Power Allocation for Distributed Passive Radar Systems with Occasional Node Failure

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Abstract—**In this paper, we address the optimal power allocation problem for a distributed passive radar system, where occasional node failures are taken into account. The goal of the network is to provide a reliable estimation from a target signal, by collecting and combining the individual observations from the network in a centralized node. In this regard, a minimum mean squared error (MMSE) problem is formulated for unbiased class of estimators, where a stochastical model regarding sensor failure is incorporated. As it is shown, the Karush Kuhn Tucker (KKT) conditions of optimality result in a solution algorithm with a water-filling (WF) structure, which provides an analytic optimal solution. In the end, numerical simulations illustrate the effect of the different network parameters on the resulting performance.**

I. Introduction

Sensor networks are nowadays more and more applied in various fields. Their importance is growing since the technological proceedings permit the development of evermore smaller sized sensor nodes (SNs). A decreased size of SNs enables in turn the realization of high-density sensor networks with a large number of nodes. This is associated with more demand for electrical energy, which is consumed by all SNs. In this way, a smart power allocation in sensor networks receives more attention than ever. Many different approaches are proposed by scientists for special use cases. For the 'IceCube Neutrino Observatory', see [1], the publications [2] and [3] provide an optimal solution for the power allocation problem in closed-form. This solution is afterwards extended with a fast algorithm in [4]. Benefiting from the obtained power optimization mechanisms, the work in [5] investigates the life-time maximization problem in a passive distributed radar system. In case of active radars, the power allocation in sensor networks is investigated for the region of high signal-to-noise ratio in [6] and then in [7], for the general noise conditions. Other approaches like [8] explicitly try to maximize the lifetime of a battery powered sensor network while in [9] the complexity of algorithms for an optimal sensor selection is studied. Other than the limited power consumption capability, the small and inexpensive nature of SNs leads to an occasional faulty behavior, resulting from variety of issues, e.g., synchronization failure, power limits, hardware or software failures [10]. Power adjustment, among other techniques, is considered to tackle the degradation resulting from faulty SN behavior, see [11], [12] and the references therein. In this context, two approaches are common. In the first approach, the failed sensors should be detected, and excluded from the network function. This approach is addressed in [13] using the prior topological network knowledge, and in [10], via smart monitoring of the reported network measurements. In the second approach, the network performance is optimized in connection to a certain application, by incorporating the statistics of SN failure. Good examples of this approach are the works in [14], [15] which propose resilient routing strategies in the presence of occasional SN failure.

Contribution: In this work, we extend the power allocation algorithm in [2], addressing the distributed passive radar applications, into a scenario where statistics of occasional failure of SNs are taken into account. In the first step, a system model is defined where a probabilistic approach for the sensor failure is incorporated. Afterwards, an MMSE-based optimization is formulated, where both individual and collective power consumption of the SNs are limited. It is then shown that the KKT conditions of optimality result in a solution algorithm with a WF structure, which provides an analytic optimal solution. Numerical simulations illustrate the effect of the different system parameters on the resulting performance.

II. System Model

In this work we investigate a network of K amplify-and-forward (AF) passive sensor nodes, cooperating to achieve a single global observation via a fusion center (FC), see Fig 1. Both communication and sensing channels (frequency-flat fading) are assumed to be wireless and static during the observation process. The final goal of each observation is to classify (or detect) a target signal $r \in \mathbb{C}$. Each observation can be segmented into three parts: sensing, communication, and information fusion. The detailed function of each SN is discussed in [2, Section II].

A. Operation of SNs

If a target signal $r \in \mathbb{C}$ is present, each SN receives and successfully amplifies the incoming signal using an amplification coefficient $u_k \in \mathbb{C}$, with probability ζ_k. The availability probability, i.e., ζ, is introduced to describe the likeliness that a SN is available and operating in accordance to its assumed function (not defect). Furthermore, we consider two kinds of possible scenarios in case of a defect SN. Firstly, the scenario where the sensor is broken or asleep, and hence has no effect in the communication process to the FC, and secondly, the scenario where a SN is active but providing a faulty operation, due to software or hardware error. The communication with FC is performed by using orthogonal waveforms for each SN so

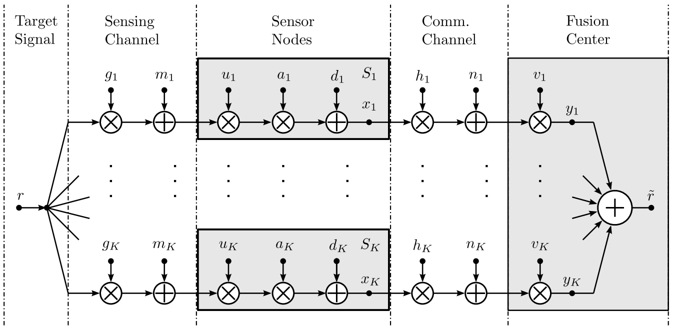

Fig. 1: Signal flow at the defined sensor network consisting of a target signal, K sensor nodes ($S_1 \cdots S_K$), and a fusion center. b_k and d_k describe the possible faulty sensor behavior, see Table I. Detailed explanation is given in Section II.

