Location Tracking of Mobiles: A Smart Filtering Method and Its Use in Practice

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***Abstract*— Accurate location tracking of mobile stations in cellular radio networks is of tremendous interest for many applications. In this work, we analyze the performance of a speed and location tracking algorithm using data from a field trial. The algorithm fits received signal strengths of surrounding base stations to corresponding predictions. These raw location estimates, in GSM available each 0.48s, are subsequently smoothed by a model-based Kalman filter. An essential ingredient of our method is to find suitable initial parameters.**

The method is tested with measurement data from a field trial by Siemens. Although the field strength prediction method is rather simple, the location algorithm itself yields promising results. Typical average deviations from the true positions were 173.5m for indoor, 117.7m for walking, and 104.9m for driving scenarios. This shows that the method is robust against moderate errors in the prediction model and leads to good results in a real GSM network.

I. Introduction

Location dependent services, but also cell assignment and access control strategies for layered structures, need information about the position and velocity of mobiles.

Various methods have been proposed to estimate the position of mobiles in cellular radio networks analyzing the radio signals from the base stations. This includes angle of arrival, time of arrival and received signal strength measurements. However, all measurements are subject to strong stochastic variations causing random deviations in the position estimation.

Filtering the initial location estimation can help to reduce the location error. In [1] Kalman filtering based on a locally linear model for the mobiles motion was suggested. Various papers extend this work by adapting the underlying motion model or by adding human control factors, like [2]–[4].

The performance of the location methods is, however, mostly evaluated by computer simulation. Only a few papers analyze the performance of location tracking on the basis of real measurement data, like [5] which evaluates data collected during a single call.

In this paper, we use position estimation based on signal strength analysis and successive Kalman filtering as proposed in [1]. Initially, rough location estimates are obtained by fitting the measured field strength pattern of some received base stations to a position with maximum coincidence to the corresponding predicted values. We improve the least squares estimation by adding a global path loss constant d, which models unpredictable attenuation by changing weather conditions, e.g., geometrical obstacles like walls or systematic inaccuracies in the predictions. In a second step, the successive initial guesses are smoothed by Kalman filtering. The Kalman filter is based on a locally linear model of a mobile's motion. The parameters describing the mobile's dynamic are unknown in advance, and are estimated within the filtering procedure. However, even with only a few consecutive measurements, the Kalman filter approach proves to be successful.

The accuracy of the improved algorithm is verified by measurements, which have been taken by Siemens in an existing GSM network in a suburban area. Measurement reports were recorded from numerous mobiles for three different scenarios. The mobile was moving on streets with high (typical driving speed) and low speed (typical walking speed) in the driving and walking scenario respectively. In the indoor scenario the position of the mobile was kept fixed inside a building. The mobiles' true positions were measured via GPS, which allows for creating error statistics. About 10000 measurement samples were recorded altogether. Furthermore, predictions for 17 BTS's supplying the test area are available.

Section II discusses the improved initial position estimation and its accuracy on the measured data set. Section III describes the Kalman filtering, and Section IV discusses the accuracy of the whole location method. Our conclusions are summarized in Section V.

II. Initial Position Estimation

We use the "$\mathrm{RX}_{\mathrm{lev}}$_DOWN" value to determine the field strength of the serving cell and the corresponding "$\mathrm{RX}_{\mathrm{lev}}$_$i$" values, $i = 1, 2, \ldots, 8$ to get the field strengths of the neighboring cells. These values are reported to the network by the mobile every 480ms (cf. [6]).

