Fifty-second Annual Allerton Conference
Allerton House, UIUC, Illinois, USA
October 1 - 3, 2014

Capacity Region of the Reciprocal Deterministic 3-Way Channel via Δ-Y Transformation

Henning Maier*, Anas Chaaban†, Rudolf Mathar*, Aydin Sezgin†
*Institute for Theoretical Information Technology, RWTH Aachen University, 52056 Aachen, Germany
†Institute of Digital Communication Systems, Ruhr-Universität Bochum, 44780 Bochum, Germany
Email: {maier, mathar}@ti.rwth-aachen.de, {anas.chaaban, aydin.sezgin}@rub.de

***Abstract*—A linear shift deterministic 3-way channel with reciprocal channel gains is considered in this work. The 3-way channel is an extension of the 2-way channel introduced by Shannon. Here, a number of six messages is exchanged, one message from each user to the two other users. Each user operates in a full-duplex mode. We derive the capacity region of this 3-way channel w.r.t. the linear shift deterministic channel model. To this end, first, an outer bound is derived using cut-set and genie-aided upper bounds. Then, it is noted that the outer bound bears a resemblance to the capacity region of a related linear shift deterministic Y-channel. Utilizing a Δ-Y transformation, the optimal scheme for the related Y-channel is modified in a way that achieves the outer bound of the 3-way channel. Mainly, the capacity achieving communication schemes are based on multi-way relaying by signal alignment, interference neutralization and backward decoding. We also consider a scheme which is based on interference alignment only. It turns out, that for the symmetric linear deterministic 3-way channel, this scheme is optimal. Thus, backward decoding and the resulting delays are avoided.**

I. Introduction

The linear deterministic channel model (LDCM) is a conceptual model to describe the influence of interference in a multi-user network as introduced in [1]. While noise is de-emphasized, the LDCM focuses on the impact of signal strength and superposition of multiple interfering signals. Capacity-achieving communication schemes derived in terms of the LDCM are useful tools to approximate the corresponding Gaussian channel capacity within a limited number of bits [2] at high signal-to-noise ratio. Furthermore, the principle of interference alignment (IA) [3] is also applicable for linear deterministic multi-user channels, as, e. g., shown in [4], [5]. A solid basis of works treating several different multi-user networking problems in terms of the linear deterministic channel model is already available, e. g., the sum capacity of the 2-user X-channel is treated in [5], and the sum-capacity of the K-user symmetric interference channel in [4].

The main focus of this paper is to characterize the capacity region of the linear deterministic 3-way channel. The 3-way channel is an extension of the two-way channel [6], a common setup in wireless communications where two

The work of H. Maier and R. Mathar is supported by the DFG within the project PACIA - Ma 1184/15-3 of the DFG program COIN and furthermore by the UMIC Research Centre, RWTH Aachen University. The work of A. Chaaban and A. Sezgin is supported by the DFG under grant SE 1697/5.

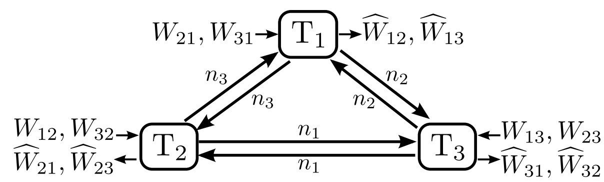


Fig. 1. The reciprocal linear deterministic 3-way channel (or Δ-channel) with three transceivers $\mathrm{T}_1, \mathrm{T}_2$ and T_3 has six independent messages W_{ji} transmitted and six corresponding estimated messages $\widehat{W}_{ji}$ received by the nodes, for $i \neq j \in \mathcal{K}$. The channel gains are parametrized by $n_j \in \mathbb{N}$, $j \in \mathcal{K}$.

users communicate with each other simultaneously over the same channel. This is clearly a very natural communication situation with 2 users. In the light of the LDCM, the capacity region of the two-way multiple access channel, the two-way broadcast channel and the two-way interference channel is discussed in [7].

The 3-way channel with perfect full-duplex operation is a basic extension of the two-way channel, in which all three users communicate with each other simultaneously. The reciprocal linear deterministic 3-way channel is depicted in Fig. 1. There is a total number of six messages involved. The sum-rate of the Gaussian 3-way channel has already been approximated within 2 bits in [8], recently. Note that a related conferencing 3-way channel is discussed in [9] for a finite-field channel model. But in that case, each transmitter has only one message that is dedicated for both receivers, i. e., three messages in total. The deterministic 3-way channel is similar to the deterministic 3-user Y-channel in [10], [11], [12], but it has no intermediate relay for the purpose of two-way relaying and signal alignment (SA) [13], as also discussed for MIMO channels in [14] and [15].

In our work presented here, we consider the capacity region of a linear deterministic 3-way channel (or Δ-channel) as depicted in Fig. 1. For the 3-way channel, we first provide genie-aided upper bounds for the capacity region. Achieving this capacity region relies upon SA, interference neutralization (IN) [16] and backward decoding [17].

Our main tool is a Δ-Y transformation of the 3-way channel to an (extended) Y-channel, motivated by elementary electrical circuit theory [18], [19]. Furthermore, we integrate a capacity-achieving scheme for a symmetric 3-way channel that is purely based on signal-scale interference alignment (IA) as in [3]–[5]. For that case we also avoid backward decoding.

Organization. The reciprocal linear deterministic 3-way channel is introduced in Sect. II-A. Cut-set and genie-aided upper bounds on the capacity region are derived in Sect. II-B. We briefly describe the closely related Y-channel and its capacity region in Sect. III. In Sect. IV, the achievability of the capacity region of the 3-way channel is derived by means of the aforementioned Δ-Y transformation. Furthermore, we discuss in Sect. V to what extent a purely IA based scheme can be used to achieve the capacity region. We conclude in Sect. VI.

II. 3-Way Channel

A. System Model: 3-Way Channel

In the 3-way channel, a user T_i is a combined full-duplex transmitter Tx_i and receiver Rx_i. We consider six independent messages W_{ji} dedicated to be conveyed from T_i to T_j with $W_{ji} \in \mathcal{W}_{ji} = \{1, \dots, 2^{nR_{ji}}\}$, $R_{ji} \in \mathbb{R}^+$, for distinct $i, j \in \mathcal{K} := \{1, 2, 3\}$. The vector of all messages is denoted by:

$$\boldsymbol{w} = (W_{12}, W_{21}, W_{13}, W_{31}, W_{23}, W_{32}). \tag{1}$$

The rate tuple $\boldsymbol{R}$ and the total sum-rate R_Σ are defined for rates $R_{ji} \in \mathbb{R}^+$ between Tx_i and Rx_j by:

$$\boldsymbol{R} = (R_{12}, R_{21}, R_{13}, R_{31}, R_{23}, R_{32}), \tag{2}$$

$$R_\Sigma = R_{12} + R_{21} + R_{13} + R_{31} + R_{23} + R_{32}. \tag{3}$$

T_j encodes its messages into a codeword[1] $\boldsymbol{x}_j^N$. The l-th symbol of $\boldsymbol{x}_j^N$ is an element of an alphabet $\mathcal{X}$ encoded as $\boldsymbol{x}_j(l) = f_{j,l}(W_{ij}, W_{kj}, \boldsymbol{y}_j^{l-1})$ for distinct $i, j, k \in \mathcal{K}$. Therein, $\boldsymbol{y}_j^{l-1}$ are all received symbols at T_j until time instant $l-1$ with encoding function $f_{j,l}(\cdot)$. A receiving T_i decodes $(\hat{W}_{ij}, \hat{W}_{ik}) = g_i(\boldsymbol{y}_i^N, W_{ij}, W_{ik})$ with the decoding function $g_i(\cdot)$. An error occurs if $(\hat{W}_{ij}, \hat{W}_{ik}) \neq (W_{ij}, W_{ik})$. The collection of messages, encoders, and decoders defines a code for the 3-way channel. Furthermore, rate tuple $\boldsymbol{R}$ is called *achievable* if there is a sequence of codes such that the average error probability ϵ_N becomes arbitrarily small by increasing N. The set of all achievable rate tuples is the capacity region $\mathcal{C}_\Delta$.