that the data from different SNs can be separated and processed in FC. The process of each SN can be hence described as

$$x_k \coloneqq a_k \left(rg_k + m_k\right) u_k + \underbrace{(1-a_k)b_k w_k}_{=:d_k}, \quad k \in \mathbb{F}_K, \qquad (1)$$

and

$$X_k \coloneqq \mathcal{E}\{|x_k|^2\}, \quad R \coloneqq \mathcal{E}\{|r|^2\}, \qquad (2)$$

where $\mathcal{E}\{\cdot\}$ represents mathematical expectation, and $\mathbb{F}_K$ represents the index set of al sensor nodes. The sensing channel coefficient, the transmit signal from the SN with index k and its power are respectively denoted by $g_k \in \mathbb{C}$, $x_k \in \mathbb{C}$ and X_k. The availability factor, i.e., $a_k \in \{0, 1\}$, where $\mathcal{E}\{a_k\} = \zeta_k$, determines if a SN is faulty, i.e., $a_k = 0$, or available, i.e., $a_k = 1$. Furthermore, the factor $b_k \in \{0, 1\}$, $\mathcal{E}\{b_k\} = \gamma_k$, determines if a faulty sensor is inactive or asleep, $b_k = 0$, or is providing a faulty signal, $b_k = 1$. As it can be observed from (1), the transmitted signal from a sensor can be simplified to $(rg_k + m_k)\, u_k$, for a correct SN operation, and into $b_k w_k$ for faulty SN operation, where b_k represents the type of sensor failure and w_k is the zero-mean faulty signal transmission with variance W_k. The additive white Gaussian noise (AWGN) on the sensing process and its variance are respectively denoted as $m_k \in \mathbb{C}$ and M_k. Furthermore, it is assumed that the power consumption of each SN may not exceed a certain limit, namely P_k, where the total average power consumption of the network is limited by P_{tot}:

$$X_k \leq P_k, \; k \in \mathbb{F}_K, \; \sum_{k \in \mathbb{F}_K} X_k \leq P_{\text{tot}}. \qquad (3)$$

B. Fusion Center

The transmitted signal from each SN passes through the communication channel, with coefficient $h_k \in \mathbb{C}$, and arrives at the FC combined with an AWGN component $n_k \in \mathbb{C}$, with variance N_k. A linear combination rule with weights $v_k \in \mathbb{C}$ is then applied at the FC to achieve an estimation, $\tilde{r}$, from the observed target signal. This is described as

$$y_k \coloneqq \left(h_k x_k + n_k\right) v_k, \qquad (4)$$

TABLE I: Used symbols and notations

Notation	Description
K	number of all SNs
$r,\ R$	target (reference) signal and its power
$\tilde{r}$	the estimate of r
g_k, h_k	complex-valued sensing and communication channel coefficients
m_k, n_k	complex-valued zero-mean AWGN at each SN and at FC
M_k, N_k	variances of m_k and n_k
u_k, v_k	complex-valued amplification factors and fusion weights
X_k	communication power of k^{th} SN
P_k	maximum allowed individual SN power
P_{tot}	maximum allowed total network power
a_k, ζ_k	coefficient representing SN correct behavior, and its expected value
b_k, γ_k	coefficient representing the type of SN failure and its expected value
w_k, W_k	possible faulty signal transmission and its variance
d_k, D_k	combined faulty signal transmission and its variance
$\mathbb{F}_K$	the index-set of all K nodes

and results in

$$\tilde{r} \coloneqq \sum_{k \in \mathbb{F}_K} y_k = r \sum_{k \in \mathbb{F}_K} a_k g_k u_k h_k v_k + \sum_{k \in \mathbb{F}_K} \left(a_k m_k u_k h_k + n_k + d_k h_k\right) v_k. \qquad (5)$$

where $\tilde{r}$ represents the estimated target signal at the fusion center.