The reported field strength pattern is compared with the predicted field strength values of the received base stations. The predictions are based on a semi-empirical two-dimensional prediction model with a resolution of 12.5m. The search area

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scenario	drive			walk			indoor		
	mean	std	max	mean	std	max	mean	std	max
LSQ	144.3	119.2	1218.6	166.8	81.4	688.5	365.7	110.9	1279.2
LSQ with $d \in \mathbb{R}$	185.8	184.3	1359.8	145.9	84.8	1359.8	213.4	158.8	1381.6
LSQ with $d \geq \mathbb{R}$	134.9	113.3	1218.6	127.7	54.4	442.2	185.2	96.6	797.7

TABLE I

COMPARISON OF MEAN ERROR, ITS STANDARD DEVIATION AND THE MAXIMUM ERROR FOR VARIOUS INITIAL ESTIMATION METHODS.

is restricted with help of timing advance information, which is known for the serving base station and reported by the mobile together with the field strength. With the serving base stations located at $\mathbf{B}_0$ and a timing advance value $T_a \in \{0, 1, \ldots, 63\}$, the mobile's position can be restricted to the area

$$A = \{\mathbf{x} \mid |\mathbf{x} - \mathbf{B}_0| \leq (T_a + 1 + \Delta)\, 554\mathrm{m}\}. \quad (1)$$

Δ represents the uncertainty of the value T_a. A value of $\Delta = 0.8$ proved to give acceptable results without unnecessary computational overhead.

In [1], a least squares estimation (LSQ) is suggested to find the best approximation of the mobiles position $\hat{\mathbf{y}}(t)$ at time t,

$$\hat{\mathbf{y}}(t) = \arg \min_{\mathbf{x} \in A} \sum_{i=1}^{N} (s_i(\mathbf{x}) - \gamma_i(t))^2, \quad (2)$$

where $s_i(\mathbf{x})$ denotes the predicted field strength of base station i at location $\mathbf{x}$. $\gamma_i(t)$ denotes the measured field strength of station i at time t, and A is the search area.

In computer simulations it turned out that the LSQ approach is rather accurate (cf. [7]). However, the results of the pure LSQ method should be improved for accurate location tracking with real measurement data, as can be seen from Table I. Especially for indoor measurements the error is large because of additional shadowing by walls disregarded by the prediction model. Similar problems may occur for outdoor scenarios, e.g., because of trees, cars and obstacles unknown to the prediction model.

It is reasonable to assume that the signal of each received base station is attenuated by the same constant $d \geq 0$. The least squares approach leads to the optimization problem

$$\hat{\mathbf{y}}(t) = \arg \min_{\mathbf{x} \in A} \left(\min_{d \geq 0} \sum_{i=1}^{N} (s_i(\mathbf{x}) - \gamma_i(t) - d)^2 \right). \quad (3)$$

The solution of the inner optimization problem with respect to d, with $\mathbf{x}$ fixed, is given by

$$d = \max \left\{ 0, \frac{1}{N} \sum_{i=1}^{N} (s_i(\mathbf{x}) - \gamma_i(t)) \right\}. \quad (4)$$

By introducing the global attenuation constant d the average dislocation of the initial estimation $\hat{\mathbf{y}}(t)$ is reduced from 119.2m to 113.3m in the driving scenario, and from 365.7m to 185.2m for indoor location (cp. Table I). Especially the estimation in the indoor scenario is improved by the constant d, as it helps to overcome the problems of the usually unpredictable additional attenuation by walls.

One might argue that omitting the nonnegativity constraint $d \geq 0$, and minimizing over all real $d \in \mathbb{R}$ would even further improve the raw estimates from (3). However, when applying the unconstrained minimizations the derivation between real and estimated positions grew even worse for the presented data, as can be seen from the second row in Table I.

In the following, attenuated LSQ estimation with $d \geq 0$ is used as input for the Kalman filter, as it provides the most accurate results.

III. KALMAN FILTERING

The Kalman filter is based on a locally linear model for the motion of the mobile. A description of the theoretical foundations can be found in [1] and [7]. The Kalman filter gives an optimal recursive estimator of minimal variance.