In the LDCM, the physical channel between T_i and T_j is modelled by $n_{ji} \in \mathbb{N}$ bit pipes, and the transmitted symbols $\boldsymbol{x}_j(l)$ are bit-vectors in $\mathcal{X} = \mathbb{F}_2^q$ with $q = \max_{i \neq j \in \mathcal{K}}\{n_{ji}\}$. The received signals $\boldsymbol{y}_i$ at receivers Rx_i, $i \in \mathcal{K}$ are deterministic functions of the transmitted signals for distinct $i, j, k \in \mathcal{K}$:

$$\boldsymbol{y}_i = \boldsymbol{S}^{q-n_{ij}} \boldsymbol{x}_j \oplus \boldsymbol{S}^{q-n_{ik}} \boldsymbol{x}_k, \tag{4}$$

where $\boldsymbol{S}$ is a $q \times q$ lower shift matrix, having unit entries on the lower side-diagonal. The effect of noise is mimicked by clipping linearly shifted symbols. Note that loop-back self-interference is entirely cancelled from (4) due to the perfect full-duplex operation.

In a wireless channel, it is valid to assume reciprocity for the bidirectional links such that we may use the following parametrization $n_{ij} = n_{ji} =: n_k$ holds for distinct $i, j, k \in \mathcal{K}$. We also assume:

$$n_3 \geq n_2 \geq n_1, \tag{5}$$

as an ordering of parameters w.l.o.g., and obtain $n_3 = q$. We denote this reciprocal 3-way channel by $\mathrm{D3C}(n_1, n_2, n_3)$.

B. Upper Bounds: Linear Deterministic 3-Way Channel

Cut-set upper bounds: The cut-set bounds of broadcast and multiple-access channels as in [1] state that users T_i can not receive more bits than the number of incoming bit-levels and they cannot transmit more bits than the number of outgoing bit-levels available:

$$R_{ij} + R_{ik} \leq \max\{n_k, n_j\}, \tag{6}$$

$$R_{ji} + R_{ki} \leq \max\{n_k, n_j\}, \tag{7}$$

for distinct $i, j, k \in \mathcal{K}$. These cut-set bounds already provide a loose upper bound $\overline{\mathcal{C}}_{\Delta,\text{cut-set}}$ on the actual capacity region $\mathcal{C}_\Delta$. To obtain a tight capacity characterization, we include further genie-aided upper bounds similar to those derived in [10].

Genie-aided upper bounds: Receiver T_1 intends to decode the dedicated messages W_{12} and W_{13} using its received signal $\boldsymbol{y}_1^N$ and its own messages W_{21}, W_{31}, with a reliable decoding strategy. Let the interfering message W_{23} be provided to node T_1 as genie-aided side-information. Since T_1 already knows W_{13} and W_{23}, it can reconstruct $\boldsymbol{x}_3(1)$. From $\boldsymbol{y}_1^N$ and $\boldsymbol{x}_3(1)$, T_1 can derive $\boldsymbol{x}_2(1)$ from the deterministic function (4). T_1 then constructs $\boldsymbol{y}_3(1)$ from $\boldsymbol{x}_1(1)$ and $\boldsymbol{x}_2(1)$. With W_{13}, W_{23} and $\boldsymbol{y}_3(1)$, T_1 can generate $\boldsymbol{x}_3(2)$. These steps are repeated accordingly for all time instants from 2 to N until $\boldsymbol{y}_3^N$ is completely constructed. Therefore, by knowing $\boldsymbol{y}_1^N, W_{21}, W_{31}$ and W_{23} at T_1, it can reliably decode W_{12} and W_{13}, and then reconstruct $\boldsymbol{y}_3^N$ to reliably decode W_{32}. All messages are known at node T_1 now. Thus, for the genie-aided channel, any reliable code allows decoding W_{32}. From Fano's inequality, we can derive:

$$\begin{aligned} &N(R_{12} + R_{13} + R_{32}) \\ &\leq I(W_{12}, W_{13}, W_{32}; \boldsymbol{y}_1^N, W_{21}, W_{31}, W_{23}) + N\epsilon_N \\ &\leq \mathrm{H}(\boldsymbol{y}_1^N) - \mathrm{H}(\boldsymbol{y}_1^N | \boldsymbol{w}) + N\epsilon_N \\ &\leq \mathrm{H}(\boldsymbol{y}_1^N) + N\epsilon_N \\ &\leq N(\max\{n_3, n_2\} + \epsilon_N) \\ &= N(n_3 + \epsilon_N), \end{aligned} \tag{8}$$

where $\epsilon_N \to 0$ as $N \to \infty$. By letting $N \to \infty$, we get the bound $R_{12} + R_{13} + R_{32} \leq n_3$. Similar bounds can be derived by considering different receivers and side-information (cf. Appendix). By considering genie-aided and non-redundant cut-set bounds jointly, we obtain the following set of upper bounds on the capacity region $\mathcal{C}_\Delta$:

$$R_{31} + R_{32} \leq n_2, \tag{9}$$

$$R_{13} + R_{23} \leq n_2, \tag{10}$$

[1] We use $\boldsymbol{v}^N$ denote a sequence of N-vectors $(\boldsymbol{v}(1), \dots, \boldsymbol{v}(N))$.

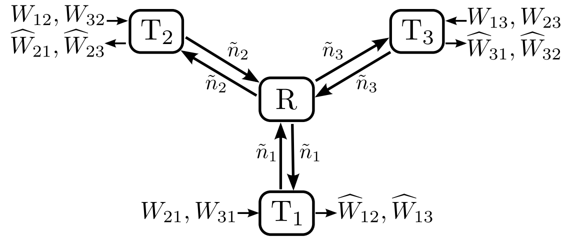


Fig. 2. The reciprocal Y-channel with three transceivers $\mathrm{T}_1, \mathrm{T}_2$ and T_3 has six independent messages W_{ji} transmitted and six corresponding estimated messages $\widehat{W}_{ji}$ received by the nodes, $i \neq j \in \mathcal{K}$. The channel gains are parameterized by $\tilde{n}_j \in \mathbb{N}$, for $j \in \mathcal{K}$.

$$R_{12} + R_{13} + R_{32} \leq n_3, \quad (11)$$
$$R_{12} + R_{13} + R_{23} \leq n_3, \quad (12)$$
$$R_{21} + R_{23} + R_{13} \leq n_3 + n_2 - n_1, \quad (13)$$
$$R_{21} + R_{23} + R_{31} \leq n_3, \quad (14)$$
$$R_{31} + R_{32} + R_{21} \leq n_3, \quad (15)$$
$$R_{31} + R_{32} + R_{12} \leq n_3 + n_2 - n_1. \quad (16)$$

The sum-capacity upper bound yields $R_\Sigma \leq 2n_3$. This set of bounds leads to the following lemma.