C. Remarks

In the present work, we assume the availability of perfect channel information for both sensing and communication channels. Furthermore, it is assumed that the network sensor failures follow the defined model with a known statistics, i.e., ζ_k, γ_k, W_k are known. Such statistics usually follow a Markov chain model, in which the values of sensor failure probability can be updated in every observation, taking into account the validity of network measurements in different nodes, see [10], [16]. On the other hand, in general, it is rather difficult to estimate the sensing channel in an accurate way unless the channel has a highly stationary nature, see, e.g., [1]. Hence, for scenarios where the sensing channel is not stationary, or the statistics regarding node failure is not tractable, the results of this paper can be treated as theoretical limits. In the following parts of this paper, we aim at providing an MMSE-based design of the system parameters, focusing on the unbiased class of estimators. Table I presents the used notations for different signals and system parameters.

III. MMSE Design of Network Parameters Under Occasional Sensor Node Failures

In this section we propose an MMSE design of the network parameters for unbiased class of estimators. The statistics of the SN failure are taken into account, as defined in Section II. In the first step, we observe that the unbiased estimation property can be written as

$$\mathcal{E}\left\{\sum_{k \in \mathbb{F}_K} a_k g_k u_k h_k v_k\right\} = \sum_{k \in \mathbb{F}_K} \mathcal{E}\left\{a_k\right\} g_k u_k h_k v_k = \sum_{k \in \mathbb{F}_K} \zeta_k g_k u_k h_k v_k = 1, \qquad (6)$$

using identity (5), and the the fact that m_k, n_k, and w_k are zero mean. Subsequently, the mean squared error of the estimation can be written as

$$\begin{aligned} V :&= \mathcal{E}\{|\tilde{r}-r|^2\} \\ &= \mathcal{E}\Bigg\{\Bigg| -r + r\sum_{k\in\mathbb{F}_K} a_k g_k u_k h_k v_k \\ &\quad + \sum_{k\in\mathbb{F}_K} \left(a_k m_k u_k h_k + d_k h_k + n_k\right) v_k \Bigg|^2\Bigg\} \\ &= \mathcal{E}\Bigg\{\Bigg| r\sum_{k\in\mathbb{F}_K} (a_k - \zeta_k) g_k u_k h_k v_k \\ &\quad + \sum_{k\in\mathbb{F}_K} \left(a_k m_k u_k h_k + d_k h_k + n_k\right) v_k \Bigg|^2\Bigg\}, \end{aligned} \tag{7}$$

where V represents the estimation mean squared error (MSE). By exploiting the fact that the elements of noise, target signal, and the sensor failure coefficients are mutually independent, the formulated MSE can be simplified as

$$\begin{aligned} V =& R\sum_{k\in\mathbb{F}_K} \zeta_k(1-\zeta_k)|g_k h_k|^2 |u_k v_k|^2 \\ &+ \sum_{k\in\mathbb{F}_K} |v_k|^2 \left(\zeta_k M_k |u_k h_k|^2 + N_k + D_k|h_k|^2\right) \\ =& \sum_{k\in\mathbb{F}_K} |u_k v_k|^2 \Big(R\zeta_k(1-\zeta_k)|g_k h_k|^2 + \zeta_k M_k |h_k|^2\Big) \\ &+ |v_k|^2\left(N_k + D_k|h_k|^2\right), \end{aligned} \tag{8}$$

where $D_k = \gamma_k(1-\zeta_k)W_k$. The identity (8) provides an explicit expression of our optimization objective, i.e., estimation MSE, in terms of the adjustable network parameters u_k and v_k. Furthermore, the individual and collective power consumption constraints, see (3), can be respectively formulated as

$$\begin{aligned} & X_k \le P_k \quad \Leftrightarrow \\ & \zeta_k(R|g_k|^2 + M_k)|u_k|^2 + D_k \le P_k, \end{aligned} \tag{9}$$

and

$$\begin{aligned} & \sum_{k\in\mathbb{F}_K} X_k \le P_{\text{tot}} \quad \Leftrightarrow \\ & \sum_{k\in\mathbb{F}_K} \zeta_k(R|g_k|^2 + M_k)|u_k|^2 + D_k \le P_{\text{tot}}. \end{aligned} \tag{10}$$

As a result of the above analysis, the MSE minimization problem can be formulated as

$$\min_{u_k, v_k, \, \forall k} \quad V \tag{11a}$$

$$\text{s.t.} \quad (6), (9), (10), \tag{11b}$$

where (6), (9), (10) respectively describe the unbiased estimation constraint, and the individual and total network power constraint. The following lemma provides important information on the phase of our system parameters in the optimum point:

Lemma 1: For any optimal choice of system parameters, $u_k, v_k, \forall k \in \mathbb{F}_K$, the following parameter update is feasible and does not degrade (increase) the objective value in (11a):

$$\forall k \in \mathbb{F}_k :$$

$$v_{k,\text{new}} := |v_k| \frac{(g_k h_k)^*}{|g_k h_k| \left(\sum_{k\in\mathbb{F}_K} |\zeta_k g_k h_k u_k v_k|\right)}, \quad u_{k,\text{new}} := |u_k|, \tag{12}$$

where $(\cdot)^*$ represents conjugation.