First, we use the following stochastic model to describe the random nature of the measurements $\mathbf{y}(t_k) = (y_1(t_k), y_2(t_k))$. Let $'$ denote the transpose of a vector or a matrix. Define the four-dimensional stochastic process

$$\mathbf{X}(t) = (X_1(t), X_2(t), V_1(t), V_2(t))', \quad t \in \mathbb{R}.$$

$X_1(t), X_2(t)$ denote the x- and y-coordinate of a mobile's random position, and $V_1(t), V_2(t)$ the x- and y-coordinate of the velocity vector at time t. Observations are taken at discrete time points $t_k = t_0 + \Delta t \cdot k$, $k \in \mathbb{N}_0$. We assume that $\mathbf{X}(t_k)$ satisfies the discrete linear recursion

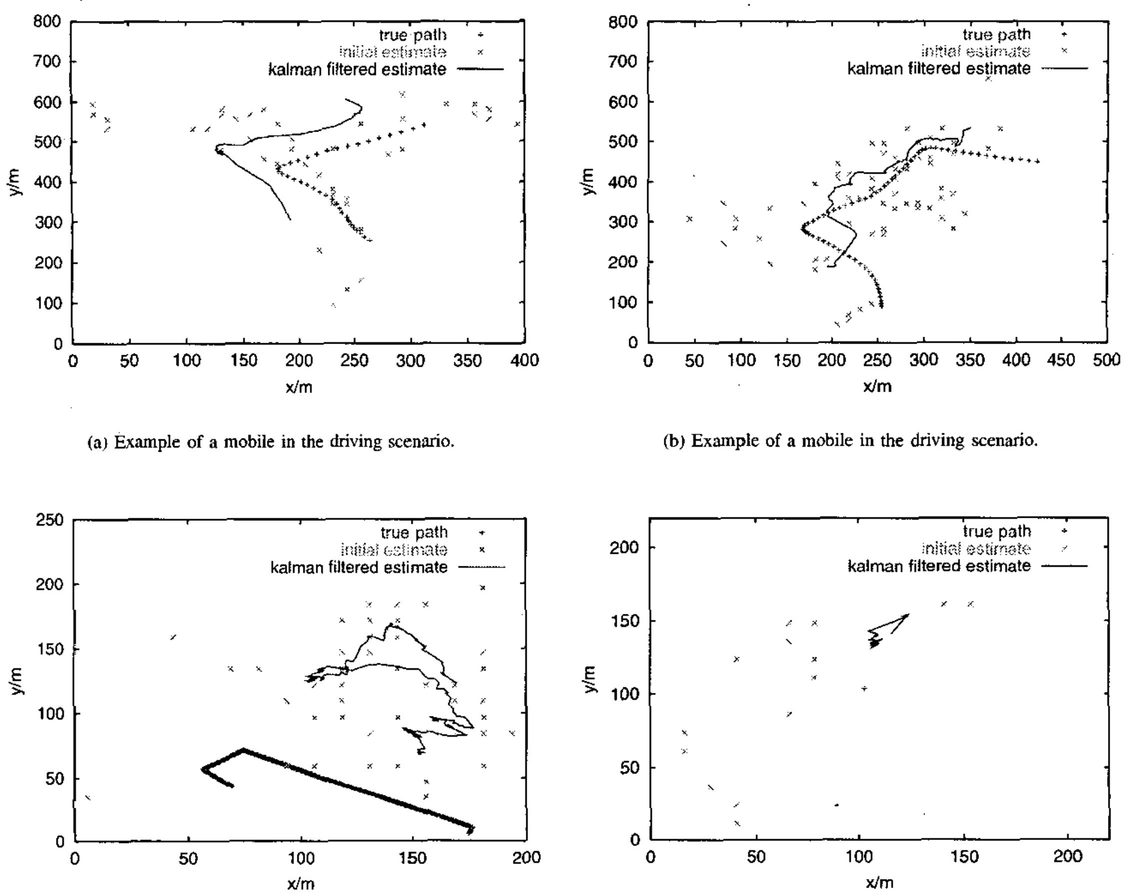
$$\mathbf{X}(t_{k+1}) = \mathbf{\Phi}\, \mathbf{X}(t_k) + \mathbf{\Gamma}\, \mathbf{W}(t_k), \; k \in \mathbb{N}_0, \quad (5)$$

where $\mathbf{\Phi}$ and $\mathbf{\Gamma}$ are the following matrices

$$\mathbf{\Phi} = \begin{pmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{\Gamma} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \Delta t & 0 \\ 0 & \Delta t \end{pmatrix}.$$

