# Comparing Several Power Allocation Strategies for Sensor Networks

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Abstract—The power consumption of sensors is a crucial point for developing large-scale sensor networks nowadays. Many methods are proposed in the literature in order to optimize the power allocation to the sensor nodes under several constraints and different system aspects. In the present publication, we present a uniform framework to enable the comparison of different power allocation methods within the same scenario. Five state-of-the-art optimization techniques are discussed and their performance is compared based on simulation results. Since the system performance is a function of the service lifetime, power consumption, parameter limitations and system reliability, particular scenarios are considered in order to obtain meaningful results for the sensitivity of all five optimization strategies.

*Index Terms*—Non-convex optimization; lifetime maximization; passive radar; distributed sensing;

# I. INTRODUCTION

The fast rise of 5<sup>th</sup> generation wireless systems (5G) pushes massively the development of sensor networks for various applications. In most of these applications, the sensor nodes (SNs) are used for sensing the environment and monitoring of targets. These tasks are performed independently and in a distributed fashion by the SNs, which are rather weak in their abilities, e.g., poor signal processing units, low signal-to-noise ratio (SNR) regions for data transmission, and limited power supplies or non-renewable energy sources. The bottleneck of such networks, especially when the number of SNs is huge and the size of each SN is tiny, is the limited service lifetime. In order to prolong the lifetime, two different strategies are pursued by the scientists. The first strategy is to minimize the power consumption of each SN by smart hardware implementations while the second one is to minimize the power consumption or to maximize the lifetime of the whole network by performing intelligent power optimization techniques. In general, both strategies benefit from each other and are thus conclusively justified for common applications. Since the first strategy is specific for a single SN, a comparison of different hardware implementations is less difficult compared to the second strategy in which all SNs with many system parameters are involved. These system parameters are usually scenario dependent and hence can influence the system performance massively. In addition, a comparison in real scenarios is further exacerbated, since the important parameters are often random, e.g., the position of the distributed SNs, the distances between the SNs and the moving targets, the fluctuation of noise and channel realizations, and so on. Hence, reliable frameworks

for comparing different methods of power minimization or lifetime maximization techniques are rarely developed and seldom proposed by scientists. We aim to close this gap by establishing a solid framework in the present work, in order to enable a fair assessment of different optimization techniques.

In practice, the mostly engaged power allocation technique is the uniform scheme such that all SNs are allocated with equal power. As an example, we can mention the neutrino telescope 'IceCube Neutrino Observatory' at the Amundsen-Scott South Pole Station [1] in Antarctica, where a network with over 5000 nodes is implemented. The uniform allocation of power is the simplest method, since neither a channel state information nor a complicated optimization procedure is needed for the power distribution. For a smart optimization technique usually an objective along with few system constraints as well as a centralized unit or a decentralized approach for regulation and controlling of the sensor network are needed. Furthermore, the optimization procedure will be successful when additional information is available such as the channel state information or the statistical properties of involved random variables. In the literature, the most well-known objectives are the minimization of the mean squared error [2], [3] (MSE) and the maximization of the data rate or system capacity [4]-[8]. The latter one is a challenging approach, since a closed-form formula describing the capacity of general sensor networks is missing in the literature. Hence, the former approach receives more attention for investigation. For example, in [9]-[16] the MSE approach is applied to achieve optimal solutions in closed-form to the power allocation problem. The optimization problems there are subject to individual power and sum power constraints. Furthermore, a fusion center is considered in order to combine the independent observations of the SNs into a single reliable estimate of the target. Apart from these techniques, several powerful optimization strategies are proposed, which however have a minor importance for the present work. Nevertheless, we want to mention the work [17], where a cluster-based approach and a centralized routing protocol is used to improve the performance. Moreover, a theoretical upper bound is investigated in [18], which is in practice not achievable. Finally, a further notable publication is [19] in which different heuristics are proposed.