Proof: It is clear that (11) does not violate the power constraints (9) and (10) as the absolute value of amplification factor u_k and consequently X_k are kept constant. Furthermore it is easily verified that the unbiased condition (6) still holds:

$$\sum_{k\in\mathbb{F}_K} \zeta_k g_k h_k u_{k,\text{new}} v_{k,\text{new}} = \frac{\sum_{k\in\mathbb{F}_K} |\zeta_k g_k h_k u_k v_k|}{\sum_{k\in\mathbb{F}_K} |\zeta_k g_k h_k u_k v_k|} = 1. \tag{13}$$

On the other hand, due to (6) and the triangular inequality we have:

$$\sum_{k\in\mathbb{F}_K} |\zeta_k g_k h_k u_k v_k| \ge \sum_{k\in\mathbb{F}_K} \zeta_k g_k h_k u_k v_k = 1, \tag{14}$$

which shows that the variable update (12) does not increase the norms of v_k and u_k and hence does not increase the objective value (8). ■

The above lemma shows that the real-valued assumption for $u_k, \ k \in \mathbb{F}_K$ does not reduce the optimality. Furthermore, it provides us with an optimal choice of $\angle v_k$ and simplifies our optimization problem into finding $|u_k|, |v_k|, k \in \mathbb{F}_K$

$$u_k \in \mathbb{R}^+, \quad v_k = |v_k| \angle \left(g_k h_k\right)^*, \tag{15}$$

where $\angle(\cdot)$ represents the phase. In the following parts of this section, we aim at providing an analytical solution to (11). In the first step, we recognize that the objective, as well as the constraints in (11) are separately convex (and not jointly convex) with respect to u_k and v_k. Furthermore, the defined power constraints in (9) and (10) are invariant to the choice of v_k. Hence, for any arbitrary value of u_k which satisfies the power constraints (9) and (10), the optimal v_k can be found as

$$\min_{v_k, \, k\in\mathbb{F}_K} \quad V \tag{16a}$$

$$\text{s.t.} \quad \sum_{k\in\mathbb{F}_K} \zeta_k g_k u_k h_k v_k = 1, \tag{16b}$$

where V is formulated in (8). Note that due to the convexity of (16), the optimal solution can be obtained by constructing the Lagrangian function and locating the corresponding stationary points, see [17]. The corresponding Lagrangian function to (16) can be hence formulated as

$$\begin{aligned} L\left(|v_k|, \lambda\right) = & \sum_{k\in\mathbb{F}_K} |u_k v_k|^2 \Big(R\zeta_k(1-\zeta_k)|g_k h_k|^2 + \zeta_k M_k |h_k|^2\Big) \\ & + |v_k|^2\left(N_k + D_k|h_k|^2\right) \\ & + \lambda\left(1 - \sum_{k\in\mathbb{F}_K} \zeta_k |g_k h_k| \cdot u_k \cdot |v_k|\right). \end{aligned} \tag{17}$$

By putting the derivative of (17) to zero, and following the similar procedure as [2, equation 19-21], we obtain

$$\lambda = 2V,$$
$$|v_k| = \frac{\zeta_k u_k |h_k g_k| V}{u_k^2 \zeta_k |h_k|^2 \big(M_k + R(1-\zeta_k)|g_k|^2\big) + N + D|h_k|^2}, \quad (18)$$

and

$$V = \left(\sum_{k\in\mathbb{F}_K} \frac{\tilde{u_k}\zeta_k^2}{\tilde{u_k} \underbrace{\left(\frac{\zeta_k M_k}{|g_k|^2} + R\zeta_k(1-\zeta_k) \right)}_{=:\alpha_k} + \underbrace{\frac{(N_k + D_k|h_k|^2)}{|g_k h_k|^2}}_{=:\beta_k}} \right)^{-1} \quad (19)$$

where $\tilde{u_k} := u_k^2$, and the constants α_k and β_k are defined in (19). Note that the identity (19) describes the minimum obtainable MSE value for a given set of u_k, when $v_k, k \in \mathbb{F}_K$ are optimally chosen with unbiased estimation constraint, see (6). The optimal u_k can be subsequently obtained via the optimization problem

$$\min_{\tilde{u_k},\, k\in\mathbb{F}_K} \quad -\sum_{k\in\mathbb{F}_K} \zeta_k^2 \frac{\tilde{u_k}}{\tilde{u_k}\alpha_k + \beta_k} \quad (20a)$$
$$\text{s.t.} \quad \sum_{k\in\mathbb{F}_K} c_k \tilde{u_k} \le U_{\text{tot}}, \quad (20b)$$
$$c_k \tilde{u_k} \le U_k, \;\; \tilde{u_k} \ge 0, \;\; k \in \mathbb{F}_K, \quad (20c)$$