Lemma 1. *The capacity region $\mathcal{C}_\Delta$ of the* $\mathrm{D3C}(n_1, n_2, n_3)$ *is outer bounded by $\overline{\mathcal{C}}_\Delta$, i.e., $\mathcal{C}_\Delta \subseteq \overline{\mathcal{C}}_\Delta$, where:*

$$\overline{\mathcal{C}}_\Delta = \{\boldsymbol{R} \in \mathbb{R}_+^6 \mid \boldsymbol{R} \textit{ satisfies } (9)\text{-}(16)\}.$$

This outer bound is in fact achievable. The achievability of this bound is proved via a Δ-Y transformation utilizing the optimal scheme for the Y-channel as a building block. Next, we briefly introduce the Y-channel.

III. Y-CHANNEL

Before we prove the achievability of Lemma 1, we briefly recapitulate the Y-channel whose capacity in terms of the LDCM is characterized in [10].

A. System Model: Linear Deterministic Y-Channel

The deterministic reciprocal Y-channel[2] $\mathrm{DYC}(\tilde{n}_1, \tilde{n}_2, \tilde{n}_3)$ is depicted in Fig. 2. The definitions of the message vector, rate tuple, the transmission symbols and the encoding/decoding functions carry over from those given in Sect. II-A, but are denoted with the tilde-notation to distinguish between the two models. In contrast to the 3-way channel, all users T_j are connected via bidirectional reciprocal links to an intermediate relay R. The channel gain from R to user T_j is denoted by $\tilde{n}_j$. The gains are ordered w.l.o.g. by:

$$\tilde{n}_1 \geq \tilde{n}_2 \geq \tilde{n}_3, \quad (17)$$

so that $q = \max_{i \in \mathcal{K}}\{\tilde{n}_i\} = \tilde{n}_1$. Note that this ordering is reversely oriented when comparing it with (5). The transmitted signals are vectors $\boldsymbol{x}_j, \boldsymbol{x}_\mathrm{R} \in \mathbb{F}_2^q$ from T_j and R, respectively.

[2]Our notation slightly differs from [10] w.r.t. swapped indexation and tilde.

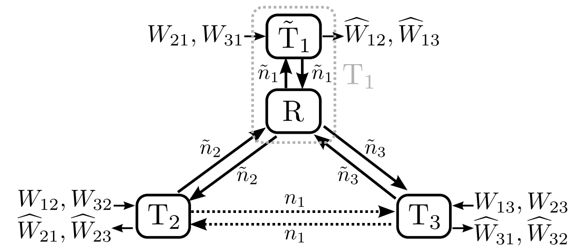


Fig. 3. The reciprocal 3-way channel $\mathrm{D3C}(n_1, n_2, n_3)$ is transformed into an extended Y-channel $\mathrm{eDYC}(\tilde{n}_1, \tilde{n}_2, \tilde{n}_3, n_1)$ including an additional bidirectional link n_1 between T_2 and T_3.

The received signal at R and the received signals at T_j are given by:

$$\boldsymbol{y}_\mathrm{R} = \sum_{j=1}^3 \boldsymbol{S}^{q-n_j}\boldsymbol{x}_j, \quad (18)$$
$$\boldsymbol{y}_j = \boldsymbol{S}^{q-n_j}\boldsymbol{x}_\mathrm{R}, \quad (19)$$

respectively. Next, we re-state the capacity region of the linear shift deterministic Y-channel, which will be an essential part of the proof for the achievability of Lemma 1.

B. Capacity Region: Linear Deterministic Y-Channel

The capacity region $\mathcal{C}_Y$ of the $\mathrm{DYC}(\tilde{n}_1, \tilde{n}_2, \tilde{n}_3)$ has already been characterized in [10], and is given by the set of rate tuples $\tilde{\boldsymbol{R}} = (\tilde{R}_{12}, \tilde{R}_{21}, \tilde{R}_{13}, \tilde{R}_{31}, \tilde{R}_{23}, \tilde{R}_{32})$, satisfying:

$$\tilde{R}_{31} + \tilde{R}_{32} \leq \tilde{n}_3, \quad (20)$$
$$\tilde{R}_{13} + \tilde{R}_{23} \leq \tilde{n}_3, \quad (21)$$
$$\tilde{R}_{12} + \tilde{R}_{13} + \tilde{R}_{32} \leq \tilde{n}_2, \quad (22)$$
$$\tilde{R}_{12} + \tilde{R}_{13} + \tilde{R}_{23} \leq \tilde{n}_2, \quad (23)$$
$$\tilde{R}_{21} + \tilde{R}_{23} + \tilde{R}_{13} \leq \tilde{n}_1, \quad (24)$$
$$\tilde{R}_{21} + \tilde{R}_{23} + \tilde{R}_{31} \leq \tilde{n}_2, \quad (25)$$
$$\tilde{R}_{31} + \tilde{R}_{32} + \tilde{R}_{21} \leq \tilde{n}_2, \quad (26)$$
$$\tilde{R}_{31} + \tilde{R}_{32} + \tilde{R}_{12} \leq \tilde{n}_1. \quad (27)$$

There is an interesting resemblance between the bounds (20)-(27) and (9)-(16). This resemblance will be exploited to design an optimal scheme for the 3-way channel next.

IV. Δ-Y TRANSFORMATION

Equating the upper bounds of the $\mathrm{D3C}(n_1, n_2, n_3)$ in (9)-(16) and the $\mathrm{DYC}(\tilde{n}_1, \tilde{n}_2, \tilde{n}_3)$ in (20)-(27) yields:

$$\tilde{n}_1 = n_2 + n_3 - n_1 \quad (28)$$
$$\tilde{n}_2 = n_3, \quad (29)$$
$$\tilde{n}_3 = n_2. \quad (30)$$

In other words, the outer bound for the $\mathrm{D3C}(n_1, n_2, n_3)$ coincides with the capacity region of a $\mathrm{DYC}(\tilde{n}_1, \tilde{n}_2, \tilde{n}_3)$. Note that the ordering of the channel gains in (5) and (17) still holds.

In order to show the achievability of the outer bound in Lemma 1, we first express the 3-way channel in terms of an extended Y-channel as depicted in Fig. 3. User T_1 is extended such that it behaves like a virtual relay R which is also connected to a virtual user $\tilde{\mathrm{T}}_1$ via an artificial sub-channel parametrized by $\tilde{n}_1$. At R, the topmost levels $\tilde{n}_2+1,\cdots,\tilde{n}_1$ are only accessible by $\tilde{\mathrm{T}}_1$ and not visible for T_2 and T_3 (see Fig. 4(a)) and hence only virtual within T_1. The residual link with n_1 from the previous $\mathrm{D3C}(n_1,n_2,n_3)$ remains as a weak bidirectional link between T_2 and T_3 in the extended Y-channel $\mathrm{eDYC}(\tilde{n}_1,\tilde{n}_2,\tilde{n}_3,n_1)$. The channel gain n_1 is still the weakest one, since:

$$n_1 \le n_2 = \tilde{n}_3 \le \tilde{n}_2 = n_3 \le n_3 + n_2 - n_1 = \tilde{n}_1. \qquad (31)$$

The optimal scheme for the Y-channel already achieves the outer bound $\overline{\mathcal{C}}_\Delta$ for $n_1 = 0$. However, in general we have $n_1 \ge 0$. Hence, we have to modify our scheme to deal with the additional interference over n_1.