In the present work, our goal is the comparison of five state-of-the-art optimization techniques for allocation of power in sensor networks. We start with the presentation of the optimal power allocation strategy, then the uniform power allocation strategy is discussed, and finally we treat the singlenode-selection strategy. In between, both the optimal and the uniform power allocation strategies with occasional node failures are considered. We show the performance of each optimization technique via numerical simulations and discuss their advantages and disadvantages.

We start our investigations with the description of the underlying system model and scenarios in the next section. Subsequently, all optimization strategies are explained and presented. Then, simulation results are shown and discussed. Afterwards, we summarize our contributions in the conclusion.

# Mathematical Notations:

Throughout this paper, we denote the sets of natural, real and complex numbers by  $\mathbb{N}$ ,  $\mathbb{R}$  and  $\mathbb{C}$ , respectively. Note that the set of natural numbers does not include the element zero. Moreover,  $\mathbb{R}_+$  denotes the set of non-negative real numbers. Furthermore, we use the subset  $\mathbb{F}_N \subseteq \mathbb{N}$ , which is defined as  $\mathbb{F}_N \coloneqq \{1, \ldots, N\}$  for any given natural number N. We denote the absolute value of a real or complex-valued number z by |z|. The expected value of a random variable v is denoted by  $\mathcal{E}[v]$  while the probability that an event A is occurred is described by  $\operatorname{Prob}(A)$ . Moreover, the notation  $V^*$  stands for the value of an optimization variable V where the optimum is attained.

#### II. THE SYSTEM MODEL

In this section, we use the same system model that is elaborated in [11] for the most optimization strategies. This system model is depicted in Figure 1 and is briefly presented in the following. For a system in which occasional node failures are allowed, we use an extended system model, which is described in the next subsection. An overview of all system parameters is given in Table I.

We assume a discrete time system with perfect time, phase and frequency synchronization. A sensor network consisting of  $K \in \mathbb{N}$  independent and spatially distributed SNs is considered, where each SN receives random observations from a jointly observed target in each observation process. If a target signal  $r \in \mathbb{C}$  with  $R := \mathcal{E}[|r|^2]$  and  $0 < R < \infty$  is present, then the received power at the SN  $S_k$  is a part of the emitted power from the actual target. Each received signal is weighted by the corresponding channel coefficient  $g_k \in \mathbb{C}$ and is disturbed by additive white Gaussian noise (AWGN)  $m_k \in \mathbb{C}$  with  $M_k := \mathcal{E}[|m_k|^2] < \infty$ . In this paper, we assume that the sensing channel is constant, i.e.,  $\mathcal{E}[g_k] = g_k$  and  $\mathcal{E}[|g_k|^2] = |g_k|^2$ . The sensing channel is obviously wireless.

All SNs continuously take samples from the disturbed received signal and amplify them by the amplification factors  $u_k \in \mathbb{R}_+$  without any additional data processing. Thus, the output signal and the expected value of its transmission power are described by

$$x_k \coloneqq (rg_k + m_k)u_k, \ k \in \mathbb{F}_K, \tag{1}$$



Fig. 1. The system model of the distributed sensor network, which shows the signal flow from a target signal over the sensor nodes to a fusion center.

and

$$X_k \coloneqq \mathcal{E}[|x_k|^2] = (R|g_k|^2 + M_k)u_k^2, \ k \in \mathbb{F}_K,$$
(2)

respectively. The local measurements are then transmitted to a fusion center which is placed at a remote location. The data communication between each SN and the fusion center can be either wired or wireless. In the latter case, a distinct waveform for each SN is used to distinguish the communication of different SNs and to suppress inter-user (internode) interferences at the fusion center. Hence, all K received signals at the fusion center are pairwise uncorrelated and are assumed to be conditionally independent. Each received signal at the fusion center is also weighted by the corresponding channel coefficient  $h_k \in \mathbb{C}$  and is disturbed by additive white Gaussian noise  $n_k \in \mathbb{C}$  with  $N_k := \mathcal{E}[|n_k|^2] < \infty$ , as well. We assume that the communication channel is also constant, i.e.,  $\mathcal{E}[h_k] = h_k$  and  $\mathcal{E}[|h_k|^2] = |h_k|^2$ .