where the $c_k := \zeta_k(R|g_k|^2 + M_k)$ and $U_k := P_k - D_k$ and $U_{\text{tot}} := P_{\text{tot}} - \sum_{k\in\mathbb{F}_K} D_k$, see (9), (10). As it can be observed, the defined problem in (20) is a convex optimization problem, due to the convexity of the objective and the affine nature of its constraints over $\tilde{u_k}$. As a result, the well known KKT conditions of optimality, provide a globally optimal solution, see [17], [18]. The corresponding Lagrangian function for (20) can be hence formulated as

$$L(\tilde{u_k}, \lambda, \tau_k, \varrho_k) := -\tau_k \tilde{u_k} - \sum_{k\in\mathbb{F}_K} \zeta_k^2 \frac{\tilde{u_k}}{\tilde{u_k}\alpha_k + \beta_k} + \lambda \left(-U_{\text{tot}} + \sum_{k\in\mathbb{F}_K} c_k \tilde{u_k} \right) + \sum_{k\in\mathbb{F}_K} \varrho_k (c_k \tilde{u_k} - U_k), \quad (21)$$

where λ, τ_k, and ϱ_k represent slack variables. The KKT optimality conditions can be subsequently expressed as

$$\varrho_k^\star \ge 0, \;\; k \in \mathbb{F}_K, \quad (22a)$$
$$\tau_k^\star \ge 0, \;\; k \in \mathbb{F}_K, \quad (22b)$$
$$\lambda^\star \ge 0, \;\; k \in \mathbb{F}_K, \quad (22c)$$
$$\tau_k^\star \tilde{u_k}^\star = 0, \;\; k \in \mathbb{F}_K, \quad (22d)$$
$$\varrho_k (c_k \tilde{u_k}^\star - U_k) = 0, \;\; k \in \mathbb{F}_K, \quad (22e)$$
$$\lambda \left(-U_{\text{tot}} + \sum_{k\in\mathbb{F}_K} c_k \tilde{u_k}^\star \right) = 0, \quad (22f)$$
$$0 \le \tilde{u_k}^\star \le U_K / c_k, \;\; k \in \mathbb{F}_K, \quad (22g)$$
$$\sum_{k\in\mathbb{F}_K} c_k \tilde{u_k}^\star \le U_{\text{tot}}, \quad (22h)$$

and

$$\frac{\partial}{\partial \tilde{u_k}} L\Big(\tilde{u_k}^\star, \lambda^\star, \tau_k^\star, \varrho_k^\star\Big) = 0 \;\; \Leftrightarrow$$
$$c_k(\lambda^\star + \varrho_k^\star) - \tau_k^\star = \frac{\zeta_k^2 \beta_k}{(\tilde{u_k}^\star \alpha_k + \beta_k)^2} \;\; \Leftrightarrow$$
$$\lambda^\star = \frac{\tau_k^\star}{c_k} - \varrho_k^\star + J_k(\tilde{u_k}^\star), \quad (22i)$$

where $J_k(\tilde{u_k}) := \frac{\zeta_k^2 \beta_k / c_k}{(\tilde{u_k}\alpha_k + \beta_k)^2}$, and $(\cdot)^\star$ indicates optimality. In the following lemma, we provide a few observations on the conditions (22a)-(22i) which lead us to the final solution.

Lemma 2: The following conditional arguments hold at the optimality:

$$\tilde{u_k}^\star = 0 \;\; \Leftrightarrow \;\; \lambda^\star \ge J_k(0), \quad (23a)$$
$$\tilde{u_k}^\star = U_k / c_k \;\; \Leftrightarrow \;\; \lambda^\star \le J_k(U_k/c_k), \quad (23b)$$
$$0 < \tilde{u_k}^\star < U_k/c_k \;\; \Rightarrow \;\; \lambda^\star = J_k(\tilde{u_k}^\star). \quad (23c)$$

In order to emphasize the special role of $\lambda^\star$ in (22a)-(22i) we name it as *water-level* hereinafter. Furthermore, we name the SN indices for which $\tilde{u_k}^\star = 0$, $\tilde{u_k}^\star = U_k/c_k$, and $0 < \tilde{u_k}^\star < U_k/c_k$, respectively as $\mathbb{K}_0$, $\mathbb{K}_{\text{sat}}$, and $\mathbb{K}$.