A. Achievability of $\overline{\mathcal{C}}_\Delta$

Let all (virtual) users $\tilde{\mathrm{T}}_1$, T_2, T_3 and R apply the capacity-achieving SA scheme described in [10] as if it would be applied on a $\mathrm{DYC}(\tilde{n}_1,\tilde{n}_2,\tilde{n}_3)$, but without decoding at the receivers yet. We consider N uplink and N downlink transmissions, over $N+1$ time-instants. We call this scheme the 'original' scheme. For illustration, consider the uplink, downlink and the additional bidirectional link n_1 for the $\mathrm{eDYC}(\tilde{n}_1,\tilde{n}_2,\tilde{n}_3,n_1)$ as depicted in Fig. 4(a).

In contrast to the original scheme for the $\mathrm{DYC}(\tilde{n}_1,\tilde{n}_2,\tilde{n}_3)$, the scheme for the $\mathrm{eDYC}(\tilde{n}_1,\tilde{n}_2,\tilde{n}_3,n_1)$ must be adapted to deal with the signals inherently transmitted over n_1. We will overlay the adapted scheme on top of the original scheme.

In particular, any signal transmitted by T_i, $i\in\{2,3\}$, on the topmost levels $\tilde{n}_1-n_1+1,\cdots,\tilde{n}_1$, will interfere at receiver T_j on the lowermost levels $1,\cdots,n_1$. We discern three classes of interference over n_1 that are potentially received at T_2 and T_3 when applying the original scheme:

(a) The interference over n_1 received at T_i is a dedicated signal from T_j to T_i, which will also be forwarded from R to T_i in the next time-instant.
(b) The interference over n_1 received at T_3 is a dedicated signal from T_2 to T_1.
(c) The interference over n_1 received at T_2 is a dedicated signal from T_3 to T_1.

Class (a): To compensate interference of class (a), we postpone decoding until the last signals of the $(N+1)$-th time-instant are received. The transmission scheme does not change w.r.t. the original one. Since the (uplink) transmitters of T_2 and T_3 are silent on the $(N+1)$-th time-instant, the (downlink) receivers of T_2 and T_3 receive no signal over n_1 at the $(N+1)$-th time-instant. Hence, T_2 and T_3 can decode their dedicated signals as received in the last hop. In fact, the downlink signals of the $(N+1)$-th time-instant behave analogously to the $(N+1)$-th hop of the original scheme. Since the class (a) interference of the N-th hop is a subset of the dedicated signals in the $(N+1)$-th hop, it is cancelled after decoding the dedicated signals of the $(N+1)$-th hop. With such a backward decoding scheme, the interference of class (a) is cancelled analogously for all preceding time-instants $N-1,\cdots,2,1$.

Class (b): To compensate the interference of class (b), i.e., those bits received at T_3 over n_1 carrying a dedicated signal from T_2 to T_1, say $\boldsymbol{x}_{12}$, we apply an IN scheme. In detail, T_2 *pre-transmits* the interference signal $(\boldsymbol{x}_{12}(l))$ one time-instant in advance (in time-instant $l-1$) as follows. Assume that T_3 receives $[\boldsymbol{x}'^{\mathsf{T}}_{R,3}(l),\boldsymbol{x}^{\mathsf{T}}_{R,3}(l)]^{\mathsf{T}}$ from R in the downlink at time-instant l, where $\boldsymbol{x}'_{R,3}(l)$ and $\boldsymbol{x}_{R,3}(l)$ are binary vectors of lengths n_1 and $\tilde{n}_3-n_1$, respectively (see Fig. 4(a)). Moreover, assume that T_3 receives interference from $\boldsymbol{x}_{12}(l)$ over some bits of $\boldsymbol{x}_{R,3}(l)$. To deal with this interference, T_2 pre-transmits $\boldsymbol{x}_{12}(l)$ in time-instant $l-1$ in the uplink, over exactly the same levels where $\boldsymbol{x}_{R,3}(l)$ is received in the uplink[3]. By doing so, T_3 receives $\boldsymbol{x}_{12}(l)$ twice over $\boldsymbol{x}_{R,3}(l)$ in the downlink, once from T_2 and once from R. Since $\boldsymbol{x}_{12}(l)$ is a binary vector, the addition of $\boldsymbol{x}_{12}(l)$ to itself results in interference neutralization.

It remains to make sure that the pre-transmission does not disturb any other node. Clearly, $\boldsymbol{x}_{12}$ does not disturb T_2 since it originates from the same node T_2. Also, $\boldsymbol{x}_{12}$ does not disturb T_1 since $\boldsymbol{x}_{12}$ is a desired signal at T_1, and thus the interfering $\boldsymbol{x}_{12}$ is removed by backward decoding.

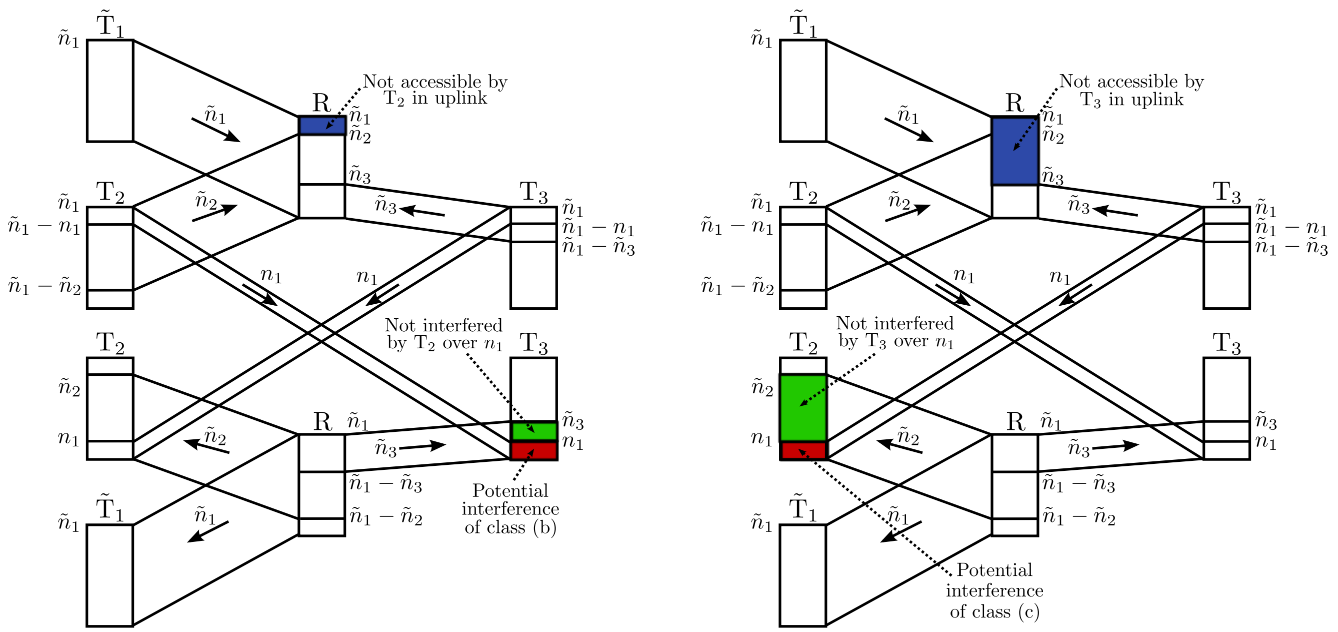
One more problem remains. Our approach using IN only works if $\boldsymbol{x}_{R,3}(l)$ is received over levels that are accessible by T_2 in the uplink, i.e., the levels $1,\cdots,\tilde{n}_2$ at R. However, $\boldsymbol{x}_{R,3}(l)$ might contain information from T_1, say $\boldsymbol{x}_{31}$, which might not be accessible by T_2 in the uplink. This is exactly the case if T_1 sends $\boldsymbol{x}_{31}$ over levels $\tilde{n}_2+1,\cdots,\tilde{n}_1$ at R (blue area in Fig. 4(a)). However, the given problem can be solved easily by noting that the number of levels in the blue area in Fig. 4(a) is $\tilde{n}_1-\tilde{n}_2$. We have $\tilde{n}_1-\tilde{n}_2=\tilde{n}_3-n_1$ by (28), i.e., the same number of levels in the non-interfered downlink levels at T_3 (green area in Fig. 4(a)). Therefore, we exploit this interesting equality of the transformation: R forwards $\boldsymbol{x}_{31}$ over the non-interfered downlink levels at T_3 and the given problem is avoided. By pursuing such an approach, the impact of class (b) interference is completely eliminated.