$\mathbf{W}(t_k) = (W_1(t_k), W_2(t_k))'$, $k \in \mathbb{N}_0$, are stochastically independent random errors, two-dimensional normally distributed with expectation $\mathbf{0}$ and covariance matrix $\mathbf{Q} = \sigma^2 \mathbf{I}_2$, denoted by $\mathbf{W}(t_k) \sim \mathrm{N}(\mathbf{0}, \mathbf{Q})$. $\mathbf{I}_\ell$ denotes the identity matrix of order ℓ.

$\mathbf{X}(t_k)$ cannot be observed directly and is subject to further inaccuracies by the raw position estimation procedure. To take these effects into account the estimated positions



(a) Example of a mobile in the driving scenario.

(b) Example of a mobile in the driving scenario.

(c) Example of a mobile in the walking scenario.

(d) Example of a mobile in the walking scenario.

Fig. 1. Comparison of the mobiles true path ("+"), the initial position estimations ("×") and the Kalman filtered estimation (solid line).

$(y_1(t_k), y_2(t_k))'$ are modeled by independent additive random errors as

$$\mathbf{Y}(t_k) = \mathbf{M}\,\mathbf{X}(t_k) + \mathbf{U}_k,\ \ k \in \mathbb{N}_0, \tag{6}$$

where $\mathbf{M} = \left(\begin{smallmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{smallmatrix}\right)$ and $\mathbf{U}_k \sim \mathrm{N}(0, \mathbf{R})$.

(5) and (6) form a discrete linear difference equation with white Gaussian noise. The state at time t_k is estimated by the variance minimal conditional expectation of $\mathbf{X}(t_k)$ given previous observations $\tilde{\mathbf{Y}}(t_k) = \big(\mathbf{Y}(t_0), \dots, \mathbf{Y}(t_k)\big)$ as

$$\hat{\mathbf{X}}_k(t_k) = \mathrm{E}\big[\mathbf{X}(t_k) \mid \tilde{\mathbf{Y}}(t_k)\big],$$

and the predicted value at time t_k as

$$\hat{\mathbf{X}}_{k-1}(t_k) = \mathbf{\Phi}\hat{\mathbf{X}}_{k-1}(t_{k-1}). \tag{7}$$

Corresponding covariance matrices are denoted by

$$\mathbf{C}_k(t_k) = \mathrm{Cov}\big[\mathbf{X}(t_k) \mid \tilde{\mathbf{Y}}(t_k)\big],$$

and

$$\mathbf{C}_{k-1}(t_k) = \mathbf{\Phi}\mathbf{C}_{k-1}(t_{k-1})\,\mathbf{\Phi}' + \mathbf{\Gamma}\mathbf{Q}\mathbf{\Gamma}'. \tag{8}$$

Optimal recursive estimators of minimal variance are given by the Kalman-Bucy filter.

The minimum variance estimator of the state at time t_k is given by

$$\hat{\mathbf{X}}_k(t_k) = \hat{\mathbf{X}}_{k-1}(t_k) + \mathbf{K}(t_k)\big(\mathbf{Y}(t_k) - \mathbf{M}\hat{\mathbf{X}}_{k-1}(t_k)\big). \tag{9}$$

Covariance matrices are updated by

$$\mathbf{C}_k(t_k) = \mathbf{C}_{k-1}(t_k) - \mathbf{K}(t_k)\mathbf{M}\mathbf{C}_{k-1}(t_k), \tag{10}$$

where

$$\mathbf{K}(t_k) = \mathbf{C}_{k-1}(t_k)\mathbf{M}'\big(\mathbf{M}\mathbf{C}_{k-1}(t_k)\mathbf{M}' + \mathbf{R}\big)^{-1}. \tag{11}$$

is the Kalman gain.