Proof: If $\tilde{u_k}^\star = 0$, then we have $\varrho_k^\star = 0$ due to (22e) and $\tau_k^\star \ge 0$ due to (22b). This concludes $\lambda^\star \ge J_k(0)$ according to (22i). Furthermore, if $\tilde{u_k}^\star \neq 0$, together with (22g) it results in $\tilde{u_k}^\star > 0$. This results in $\tau_k^\star = 0$, due to (22d) and subsequently $\lambda_k^\star \le J_k(\tilde{u_k}^\star) < J_k(0)$. This concludes the conditional argument in (23a). The proof to (23b) is similar to that of (23a), by exchanging the role of (22a) with (22b) and the role of (22d) with (22e). In the case where $0 < \tilde{u_k}^\star < U_k/c_k$, we have $\varrho_k^\star = 0$ and $\tau_k^\star = 0$, from (22d) and (22e). This readily results in $\lambda^\star = J_k(\tilde{u_k}^\star)$ from (22i). ■

Lemma 3: The constraint on total network power consumption, i.e., (20b), is tight in the optimality, unless we have $\sum_{k\in\mathbb{F}_K} P_k < P_{\text{tot}}$, in which case we have $\tilde{u_k}^\star = U_k/c_k, \; k \in \mathbb{F}_K$.

Proof: If (20b) is not tight in the optimum point, then we have $\lambda^\star = 0$, due to (22f). Hence, due to (23b), we have $\tilde{u_k}^\star = U_k/c_k$, which corresponds to maximum power consumption at each node. The latest statement results in $\sum_{k\in\mathbb{F}_K} P_k < P_{\text{tot}}$ when (20b) is not tight. Moreover, it is clear that if $\sum_{k\in\mathbb{F}_K} P_k < P_{\text{tot}}$, the total network consumed power may never reach P_{tot}, which concludes the proof. ■

The importance of Lemma 2, in connection to Lemma 3, lies in the fact that it defines clear borders, on how the water-level, i.e., value of $\lambda^\star$, is related to the classification of the SNs into the sets $\mathbb{K}_0, \mathbb{K}_{\text{sat}}$, and $\mathbb{K}$. As a result, for a correct classification of the nodes, the water-level is positioned such that

$$\max_{k\in\mathbb{K}_0} \Big\{ J_k(0) \Big\} \le \lambda^\star \le \min_{k\in\mathbb{K}_{\text{sat}}} \Big\{ J_k(U_k/c_k) \Big\}. \quad (24)$$

As it reasonably arises, our solution strategy is to choose $\lambda^\star$ as a search variable, and use the results of Lemma 2 to identify the correct status for all SNs. By obtaining the defined borders in (23a) and (23b) and sorting them as an increasing sequence, see Fig. 2, we obtain $2K+1$ incremental regions to look for the optimal water-level value. Nevertheless, in order to construct our search procedure, we still need an explicit criteria

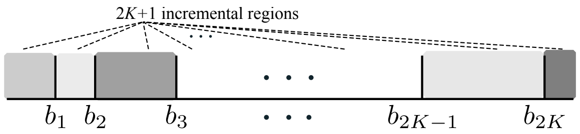

Fig. 2: We obtain $2K+1$ incremental regions in order to search for the value of water-level, i.e., $\lambda^{\star}$, where $b_1 \leq \cdots \leq b_{2K} \leftarrow \text{sort}\{J_k(0),\ J_k(U_k/c_k),\ \forall k \in \mathbb{F}_K\}$.

to determine if a value of $\lambda^{\star}$ fits into a selected region. By exploiting (23c) we have

$$\tilde{u_k}^{\star} = \frac{\zeta_k}{\alpha_k}\sqrt{\frac{\beta_k/c_k}{\lambda^{\star}}} - \frac{\beta_k}{\alpha_k}, \quad k \in \mathbb{K}, \tag{25}$$

which together with Lemma 3 and (10) results in

$$\lambda^{\star} = \left(\frac{P_{\text{tot}} - \sum_{\mathbb{K}_0 \cup \mathbb{K}} D_k - \sum_{\mathbb{K}_{\text{sat}}} P_k + \sum_{\mathbb{K}} \frac{\beta_k c_k}{\alpha_k}}{\sum_{\mathbb{K}} \frac{\zeta_k \sqrt{\beta_k c_k}}{\alpha_k}} \right)^{-2}. \tag{26}$$

Note that the obtained identity (26) can act as a test for an assumed region of water-level. This stems from the fact that an assumed region of $\lambda^{\star}$, see Fig. 2, results in a unique classification of SNs into the sets $\mathbb{K}_0, \mathbb{K}_{\text{sat}}$, and $\mathbb{K}_{\text{act}}$. As the set membership of all SNs are identified, the exact value of $\lambda^{\star}$ can be calculated from (26), which shows if the assumed region of the water-level is accurate. Algorithm 1 defines the detailed procedure to reach the optimal classification of the nodes, and consequently the optimal values for u_k and v_k.