Class (c): To compensate the interference of class (c) at T_2 received over n_1, i.e., a dedicated signal $\boldsymbol{x}_{31}$ from T_3 to T_1, we apply a similar IN scheme. T_3 likewise additionally pre-transmits class (c) interference one time-instant in advance. T_3 can access $\tilde{n}_3$ levels in the uplink to R that are potentially forwarded to T_2 in the downlink during the next time-instant.

However, the levels $\tilde{n}_3+1,\cdots,\tilde{n}_1$ (blue area in Fig. 4(b)) are not accessible by T_3 in the uplink. Thus, if the signal received by T_2 from R in the downlink over levels $1,\cdots,n_1$ are sent over relay levels $\tilde{n}_3+1,\cdots,\tilde{n}_1$ in the uplink, then, T_3 can not perform IN. However this scenario can be avoided by sending all signals received in the uplink on the blue levels in Fig. 4(b), over the green levels in the downlink. This is possible since $\tilde{n}_2-n_1=\tilde{n}_1-\tilde{n}_3$ holds by (28). In this case, these signals do not interfere with the levels $1,\cdots,n_1$ at T_2 which renders T_3 capable of performing IN.

For the downlink from R to T_3, the pre-transmitted signals which are back-propagated to T_3 are known self-interference

[3]Note that $\boldsymbol{x}_{R,3}(l)$ is received at R at time-instant $l-1$, and transmitted at time-instant l.



(a) Auxiliary illustration for interference of class (b).

(b) Auxiliary illustration for interference of class (c).

Fig. 4. A signal level representation of the $\mathrm{eDYC}(\tilde{n}_1, \tilde{n}_2, \tilde{n}_3, n_1)$- channel is depicted with the uplink on top, the downlink at the bottom, and the additional interference links with channel gain n_1. The signal levels of the uplink to R for instance comprise three components: the topmost $\{\tilde{n}_2+1, \dots, \tilde{n}_1\}$ levels accessible by T_1 only, the levels $\{\tilde{n}_3+1, \dots, \tilde{n}_2\}$ accessible by T_1 and T_2, and the lowermost $\{1, \dots, \tilde{n}_3\}$ levels accessible by all three uplink users.

and cancelled. For the downlink of T_1, these pre-transmitted signals are dedicated for T_1 and cancelled by backward decoding. Thus, the interference of class (c) is eliminated as well.

In all three classes, the interference over the bidirectional link n_1 is cancelled or neutralized and all dedicated signals are decodable by backward decoding after $N+1$ time-instants. Altogether, this proves the achievability of $\overline{\mathcal{C}}_\triangle$ leading to the following Theorem.

Theorem 2. *The capacity region* $\mathcal{C}_\triangle$ *of the* $\mathrm{D3C}(n_1, n_2, n_3)$ *is given by* $\overline{\mathcal{C}}_\triangle$ *defined in Lemma 3.*

V. Capacity Region of the Symmetric case by IA

Interestingly, signals conveyed over the weak link n_1 are not used for direct communication. The interfering signals over n_1 are cancelled or neutralized by the communication scheme proposed in Section IV-A, so that the impact of the link n_1 is effectively eliminated. A certain drawback of our previous scheme is that the receivers must wait for $N+1$ time-instants to apply the backward decoding procedure. This is a very restrictive property, especially for delay-limited communications [20].

As a contrary approach, we now propose a purely IA-based communication scheme for the symmetric $\mathrm{D3C}(m,m,m)$ that achieves the corresponding capacity region. The communication scheme for the $\mathrm{D3C}(m,m,m)$ based on IA is proven with similar methods as the one in [10]. In this case, there is no need for backward decoding and interference neutralization.

Theorem 3. *An interference alignment scheme based on bidirectional, cyclic and unidirectional communication suffices to achieve the outer bounds on the capacity region of a symmetric* $\mathrm{D3C}(m,m,m)$ *with* $m \in \mathbb{N}$.

We consider a communication scheme of 3 components:

A) **Bidirectional**: For distinct $i, j \in \mathcal{K}$, the pair of rates R_{ji}, R_{ij} is non-zero.
B) **Cyclic**: For distinct $i, j, k \in \mathcal{K}$, the triple of rates R_{ji}, R_{jk}, R_{ki} is non-zero, whereas $R_{ij} = R_{kj} = R_{ik} = 0$.
C) **Unidirectional**: None of the above cases holds.

We now outline our proposed IA scheme based on the components A, B and C. We will begin with scheme A on the $\mathrm{D3C}(m,m,m)$ operating at 2 bits per level. Pairs of users communicate bidirectionally. Then, we reduce the channel to $\mathrm{D3C}(m',m',m')$ by removing the already used levels from scheme A. Next, scheme B with $3/2$ bits per level is applied. Again we reduce the channel to $\mathrm{D3C}(m'',m'',m'')$ removing the levels occupied by scheme B. In the last step, we apply scheme C allocating 1 bit per level. If the rate tuple to be achieved does not satisfy one of conditions A, B, and C, the corresponding scheme is merely discarded. We will show in the following that these schemes suffice to achieve the outer bounds of the capacity region for the $\mathrm{D3C}(m,m,m)$.