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scenario		NMR 1	NMR 2	NMR 4	NMR 8	NMR 16	NMR 32	NMR 64	all
drive	mean	134.9	125.1	118.0	112.7	109.1	104.9	100.5	107.0
	max	1218.6	1009.8	951.3	895.8	617.3	352.4	300.7	770.7
walk	mean	127.7	124.0	121.7	120.2	119.0	117.7	117.7	120.7
	max	442.2	374.5	346.9	341.0	343.6	240.2	263.3	348.9
indoor	mean	185.2	182.1	178.6	176.1	174.3	173.5	177.9	173.21
	max	797.7	568.7	430.7	430.2	420.3	430.1	364.8	378.2

TABLE II

ACCURACY ANALYSIS OF THE PRESENTED FILTERING ALGORITHM DEPICTING MEAN AND MAXIMAL LOCATION ERROR IN METERS.

With initial values $\hat{\mathbf{X}}_0(t_0)$ and $\mathbf{C}_0(t_0)$ recursion (9) can be evaluated via (7), (8), (10), and (11). The corresponding algorithm is described below in (12), updated values are denoted by $^+$. Let $\mathbf{y}_k$ denote the actual observed values.

$$\begin{aligned}\bar{\mathbf{C}} &= \mathbf{\Phi}\mathbf{C}\mathbf{\Phi}' + \mathbf{\Gamma}\mathbf{Q}\mathbf{\Gamma}' \\ \mathbf{K} &= \bar{\mathbf{C}}\mathbf{M}'(\mathbf{M}\bar{\mathbf{C}}\mathbf{M}' + \mathbf{R})^{-1} \\ \mathbf{C}^+ &= \bar{\mathbf{C}} - \mathbf{K}\mathbf{M}\bar{\mathbf{C}} \\ \mathbf{X}^+ &= \mathbf{\Phi}\mathbf{X} + \mathbf{K}(\mathbf{y} - \mathbf{M}\mathbf{\Phi}\mathbf{X}) \\ \mathbf{X} &= \mathbf{X}^+ \\ \mathbf{C} &= \mathbf{C}^+\end{aligned} \quad (12)$$

As in [1] we choose $\mathbf{X}_0 = \mathbf{X}(t_0) = (y_1, y_2, 0, 0)$ with (y_1, y_2) being the first initial position estimation at time t_0. The iteration is started with velocity zero. As initial value for $\mathbf{C}_0 = \mathbf{C}(t_0)$ we use

$$\mathbf{C}_0 = \begin{pmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{0} & 30^2 \cdot \mathbf{I}_2 \end{pmatrix}, \quad (13)$$

with $\mathbf{R} = 22000 \cdot \mathbf{I}_2$.

30^2 $(\text{m/sec})^2$ seems to be a reasonable upper bound for the variance of the initial velocity of a mobile. The variance of the deviation between the true and the estimated position was estimated as 22000m^2 for each coordinate. This corresponds to a standard deviation of approximately 150m per measurement, which lies in between the measured values for the walking and driving scenario and the value for the indoor scenario (cp. Table I).

The uncertainty in the initial values causes large estimation errors for the first few points of the mobiles track. This reduces the accuracy of the whole method, especially if only a few consecutive measurements are available. However, the vector $\mathbf{X}$ and the matrix $\mathbf{C}$ are updated in each step of the recursion, and improve with the number of measurements. Therefor we propose the following method to improve the accuracy.

First, the recursive filter algorithm is employed with the above described a-priori initial values. After a number of measurements the process is stopped and Kalman filtering is applied with the current estimated parameters, but with the data in reversed order, i.e., $\Delta t < 0$. After reaching the initial point again, an improved estimation of $\mathbf{C}_0$ and $\mathbf{X}_0$ is achieved. Finally, the filtering algorithm is started again with these initial values to generate the ultimate location estimation sequence.