A. Algorithm description

The procedure in Algorithm 1 is based on a bi-section search on the obtained incremental regions for the position of $\lambda^{\star}$, see Fig. 2. For any selected region, the correct status of all nodes are determined according to Lemma 2. On the other hand, for the obtained status of SNs we achieve the corresponding value of water-level, i.e., $\lambda^{\star}$, via (26) which indicates whether the selected region is correct, too big or too small. The number of required iterations for obtaining the correct region is upper-bounded by $\log_2(2K+1)+1$, following the bi-section search steps. At the end, the optimal value of water-level, along with the optimal transmit power values and the subsets $\mathbb{K}, \mathbb{K}_0$ and $\mathbb{K}_{\text{sat}}$ are determined. Optimal values for u_k and v_k can be respectively obtained as $u_k^{\star} = \sqrt{\tilde{u_k}^{\star}}$, considering (15), and via (18) and (19).

IV. Simulation Results

In this part we investigate the performance of the defined system via numerical simulations. We simulate a network with $K = 300$ SNs, where all sensing and communication channels are zero-mean and follow a Gaussian distribution with variance σ_g^2 and σ_h^2, respectively. We assume that availability probabilities, ζ_k, are randomly distributed in the range $[\zeta_{\max,k}, 1]$ where γ_k values are equally distributed between $[0, 1]$. Unless stated otherwise, the given values in Table 2 are used as the simulated network parameters. For each set of channel realizations, i.e, $h_k, g_k, \forall k \in \mathbb{F}_K$, 1000 realizations of $r, n_k, m_k, w_k, a_k, b_k,\ \forall k \in \mathbb{F}_K$ are generated, following the defined statistics. The resulting network performance is then averaged over 10000 channel realizations. In Fig. 3, the performance of the proposed Algorithm 1, in terms of estimation MSE, is compared to an optimal power allocation scheme in which the occasional SN failures are not taken into account [2]. The curves with 'Ref' legend, represent a curve following the reference values in Table II. For each of the other curves, the used parameters are similar to that of the reference curve, with the exception of the parameters specified in the legend. For both figures, dashed lines represent the performance of the algorithm in [2], where solid lines represent the performance result of the Algorithm 1. It is observed that the proposed robust algorithm consistently outperforms the optimal power allocation without failure considerations. Furthermore, the obtained performance gain increases, as the availability probability is reduced. As expected, we observe that the resulting MSE is increased for higher noise level, higher target signal power, as well as higher power of the faulty signal at the SNs (and vice versa). Note that the extreme point where $\zeta_{\max,k} = 1$, results in $\zeta_k = 1,\ \forall k \in \mathbb{F}_K$, where $\zeta_{\max,k} = 0$ means that the values of $\zeta_k,\ \forall k \in \mathbb{F}_K$ are randomly taken from the region $[0, 1]$. In Fig. 4, the target signal to error power ratio (SER), i.e., RV^{-1}, is plotted with respect to the different values of target signal power, R. Different curves represent different scenarios of noise ($M_k = N_k$), maximum individual transmit power (P_k), and W_k and ζ_k values. While Fig. 3 shows an increase in the estimation MSE when R is increased, it is shown that the value of SER is increasing with R if the proposed robust design is applied. Furthermore, an increasing gain is observed compared to the non-robust design, as the target signal becomes more powerful. It is expected, since for high values of R, the value of MSE is dominated with the R-dependent mismatch, see first part in (7) or (8), which should be correctly addressed via a design with SN-failure awareness.

TABLE II: Reference simulation parameters

K	R	M_k	N_k	W_k	$\zeta_{\min,k}$	σ_g^2	σ_h^2	P_k	P_{tot}
300	1	1	1	0.2	0.5	1	1	1	60

V. Conclusion

Small and inexpensive nature of the sensor nodes allow for a dense and distributed deployment of sensors, which serves best for many applications. At the same time, it rises issues of smart energy management, and dealing with occasional sensor failures. In this paper, we addressed the problem of optimal power allocation for a distributed passive radar system, under a known statistics of noise and sensor failure. An optimal MMSE-based solution is presented for unbiased class of estimators, applying the known KKT conditions of optimality. As it is expected, numerical simulations show that the gain of the proposed algorithm is significant for a network with high node failure probability, given that the statistics of node failure are known.