The noisy received signals at the fusion center are weighted by the fusion weights  $v_k \in \mathbb{C}$  and combined together in order to obtain a single reliable observation  $\tilde{r}$  of the actual target signal r. In this way, we obtain

$$y_k \coloneqq \left( (rg_k + m_k)u_k h_k + n_k \right) v_k \,, \ k \in \mathbb{F}_K \,, \tag{3}$$

and hence,

$$\tilde{r} \coloneqq \sum_{k=1}^{K} y_k = r \underbrace{\sum_{k=1}^{K} g_k u_k h_k v_k}_{\text{signal gain}} + \underbrace{\sum_{k=1}^{K} (m_k u_k h_k + n_k) v_k}_{\text{noise}} .$$
(4)

Note that the fusion center can separate the input streams because the data communication is either wired or performed by distinct waveforms for each SN.

In order to obtain a single reliable observation at the fusion center, the value  $\tilde{r}$  should be a good estimate of the present target signal r. Thus, the amplification factors  $u_k$  and the weights  $v_k$  should be chosen such as to minimize the average absolute deviation between  $\tilde{r}$  and the true target signal r. Hence, the amplification factors and the fusion weights are the only optimization parameters to accomplish a given optimization strategy.



Fig. 2. The extended system model with presentation of occasional node failure.

#### A. The Extended System Model

In the above system model, it is assumed that each SN perfectly works and can be configured by a controlling unit, e.g., the fusion center, at any time. This assumption is often not valid in practice, since the SNs are usually too weak and sensitive, and thus easily damaged. Hence, we update the previous system model by an extended one, which is depicted in Figure 2 and explained in the following.

We incorporate the binary independent random variables  $a_k \in \{0,1\}$  and  $b_k \in \{0,1\}$  with the corresponding probabilities  $\operatorname{Prob}(a_k = 1) = \zeta_k$  and  $\operatorname{Prob}(b_k = 1) = \gamma_k$  into the previous system model, respectively. If  $a_k$  of the  $k^{\text{th}}$  SN is equal to one, then this SN is sound and accurate, and will work normal. In contrast, if  $a_k$  is equal to zero, then the  $k^{\text{th}}$  SN is brocken or faulty and can act in two different ways. The first way is described with  $b_k = 0$ , when the SN is completely brocken and cannot send any information. The second way is described with  $b_k = 1$ , when the SN is defect and faulty, and will send useless information  $w_k$ to the fusion center. The signal  $w_k \in \mathbb{C}$  with the power  $W_k := \mathcal{E}[|w_k|^2] < \infty$  is assumed to be zero-mean and can be interpreted as interference, which cannot be suppressed at the fusion center. In summary, we can distinguish between three operating modes *i*) The  $k^{\text{th}}$  SN is healthy with probability  $\zeta_k$ , i.e.,  $a_k = 1$  and  $b_k \in \{0, 1\}, \mathbf{i}$  it is brocken and silent with probability  $(1 - \zeta_k)(1 - \gamma_k)$ , i.e.,  $a_k = 0$  and  $b_k = 0$ , *iii*) it is defect and acts as interference with probability  $(1 - \zeta_k)\gamma_k$ , i.e.,  $a_k = 0$  and  $b_k = 1$ .

With this new setup, the output signal and the expected value of its transmission power are described by

$$x_k \coloneqq (rg_k + m_k)u_k a_k + b_k w_k (1 - a_k), \ k \in \mathbb{F}_K, \quad (5)$$

and

$$X_{k} = (R|g_{k}|^{2} + M_{k})u_{k}^{2}\zeta_{k} + \gamma_{k}W_{k}(1 - \zeta_{k}), \ k \in \mathbb{F}_{K}, \ (6)$$

respectively. This leads to the estimate

$$\tilde{r} = r \sum_{k=1}^{K} g_k u_k h_k v_k a_k + \sum_{k=1}^{K} (m_k u_k h_k a_k + b_k w_k (1 - a_k) h_k + n_k) v_k.$$
(7)