A. Bidirectional Communication on $\mathrm{D3C}(m,m,m)$

We define the following three transmission parameters $a, b, c \in \mathbb{N}$:

$$a = \min\{R_{12}, R_{21}\},\ b = \min\{R_{13}, R_{31}\},\ c = \min\{R_{23}, R_{32}\}. \tag{32}$$

k	$\boldsymbol{x}_k$	$\boldsymbol{y}_k$	intervals of levels
1	0	$\boldsymbol{x}_{32}+\boldsymbol{x}_{23}$	$a+b+1,\cdots,a+b+c$
1	$\boldsymbol{x}_{31}$	$\boldsymbol{x}_{13}$	$a+1,\cdots,a+b$
1	$\boldsymbol{x}_{21}$	$\boldsymbol{x}_{12}$	$1,\cdots,a$
2	$\boldsymbol{x}_{32}$	$\boldsymbol{x}_{23}$	$a+b+1,\cdots,a+b+c$
2	0	$\boldsymbol{x}_{13}+\boldsymbol{x}_{31}$	$a+1,\cdots,a+b$
2	$\boldsymbol{x}_{12}$	$\boldsymbol{x}_{21}$	$1,\cdots,a$
3	$\boldsymbol{x}_{23}$	$\boldsymbol{x}_{32}$	$a+b+1,\cdots,a+b+c$
3	$\boldsymbol{x}_{13}$	$\boldsymbol{x}_{31}$	$a+1,\cdots,a+b$
3	0	$\boldsymbol{x}_{21}+\boldsymbol{x}_{12}$	$1,\cdots,a$

Fig. 5. A) Allocation of signals to bit-levels for the bidirectional scheme over the $\mathrm{D3C}(m,m,m)$. The 1st column denotes the considered user, the 2nd column the transmitted signal, the 3rd column its received signal, and the 4th column describes the interval of levels concerned. The lowest bit-level is indexed by 1.

If $a=b=c=0$ holds, scheme A is skipped and we continue with scheme B. We propose a signal allocation such that 2 bits per level are achieved. The signals are $\boldsymbol{x}_{12},\boldsymbol{x}_{21}\in\mathbb{F}_2^a$, $\boldsymbol{x}_{31},\boldsymbol{x}_{13}\in\mathbb{F}_2^b$, and $\boldsymbol{x}_{32},\boldsymbol{x}_{23}\in\mathbb{F}_2^c$. To transmit these signals, $a+b+c\leq m$ levels are allocated as depicted in Fig. 5. The interference signals $\boldsymbol{x}_{ji}$ and $\boldsymbol{x}_{ij}$ are aligned at T_k with pairwise distinct $i,j,k\in\mathcal{K}$.

This allocation scheme is only feasible if enough levels are available at the transmitters and receivers for all bidirectional streams. For $\boldsymbol{R}\in\overline{\mathcal{C}}$, the following must hold on a,b,c:

$$a+b+c\overset{(32)}{\leq}R_{12}+R_{13}+R_{32}\overset{(11)}{\leq}m. \tag{33}$$

This is also true for all other upper bounds. For the yet unused levels, we still need to achieve the residual rate-vector:

$$\begin{aligned}\boldsymbol{R}'&=(R_{12}-a,R_{21}-a,R_{13}-b,R_{31}-b,R_{23}-c,R_{32}-c)\\&=(R'_{12},R'_{21},R'_{13},R'_{31},R'_{23},R'_{32}).\end{aligned} \tag{34}$$

So far, at least three components will already be zero due to the min-expressions in (32). We remove the allocated levels so that the reduced $\mathrm{D3C}(m',m',m')$ is parameterized by:

$$m'=m-a-b-c. \tag{35}$$

Clearly, the reduced channel remains symmetric.

B. Cyclic Communication on $\mathrm{D3C}(m',m',m')$

Given that the conditions for scheme B hold, and depending on the residual rate-vector $\boldsymbol{R}'$ computed in (34), we apply either clock-wise cyclic communication $1\to2\to3\to1$ with parameter d or counter-clock-wise cyclic communication $1\to3\to2\to1$ with parameter e. The parameters $d,e\in\mathbb{N}$ are:

$$d=\min(R'_{21},R'_{13},R'_{32}),\ \ e=\min(R'_{12},R'_{31},R'_{23}). \tag{36}$$

Note that either d or e must be zero, since bidirectional communication is already taken care of by the previous scheme A. The definitions in (36) provide two cases:

$$d>0\Rightarrow e=0,\ \ a=R_{12},\ \ b=R_{31},\ \ c=R_{23}, \tag{37}$$
$$e>0\Rightarrow d=0,\ \ a=R_{21},\ \ b=R_{13},\ \ c=R_{32}. \tag{38}$$

i	$\boldsymbol{x}_i$	$\boldsymbol{y}_i$	intervals of levels
1	0	$\boldsymbol{x}_{13}+\boldsymbol{x}_{32}$	$d+1,\ldots,2d$
1	$\boldsymbol{x}_{21}$	$\boldsymbol{x}_{32}$	$1,\ldots,d$
2	$\boldsymbol{x}_{32}$	$\boldsymbol{x}_{13}$	$d+1,\ldots,2d$
2	$\boldsymbol{x}_{32}$	$\boldsymbol{x}_{21}$	$1,\ldots,d$
3	$\boldsymbol{x}_{13}$	$\boldsymbol{x}_{32}$	$d+1,\ldots,2d$
3	0	$\boldsymbol{x}_{21}+\boldsymbol{x}_{32}$	$1,\ldots,d$

Fig. 6. B) Allocation of signals to bit-levels for $\mathrm{D3C}(m',m',m')$ for clock-wise cyclic communication.

i	$\boldsymbol{x}_i$	$\boldsymbol{y}_i$	intervals of levels
1	0	$\boldsymbol{x}_{12}+\boldsymbol{x}_{23}$	$e+1,\ldots,2e$
1	$\boldsymbol{x}_{31}$	$\boldsymbol{x}_{12}$	$1,\ldots,e$
2	$\boldsymbol{x}_{12}$	$\boldsymbol{x}_{23}$	$e+1,\ldots,2e$
2	$\boldsymbol{x}_{12}$	$\boldsymbol{x}_{23}$	$1,\ldots,e$
3	$\boldsymbol{x}_{23}$	$\boldsymbol{x}_{12}$	$e+1,\ldots,2e$
3	0	$\boldsymbol{x}_{12}+\boldsymbol{x}_{31}$	$1,\ldots,e$

Fig. 7. B) Allocation of signals to bit-levels for $\mathrm{D3C}(m',m',m')$ for counter clock-wise cyclic communication.

If both $d=e=0$, this section is skipped and we continue with scheme C. In the following, we propose a signal allocation scheme such that 3/2 bits per level are achieved.

For case (37), $R'_{12}=R'_{31}=R'_{23}=0$ and R'_{21},R'_{13},R'_{32} are non-zero. The signals are $\boldsymbol{x}_{21}$, $\boldsymbol{x}_{13}$, $\boldsymbol{x}_{32}\in\mathbb{F}_2^d$, allocated in blocks of d levels as depicted in Fig. 6. The constraint $2d\leq m'$ must hold at each user to provide a feasible allocation. Signal $\boldsymbol{x}_{32}$ is transmitted by T_2 on both intervals of size d and T_1 applies interference cancellation to decode $\boldsymbol{x}_{13}$ from $\boldsymbol{x}_{13}+\boldsymbol{x}_{32}$. The interference signals $\boldsymbol{x}_{21}$ and $\boldsymbol{x}_{32}$ are aligned at T_3.