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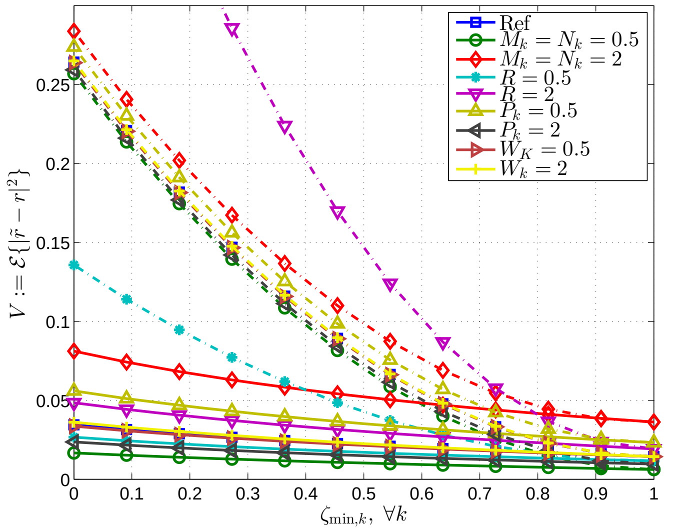

Fig. 3: Mean squared error performance vs. $\zeta_{\max,k}, \forall k \in \mathbb{F}_K$. Performance gain via robust design increases as failure probablity increases.

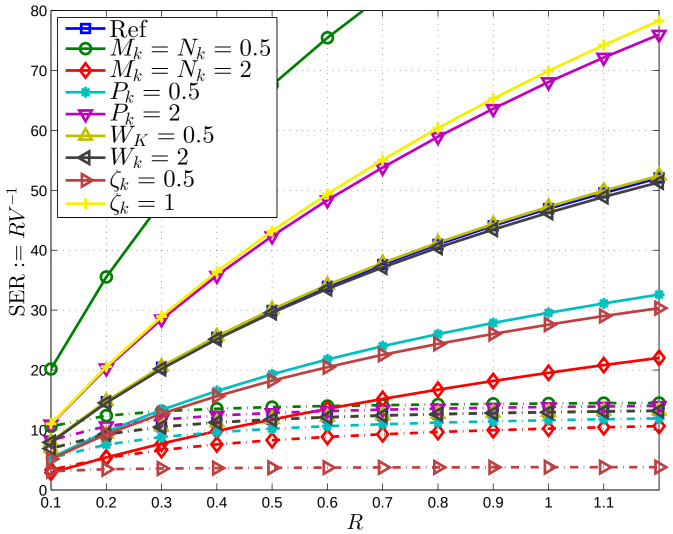

Fig. 4: Reference signal to error power ratio vs. target signal power (R). A SN failure-aware design becomes more critical for bigger values of R.

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Algorithm 1 A water-filling algorithm to achieve an optimal unbiased MMSE design.

$b_1 \le \cdots \le b_{2K} \leftarrow$ sort $\{J_k(0),\ J_k(U_k/c_k),\ \forall k \in \mathbb{F}_K\}$ ▷ see (23)
$i_{\min} \leftarrow 1,\ \ i_{\max} \leftarrow 2K+1$
repeat
  $i \leftarrow \lfloor \frac{i_{\min}+i_{\max}}{2} \rfloor$
  $\mathbb{K}_0 \leftarrow \{k \in \mathbb{F}_K \mid b_i \ge J_k(0)\}$ ▷ see (23a)
  $\mathbb{K}_{\mathrm{sat}} \leftarrow \{k \in \mathbb{F}_K \mid b_{i+1} \le J_k(U_k/c_k)\}$ ▷ see (23b)
  $\mathbb{K} \leftarrow \mathbb{F}_K \setminus (\mathbb{K}_{\mathrm{sat}} \cup \mathbb{K}_0)$
  $P_{\mathrm{remain}} \leftarrow P_{\mathrm{tot}} - \sum_{k \in \mathbb{K}_{\mathrm{sat}}} P_k - \sum_{k \in \mathbb{K}_0} D_k$
  if $\mathbb{K} = \emptyset$ **then**
    if $P_{\mathrm{remain}} = 0$ **then**
      break
    else if $P_{\mathrm{remain}} < 0$ **then**
      $i_{\min} \leftarrow i$
    else if $P_{\mathrm{remain}} > 0$ **then**
      $i_{\max} \leftarrow i$
    end if
  else
    $\lambda^\star \leftarrow$ see (26)
    if $\lambda^\star > b_{i+1}$ **or** $P_{\mathrm{remain}} < 0$ **then**
      $i_{\min} \leftarrow i$
    else
      $i_{\max} \leftarrow i$
    end if
  end if
until $(\lambda^\star > b_i$ **and** $\lambda^\star < b_{i+1})$
$\tilde{u_k}^\star \leftarrow$ see (25), $k \in \mathbb{K}$
return $(\mathbb{K}_{\mathrm{sat}}, \mathbb{K}_0,$ and $\tilde{u_k}^\star, k \in \mathbb{K})$

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