## B. Power Limitations of the System

As mentioned in the introduction, a smart hardware implementation of the SNs is highly necessary in order to prolong the lifetime and enable an energy-aware operation of the SNs. Following this trend, the average power consumption of each SN is approximately equal to its average output power  $X_k$ , if the input signal is negligible in comparison to the output signal and if the nodes have smart power components with low-power dissipation loss. We assume that equality between  $X_k$  and the average power consumption of each node is ensured. In the present work, we assume that the output power-range of each SN is in average limited by  $P_{\min} \in \mathbb{R}_+$ and  $P_{\max} \in \mathbb{R}_+$  with  $0 \le P_{\min} < P_{\max}$ . The lower limit  $P_{\min}$ denotes the minimum power which is needed to guarantee the awareness and presence of the SN while the upper limit  $P_{\text{max}}$ denotes the maximum allowed transmission power per SN due to power regulation standards or due to the functional range of the integrated circuit elements.

In addition, each SN is usually powered by weak energy supplies, e.g., batteries, such that the operation time of the entire sensor network is limited. Hence, it is wise to incorporate and consider a sum power constraint  $P_{\text{tot}} \in \mathbb{R}_+$  with  $KP_{\min} \leq P_{\text{tot}} \leq KP_{\max}$ . In this way, the sensor network shall operate under the constraints

$$P_{\min} \le X_k \le P_{\max} \,, \ k \in \mathbb{F}_K \,, \tag{8}$$

and

$$\sum_{k=1}^{K} X_k \le P_{\text{tot}} \,. \tag{9}$$

Note that the output power  $X_k$  is a function of the amplification factor  $u_k$  and thus we can adjust  $X_k$  in order to satisfy given optimization strategies.

#### **III. OPTIMIZATION STRATEGIES**

In this section, we present five state-of-the-art optimization strategies in order to enable a fair comparison and a fruitful discussion.

#### A. Optimal Power Allocation

In order to obtain an accurate estimate  $\tilde{r}$  of the target signal r, we aim at finding estimators  $\tilde{r}$  of minimum mean squared error in the class of unbiased estimators. With the aid of (4), we hence minimize the deviation

$$V \coloneqq \mathcal{E}[|\tilde{r} - r|^2] = \sum_{k=1}^{K} |v_k|^2 (u_k^2 |h_k|^2 M_k + N_k)$$
(10)

 TABLE I

 NOTATION OF SYMBOLS THAT ARE NEEDED FOR THE DESCRIPTION OF

 EACH OBSERVATION PROCESS.

Notation	Description
K	number of sensor nodes;
k	the index of $k^{\text{th}}$ sensor node;
$\mathbb{F}_{K}$	the index-set of $K$ nodes;
r, R	the present target signal and its quadratic absolute mean;
$\tilde{r}$	the estimate of r;
$g_k, h_k$	complex-valued channel coefficients;
$m_k, n_k$	complex-valued zero-mean AWGN;
$M_k, N_k$	variances of $m_k$ and $n_k$ ;
$u_k, v_k$	non-negative amplification factors and complex-valued fu-
	sion weights;
$\vartheta_k$	phase of $v_k$ ;
$\phi_k$	phase of the product $g_k h_k$ ;
$a_k, \zeta_k$	binary random variable and its expected mean to distin-
	guish between healthy and brocken sensor nodes;
$b_k, \gamma_k$	binary random variable and its expected mean to distin-
	guish between both modes of defectiveness;
$w_k, W_k$	faulty signal and its power of $k^{\text{th}}$ sensor node;
$x_k, X_k$	output signal and output power of $k^{\text{th}}$ sensor node;
$y_k$	input signals of the combiner;
$P_{\min}, P_{\max}$	lower and upper output power limitations of each sensor
	node;
Ptot	the total power consumption of the network.

with respect to  $u_k$  and  $v_k$  under the unbiasedness

$$\mathcal{E}[\tilde{r}-r] = 0 \quad \Leftrightarrow \quad \sum_{k=1}^{K} g_k u_k h_k v_k = 1.$$
(11)

In addition, the optimization problem shall be subject to both power constraints (8) and (9).