Sufficiently many levels are available for scheme B on the reduced $\mathrm{D3C}(m',m',m')$ if $\boldsymbol{R}\in\overline{\mathcal{C}}$, since:

$$\begin{aligned}2d&\overset{(36)}{\leq}R'_{13}+R'_{32}\overset{(34)}{=}R_{13}+R_{32}-b-c\\&\overset{(9)}{\leq}m-R_{12}-b-c\overset{(37)}{=}m-a-b-c\overset{(35)}{=}m'.\end{aligned} \tag{39}$$

The second case (38) for counter clock-wise communication is derived analogously, but with the indices swapped and with an adapted allocation (cf. Fig. 7). In particular, we have $R'_{21}=R'_{13}=R'_{32}=0$ and R'_{12},R'_{31},R'_{23} are non-zero. In analogy to (39), the allocations for counter-clock-wise communication with parameter e satisfies $\boldsymbol{R}\in\overline{\mathcal{C}}$:

$$2e\overset{(36)}{\leq}R'_{31}+R'_{23}\leq m-a-b-c=m'.$$

For the yet unused levels, the residual rate-vector is:

$$\begin{aligned}\boldsymbol{R}''&=(R'_{12}-d,R'_{21}-e,R'_{13}-e,R'_{31}-d,R'_{23}-d,R'_{32}-e)\\&=(R''_{12},R''_{21},R''_{13},R''_{31},R''_{23},R''_{32}),\end{aligned} \tag{40}$$

over the $\mathrm{D3C}(m'',m'',m'')$ (where either d or e is zero) with parameter:

$$m''=m'-2d-2e. \tag{41}$$

C. Unidirectional Communication on D3C(m'', m'', m'')

Six possible non-zero rate tuples remain that are not yet covered by the previous schemes A and B:

$$(R''_{21}, R''_{31}, R''_{32}) \neq \mathbf{0},\ (R''_{21}, R''_{31}, R''_{23}) \neq \mathbf{0},$$
$$(R''_{12}, R''_{13}, R''_{23}) \neq \mathbf{0},\ (R''_{12}, R''_{13}, R''_{32}) \neq \mathbf{0},$$
$$(R''_{12}, R''_{31}, R''_{32}) \neq \mathbf{0},\ (R''_{21}, R''_{13}, R''_{23}) \neq \mathbf{0}.$$

These cases pairwise exclude each other. W.l.o.g., we only consider the unidirectional case $(R''_{12}, R''_{13}, R''_{23}) \neq \mathbf{0}$, here. The remaining cases are derived by analogous steps. We have $R''_{21} = R''_{31} = R''_{32} = 0$ and we parameterize the three non-zero rates by $f, g, h \in \mathbb{N}$:

$$R''_{12} = f,\ R''_{13} = g,\ R''_{23} = h. \tag{42}$$

The signals are $\boldsymbol{x}_{12} \in \mathbb{F}_2^f$, $\boldsymbol{x}_{13} \in \mathbb{F}_2^g$, $\boldsymbol{x}_{23} \in \mathbb{F}_2^h$. A number of $f+g+h \leq m''$ levels are allocated as depicted in Fig. 8.

k	$\boldsymbol{x}_k$	$\boldsymbol{y}_k$	intervals of levels
3	0	$\boldsymbol{x}_{23}$	$f+g+1, \ldots, f+g+h$
3	0	$\boldsymbol{x}_{13}$	$f+1, \ldots, f+g$
3	0	$\boldsymbol{x}_{12}$	$1, \ldots, f$
1	0	$\boldsymbol{x}_{23}$	$f+g+1, \ldots, f+g+h$
1	0	$\boldsymbol{x}_{13}$	$f+1, \ldots, f+g$
1	$\boldsymbol{x}_{12}$	0	$1, \ldots, f$
2	$\boldsymbol{x}_{23}$	$\boldsymbol{x}_{12}$	$f+g+1, \ldots, f+g+h$
2	$\boldsymbol{x}_{13}$	0	$f+1, \ldots, f+g$
2	0	0	$1, \ldots, f$

Fig. 8. C) Allocation of signals to bit-levels in unidirectional communication over D3C(m'', m'', m'').

Since we demand $\boldsymbol{R}'' \in \overline{\mathcal{C}}$, we discern two cases depending on the previous scheme B. In the first case, we assume that clock-wise cyclic communication was applied before. Recall that either d or e must be zero. If $d > 0$ and $e = 0$, then:

$$\begin{aligned} f+g+h &= R''_{12} + R''_{13} + R''_{23} \overset{(40)}{=} R'_{12} + R'_{13} + R'_{23} - 2d \\ &\overset{(34)}{=} R_{12} + R_{13} + R_{23} - a - b - c - 2d \\ &\overset{(9)}{\leq} m - a - b - c - 2d \overset{(35)}{=} m' - 2d. \end{aligned} \tag{43}$$

Otherwise, if $e > 0$ (counter clock-wise) and $d = 0$, then:

$$\begin{aligned} f+h+g &\overset{(40),(34)}{=} R_{12} + R_{13} + R_{23} - a - b - c - e \\ &\overset{(38)}{\leq} R_{12} + R_{13} - a - b - e \overset{(14)}{\leq} m - R_{23} - a - b - e \\ &\leq m - (e + c) - a - b - e = m' - 2e. \end{aligned} \tag{44}$$

For (44), we use $e \leq R'_{23} = R_{23} - c$ from (42) and (34). Since either d or e is zero, combining (43) and (44) yields:

$$f+g+h \leq m' - 2d - 2e \overset{(41)}{=} m''. \tag{45}$$

Hence, sufficiently many levels are also available for the D3C(m'', m'', m''). Altogether, there are enough levels to communicate all $a+b+c+2d+2e+f+g+h$ bits. The application of schemes A to C achieves the upper bounds of the capacity region for the D3C(m, m, m) proving Theorem 3. The remaining steps, showing that each corner point of the capacity region is achievable is analogous to [10, Thms. 3&4] and omitted here.

VI. Conclusions

We have studied the capacity region of the linear shift deterministic 3-way channel. Tight upper bounds on the capacity region are characterized using cut-set upper bounds and genie-aided upper bounds. Our main result is that the reciprocal linear deterministic 3-way channel can be transformed into an extended linear deterministic Y-channel with an additional weak link. Then, we apply the capacity-achieving signal alignment scheme proposed by Chaaban et al. for the reciprocal linear deterministic Y-channel and further extend it with an interference neutralization and backward decoding procedure. As a result, we show that it suffices to achieve the capacity region by relaying all messages over the user with the strongest incident sub-channels using a network-coded signal-alignment scheme.

As a complementary approach, we have also proposed an interference alignment scheme that achieves the capacity region of a fully symmetric sub-channel of the 3-way channel without resorting to relaying, backward decoding and interference neutralization. For an extension of this work, it would be interesting to see whether similar transformations can be applied to transform centralized networks to decentralized ones, and vice versa.

Appendix A
Genie-aided upper bounds

The remaining upper bounds on the capacity region of the D3C(n_1, n_2, n_3) are derived similar to Section II-B. We only discuss the main differences in this appendix.

(i): To derive the bound $R_{12} + R_{13} + R_{23} \leq n_3$, we provide W_{32} as side-information to the receiver of T_1, and proceed similar to Section II-B. That is, we prove that the enhanced T_1 can construct $\boldsymbol{y}_2^N$ from which it can decode W_{23}.