This method works well for both simulations and the measured data set as the following analysis shows.

IV. RESULTS

Figures 1(a), 1(b), 1(c) and 1(d) show exemplarily how the Kalman filter works in various situations. Especially the driving scenario (Figures 1(a) and 1(b)) indicates, that the filter properly models the dynamics of the mobile. In Figure 1(a), the estimated track is shifted to the top left corner. Similar effects can be observed in other examples. This might be explained by systematic errors in the field strength predictions.

An important question is the influence of the number of available raw estimates on the accuracy of the final Kalman filtered track. For this purpose, the sequence of measurement reports is split into subsequent sample blocks of length l, with $l = 1, 2, 4, 16, 32, 64$. The raw estimates are computed according to (3), and the Kalman filter is applied to the resulting raw sequence of length l.

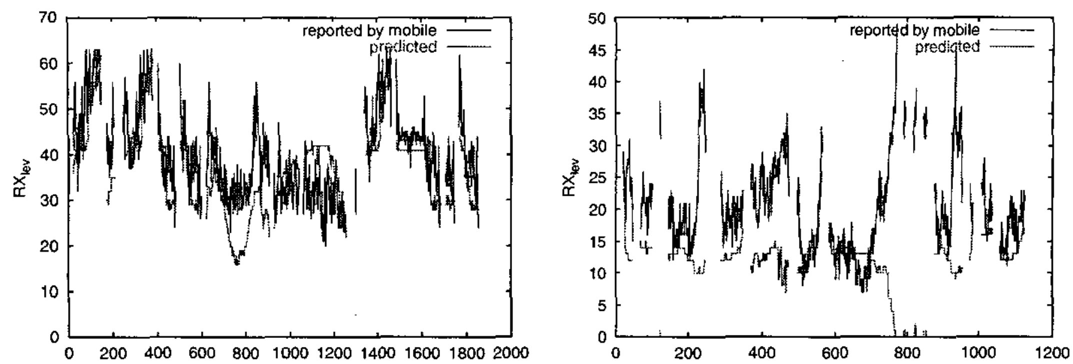
The results of this study are represented in Table II. Block lengths are referred to by NMR 1, ..., NMR 64. "All" means that the final estimate is based on all available measurement data without splitting into blocks of fixed length.

Basically, accuracy increases as the blocks grow longer. The number of blocks for computing the averages in Table II, however, varies for the different scenarios. This is because some of the basic measurement reports contained less than 32 samples, so that the data could not be included in the evaluation with $l = 32$ and $l = 64$. The averages are hence subject to higher variation which may explain the unexpected large value 177.9 for the indoor scenario with NMR 64.

A major source of deviation between the true and estimated positions is the relatively simple field strength prediction method in the present study. In Figure 2 the predictions for two base stations are compared to the measured field strength values in the driving scenario. Clearly there are large deviations which suggest the use of a more accurate, preferably three-dimensional prediction model. This is particularly necessary to reduce the prediction error for low signal strength.

V. CONCLUSIONS

The results of the presented method are quite satisfying for the purpose of location based services and hot spot detection



(a) Base station 1.

(b) Base station 2.

Fig. 2. Comparison of the RX_{lev} values reported by the mobiles with predicted values

in cellular radio networks. The optimally chosen global attenuation constant d and the subsequent Kalman filtering with the improved initial values lead to major improvements. Especially the error in indoor scenarios decreases with the introduction of the additional constant. In summary, the presented method has an accuracy in the range of 100m for the investigated outdoor, and 175m for the indoor scenario. This is much better than location estimation based only on the cell ID and the timing advance to the serving station. Further improvements are to be expected by more accurate prediction models.

The used Kalman filter does significantly improve the position estimation. The real world examples in this paper show that the predicted behavior from previous computer simulations is accomplished. Even with a rough field strength prediction and only a few measurements reliable location tracking is possible.

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