It is simple to show that this optimization problem is nonconvex and very hard to solve. Nevertheless, its optimal solution in closed-form is worked out in [11] and later extended in [20]. This solution yields the optimal amplification factors  $u_k^*$  and fusion weights  $v_k^*$  which in turn specify the optimal transmission powers  $X_k^*$ . The ratio  $\frac{R}{V^*}$  is then equivalent to the SNR of the entire sensor network and describes the quality of the estimate  $\tilde{r}$  at the output of the fusion center. This ratio will help us to fairly compare different optimization strategies by variation of different system parameters. To avoid confusion, we hereinafter denote this ratio by SNR<sub>O</sub>.

## B. Optimal Power Allocation with Occasional Node Failure

Now, we consider the more realistic system model in which occasional node failures are taken into account, cf. Subsection II-A. We again are interested in finding estimators of minimum mean squared error in the class of unbiased estimators. By using (7), the objective for minimization is given by

$$V := \mathcal{E}[|\tilde{r} - r|^{2}]$$

$$= \sum_{k=1}^{K} |v_{k}|^{2} (u_{k}^{2} |h_{k}|^{2} M_{k} \zeta_{k} + N_{k})$$

$$+ \sum_{k=1}^{K} |v_{k}|^{2} (1 - \zeta_{k}) |h_{k}|^{2} (Ru_{k}^{2} |g_{k}|^{2} \zeta_{k} + W_{k} \gamma_{k})$$
(12)

with respect to  $u_k$  and  $v_k$  under the unbiasedness

$$\mathcal{E}[\tilde{r}-r] = 0 \quad \Leftrightarrow \quad \sum_{k=1}^{K} g_k u_k h_k v_k \zeta_k = 1.$$
(13)

In addition, the optimization problem shall be subject to both power constraints (8) and (9). Note that in contrast to the objective in (10) the deviation in (10) is a function of the parameters  $\zeta_k$  and  $\gamma_k$  and has thus more degree of freedom.

Also this optimization problem is non-convex and even harder to solve than the previous one. Its optimal solution is characterized in our paper [21]. The optimal values  $u_k^*$ ,  $v_k^*$ ,  $X_k^*$  and  $V^*$  are obviously functions over  $\zeta_k \in (0, 1)$  and  $\gamma_k \in (0, 1)$ . The SNR of the entire sensor network is again described by the ratio  $\frac{R}{V^*}$ . We hereinafter denote this ratio by SNR<sub>OwF</sub>.

Note that both SNR<sub>O</sub> and SNR<sub>OwF</sub> are functions over  $P_{\min}$ ,  $P_{\max}$  and  $P_{tot}$  as well as  $M_k$ ,  $N_k$ ,  $g_k$  and  $h_k$ . This means that for the optimal allocation strategy all these parameters must be known beforehand. However, this knowledge is often not available so that such optimization strategies are rather used for theoretical considerations. This is the main reason why uniform allocation or single-node-selection strategies are often used in practice. We will consider these methods in the following.

#### C. Uniform Power Allocation with Optimal Fusion Rule

The simplest power allocation strategy is the uniformly distributed one. That means that the total power  $P_{tot}$  is equally divided and shared between all SNs such that the output power of each SN becomes

$$X_k^{\star} = \max\left\{P_{\min}, \min\{P_{\max}, \frac{P_{\text{tot}}}{K}\}\right\}.$$
 (14)

From this we can calculate the optimal amplification factors  $u_k^*$  by the relation (2). It remains to optimize the fusion weights. If the goal is again to find estimators of minimum mean squared error in the class of unbiased estimators, then we can minimize the objective

$$V \coloneqq \mathcal{E}[|\tilde{r} - r|^2] = \sum_{k=1}^{K} |v_k|^2 (u_k^2 |h_k|^2 M_k + N_k)$$
(15)

for fixed amplification factors  $u_k = u_k^*$  and with respect to  $v_k$  under the unbiasedness

$$\mathcal{E}[\tilde{r}-r] = 0 \quad \Leftrightarrow \quad \sum_{k=1}^{K} g_k u_k^{\star} h_k v_k = 1.$$
 (16)

As it is easily to see, this optimization problem is a convex program and can again be solved in closed-form. The optimization of (15) will lead to the optimal fusion weights  $v_k^{\star}$  and the deviation  $V^{\star}$ . However, this result is a particular solution of the problem (10). To see this relation, we only need to pretend  $P_{\min} = \frac{P_{\text{tot}}}{K}$  or  $P_{\max} = \frac{P_{\text{tot}}}{K}$ , due to equation (14), and solve (10) with respect to both  $u_k$  and  $v_k$  which will then lead to the same solution.