(ii): To derive the bound $R_{21} + R_{23} + R_{13} \leq n_3 + n_2 - n_1$, we provide W_{31} and $\hat{\boldsymbol{x}}_3^N$ to T_2 as side-information, where $\hat{\boldsymbol{x}}_3^N$ denotes the lowermost $n_2 - n_1$ bits of $\boldsymbol{x}_3^N$. Providing these bits is necessary since T_2 can only obtain the topmost n_1 symbols of $\boldsymbol{x}_3(1)$ from $\boldsymbol{y}_2(1)$ and $\boldsymbol{x}_1(1)$ after decoding W_{21} (recall that given W_{31} and W_{21}, T_2 can construct $\boldsymbol{x}_1(1)$). By combining the topmost n_1 bits of $\boldsymbol{x}_3(1)$ and the lowermost $n_2 - n_1$ bits provided by the side-information, T_2 can construct $\boldsymbol{y}_1(1)$ and hence also $\boldsymbol{x}_1(2)$, since it knows W_{31} from side-information and W_{21} after decoding. Similarly, all components of $\boldsymbol{y}_1^N$ can be constructed, and W_{13} can be decoded. Thus, by Fano's inequality, we can write:

$$\begin{aligned} &N(R_{21} + R_{23} + R_{13}) \\ &\leq I(W_{21}, W_{23}, W_{13}; \boldsymbol{y}_2^N, \hat{\boldsymbol{x}}_3^N, W_{12}, W_{32}, W_{31}) + N\epsilon_N \\ &\leq \mathrm{H}(\boldsymbol{y}_2^N, \hat{\boldsymbol{x}}_3^N) - \mathrm{H}(\boldsymbol{y}_2^N, \hat{\boldsymbol{x}}_3^N | \boldsymbol{w}) + N\epsilon_N \\ &\leq \mathrm{H}(\boldsymbol{y}_2^N, \hat{\boldsymbol{x}}_3^N) + N\epsilon_N \\ &\leq N(\max\{n_3 + n_2 - n_1, n_1 + n_2 - n_1\} + \epsilon_N) \\ &= N(n_3 + n_2 - n_1 + \epsilon_N), \end{aligned}$$

where $\epsilon_N \to 0$ as $N \to \infty$. This provides the upper bound $R_{21}+R_{23}+R_{13} \leq n_3+n_2-n_1$ after dividing by N and letting $N \to \infty$.

(iii): To derive $R_{21}+R_{23}+R_{31} \leq n_3$, we provide W_{13} to T_2 and proceed similar to **(i)**, by showing that T_2 can construct $\boldsymbol{y}_3^N$ and decode W_{31}.

(iv): To derive $R_{31}+R_{32}+R_{21} \leq n_3$, we give $\hat{\boldsymbol{x}}_1^N$ and W_{12} to T_3 as side-information, where $\hat{\boldsymbol{x}}_1^N$ denotes the lowermost n_3-n_2 bits of $\boldsymbol{x}_1^N$. By proceeding similar to **(ii)**, we can show that T_3 can construct $\boldsymbol{y}_2^N$ given this side-information, and then decode W_{21}, leading to the desired bound.

(v): To derive the bound $R_{31}+R_{32}+R_{12} \leq n_3+n_2-n_1$, we give W_{21} and $\hat{\boldsymbol{x}}_2^N$ to T_3, where $\hat{\boldsymbol{x}}_2^N$ denote the lowermost n_3-n_1 bits of $\boldsymbol{x}_2^N$. Similar to **(ii)**, T_3 is able to construct $\boldsymbol{y}_1^N$ given this side-information, and then to decode W_{12}, leading to the desired bound.

As a result, we obtain the upper bounds on capacity region as given by (9) to (16).

REFERENCES

[1] A. Avestimehr, S. Diggavi, and D. Tse, "A deterministic approach to wireless relay networks," in *Proc. 45th Allerton Conf. on Comm., Control, and Comp. (Allerton 2007)*, Monticello, IL, USA, Sep. 2007, pp. 721–728.

[2] R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inform. Theory*, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.

[3] V. Cadambe and S. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," *IEEE Trans. Inform. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[4] S. Jafar and S. Vishwanath, "Generalized degrees of freedom of the symmetric Gaussian K user interference channel," *IEEE Trans. Inform. Theory*, vol. 56, no. 7, pp. 3297–3303, Jul. 2010.

[5] C. Huang, V. Cadambe, and S. Jafar, "On the capacity and generalized degrees of freedom of the X channel," in *Proc. IEEE Int. Symp. on Inform. Theory (ISIT 2009)*, Seoul, South Korea, Jul. 2009, pp. 1929–1933.

[6] C. Shannon, "Two-way communication channels," in *Proc. 4th Berkeley Symp. Math. Stat. Prob.*, vol. 1, Berkeley, CA, USA, 1961, pp. 611–644.

[7] Z. Cheng and N. Devroye, "Two-way networks: when adaptation is useless," *IEEE Trans. Inform. Theory*, vol. 60, no. 3, pp. 1793–1813, Mar. 2014.

[8] A. Chaaban, H. Maier, and A. Sezgin, "The degrees of freedom of multi-way device-to-device communications is limited by 2," in *Proc. IEEE Intern. Symp. on Inform. Theory (ISIT 2014)*, Honolulu, HI, USA, Jul. 2014.

[9] L. Ong, "Capacity results for two classes of three-way channels," in *IEEE Intern. Symp. on Comm. and Inform. Techn. (ISCIT 2012)*, Gold Coast, QLD, Australia, Oct. 2012, pp. 471–476.

[10] A. Chaaban and A. Sezgin, "The capacity region of the linear shift deterministic Y-channel," in *IEEE Int. Symp. on Inform. Theory (ISIT 2011)*, St. Petersburg, Russia, 2011, pp. 2457–2461.

[11] ——, "The approximate capacity region of the Gaussian Y-channel," *CoRR*, 2013.

[12] D. Gündüz, A. Yener, A. Goldsmith, and H. Poor, "The multi-way relay channel," in *Proc. IEEE Int. Symp. on Inform. Theory (ISIT 2009)*, Seoul, South Korea, Jun. 2009, pp. 339–343.

[13] A. Chaaban, A. Sezgin, and A. S. Avestimehr, "Approximate sum capacity of the Y-channel," *IEEE Trans. on Inform. Theory*, vol. 59, no. 9, pp. 5723–5740, Sept. 2013.

[14] N. Lee, J.-B. Lim, and J. Chun, "Degrees of Freedom of the MIMO Y channel: Signal Space Alignment for Network Coding," *IEEE Trans. Inform. Theory*, vol. 56, no. 7, Jul. 2010.

[15] A. Chaaban, K. Ochs, and A. Sezgin, "The degrees of freedom of the MIMO Y-channel," in *Proc. IEEE Int. Symp. on Inform. Theory (ISIT 2013)*, Istanbul, Turkey, Jul. 2013, pp. 1581–1585.

[16] S. Mohajer, S. Diggavi, C. Fragouli, and D. Tse, "Transmission techniques for relay-interference networks," in *Proc. 46th Allerton Conf. on Comm., Control, and Comp. (Allerton 2008)*, Monticello, IL, USA, Sep. 2008, pp. 467–474.

[17] F. Willems and E. Van der Meulen, "The discrete memoryless multiple-access channel with cribbing encoders," *IEEE Trans. Inform. Theory*, vol. IT-31, no. 3, pp. 313–327, May 1985.

[18] A. Kennelly, "The equivalence of triangles and three-pointed stars in conducting networks," *Electr, World and Engin.*, vol. 34, pp. 413–414, 1899.

[19] A. Rosen, "A new network theorem," *Journal of the Inst. of Elect. Engin.*, vol. 62, no. 335, pp. 916–918, Nov. 1924.

[20] E. Jorswieck, H. Boche, and A. Sezgin, "Delay-limited capacity and maximum throughput of spatially correlated multiple antenna systems under average and peak-power constraints," in *IEEE Inform. Theory Workshop (ITW 2004)*, San Antonio, TX, USA, Oct. 2004, pp. 440–445.