpret; general lessons elusive
We drop algebra in favour of analytical geometry:

- **for link** parameters, \( a \), & symbol-SNR \( x \), \( F(x; a) \) : packet-success rate. Ex: 
  \[
  F(x; a) = [1 - P_b(x, b)]^{L/b}, \quad a = (L, b)
  \]
- For technical reasons, 
  \[
  f(x; a) := F(x; a) - F(0; a)
  \]
  replaces \( F \)
- Assume the graph of \( f(x; a) \) has the S-shape shown
- S-curves are very general ( "almost" concave, convex, linear, "ramps" etc)
Criteria for data terminal: maximise total number of information bits transferred over period of interest, \( \tau \)
- with unlimited energy, set \( \tau \) as time unit
  \( \Rightarrow \) info bits/second ("goodput") maximisation
- with energy budget \( E \), \( \tau \) is "battery life" (\( E/p \) if power=\( p \))
  \( \Rightarrow \) info bits/Joule maximisation

Transferred info bits in \( \tau \) secs, with PSR \( f(\gamma_s; a) \):

\[
\tau \frac{L-C}{L} bR_s f(\gamma_s; a)
\]  
(2)
Fact

The max no. of transferred info bits with configuration \( \textbf{a} \), energy \( E \), & normalised ch gain \( h \) is \( (hE) S(x^*; \textbf{a})/x^* \) where S-curve \( S(x; \textbf{a}) := ((L - C)/L)bf(x; \textbf{a}), \) & \( x^* \) maximises \( S(x; \textbf{a})/x \)

- with power \( p \), SNR \( x = hp/R_s \), & energy lasts \( \tau = E/p \)
- By (2), the number of transferred info bits in \( \tau \) secs is
  \[
  \frac{E}{p} \frac{L - C}{L} bR_s f \left( \frac{hp}{R_s}; \textbf{a} \right) \equiv hE \frac{L - C}{L} b \left( \frac{hp}{R_s}; \textbf{a} \right) \equiv hE \frac{S(x; \textbf{a})}{x} \tag{3}
  \]
- \( hE \) is fixed; \( \therefore \) the SNR that maximises \( S(x; \textbf{a})/x \) is optimal.
- For a given configuration, \( b(L - C)/L \) is a constant.
  \( \therefore S(x; \textbf{a}) \propto f(x; \textbf{a}), \) & if \( f \) is an S-curve, so is \( S \).
Fact

If $S$ is an $S$-curve, then, (i) $S(x)/x$ has a unique maximum, (ii) found at the tangency point ("genu") of the "tangenu" (unique tangent line from $(0,0)$ to the graph of $S$).

Proof.

See [5]
Most energy-efficient link configuration

Theorem

For each configuration $a_i$, let $S(x; a_i) = ((L - C)/L)bf(x; a_i)$. If $a_j^*$ maximises transferred info bits per Joule, then $S(\cdot; a_j^*)$ has the steepest tangenu among considered configurations.

- By previous Facts:
  (i) terminal maximises $(hE)S(x; a_i)/x$
  (ii) maximiser is $x_i^*$ (at tangency point)

∴ max no. of transferred info bits:
$(hE)S(x_i^*; a_i)/x_i^*$

∴ configuration with greatest ratio $S(x_i^*; a_i)/x_i^*$ (steepest tangenu) is best
The steeper the tangent the better the configuration
Recapitulation

- Previous work recognises the importance of link configuration (modulation, packet size, coding, etc) under higher-layer criteria for packetised communication.
- But it necessitates explicit formulae and controversial technical steps, which limits its applicability.
- Present work is grounded on analytical geometry; it postulates that the PSRF is an S-curve, and from this, it yields a sharp and general result:
  - The steeper the tangent from (0,0) to the (scaled) PSRF graph (an S-curve) the better the configuration.
  - S-curves include most (if not all) PSRF of interest. ∴ result is highly applicable.
  - Battery-fed terminal discussed; similar result for unlimited energy is in paper.
Limitations/Outlook

- Results obtained “off line” can be put in device’s memory, for link re-configuration through simple table look-ups.
- Developing such tables is possible research path.
- “Best effort” (data) traffic assumed. Similar analysis for media traffic (video) is in progress.
- Point-to-point transmission studied. Of interest: to embed analysis in network model, such as [4]’s.
THANK YOU!

QUESTIONS?
Maximising goodput (unlimited energy supply)

Fact

The max goodput with configuration $\mathbf{a}$, power limit $\hat{p}$ & normalised ch-gain $h$ is $h\hat{p}S(x^*;\mathbf{a})/x^*$ where $S$-curve $S(x;\mathbf{a}) := ((L - C)/L)bf(x;\mathbf{a})$, & $x^*$ maximises $S(x;\mathbf{a})/x$

- Unlimited energy $\Rightarrow$ optimal $p = \hat{p}$ (max power)
- $\text{SNR } x = h\hat{p}/R_s \Rightarrow R_s = h\hat{p}/x$
- By (2), the number of transferred info bits over 1 sec is

$$\left(\frac{L - C}{L}\right) bR_sf(\frac{h\hat{p}}{R_s};\mathbf{a}) \equiv h\hat{p}\left(\frac{L - C}{L}\right) b\frac{f(x;\mathbf{a})}{x} \equiv h\hat{p}\frac{S(x;\mathbf{a})}{x} \quad (4)$$

- $h\hat{p}$ is fixed; $\therefore$ the SNR that maximises $S(x;\mathbf{a})/x$ is optimal
- For a given configuration, $b(L - C)/L$ is a constant. Thus, $S(x;\mathbf{a}) \propto f(x;\mathbf{a})$, & if $f$ is an S-curve, so is $S$
- $\therefore$ main theorem also applies under unlimited energy

\[ T = \frac{L - C}{L} bR_s \left[ 1 - P_b(\gamma_s, b) \right]^{L/b} \]

for M-QAM, \( P_b \approx 4(1 - 2^{-b/2})Q \left( \sqrt{\frac{hp}{R_s} \left( \frac{3}{2^{b} - 1} \right)} \right) \)

\( \gamma_s = hp/R_s \): symbol SNR; \( p \): power; \( h \): ch gain over noise

Assume \( C \) held constant (e.g. \( C = 16 \) bits)

Some trade-offs:
- \( L \) increases \((L - C)/L\) but reduces \( \text{PSR}=[1 - P_b(\gamma_s, b)]^{L/b} \)
- \( b \) raises “raw” bps,\( bR_s \), but lowers energy/bit (\& PSR)
- \( R_s \) increases raw bps but reduces \( \gamma_s \) \& hence PSR
- power raises SNR(\& PSR) but lowers “battery life”, if appl.
For Further Reading I


For Further Reading II