We hereinafter denote the ratio  $\frac{R}{V^{\star}}$  for the uniform setup by SNR<sub>U</sub>.

TABLE II Standard value of system parameters.

Parameter:	K	R	$\sigma_g^2 = \sigma_h^2$	$M_k = N_k$	$P_{\min}$
Value:	300	1	1	0.1	0
Parameter:	P <sub>max</sub>	Ptot	W	$\zeta_{\min}$	$\gamma$
Value:	10	100	0.1	0.5	[0, 1]

# D. Uniform Power Allocation with Optimal Fusion Rule and Occasional Node Failure

Analog to the previous case, the total power  $P_{\text{tot}}$  is uniformly allocated to the SNs, which will result in  $X_k^*$  as given in (14). By pretending  $P_{\min} = \frac{P_{\text{tot}}}{K}$  or  $P_{\max} = \frac{P_{\text{tot}}}{K}$  we can use the results for the problem in (12) in order to obtain the optimal values  $u_k^*$ ,  $v_k^*$  and  $V^*$ . We hereinafter denote the ratio  $\frac{R}{V^*}$  by SNR<sub>UwF</sub>.

Note that any defect SN will have the output power  $X_k^{\star} = b_k W_k$ , which can totaly differ from the adjusted one, i.e.,  $X_k^{\star} = \frac{P_{\text{tot}}}{K}$ .

#### E. Best Single Node Selection

Another important method is the single-node strategy, in which only one powerful SN will consume the entire power  $P_{\text{tot}}$ , while all other SNs are inactive. This method is important, because it enables the comparison between a large-scale sensor network with many weak SNs and a powerful single node system which is established at the position of the most proper SN. The main question is which of the SNs is the most proper one? In [22], it is shown that the SN with the largest value of

$$z_k \coloneqq \frac{|g_k h_k|^2 P_{\text{tot}}}{M_k |h_k|^2 P_{\text{tot}} + N_k (R|g_k|^2 + M_k)}$$
(17)

is the most proper SN, in the sense that choosing the most proper SN will lead to the largest SNR at the fusion center for the given amount of  $P_{\text{tot}}$ . In this way, we can re-index all SNs, such that  $z_k \ge z_{k+1}$  for all  $k \in \mathbb{F}_{K-1}$  holds and select the first SN which should be virtually replaced by a powerful node. This node obtains the power  $X_1^* = P_{\text{tot}}$  and in turn  $u_1^*$ by the aid of (2). Keeping  $u_1 = u_1^*$  and  $u_k = u_k^* = 0$  fixed for all k > 1, we can use the same objective and constraint as in (10) and (11), respectively, in order to obtain the weight  $v_1^*$ and the deviation  $V^*$ . Note that (17) only describes the quality of healthy SNs. In scenarios, where SNs might be brocken, equation (17) must be extended for considering the impact of brocken and defect SNs. The entire SNR of this system is hereinafter denoted by SNR<sub>B</sub>.

#### **IV. SIMULATION BASED COMPARISONS**

In this section, we set out to compare the solution of the different optimization strategies from Section III via numerical simulations. We consider four different scenarios with randomly distributed SNs to demonstrate the performance of all optimization strategies. Our goal is to evaluate the values SNR<sub>O</sub>, SNR<sub>OwF</sub>, SNR<sub>U</sub>, SNR<sub>UwF</sub>, and SNR<sub>B</sub> for different ranges of parameters.



Fig. 3. The behavior of the SNR at the fusion center with respect to the sum power  $P_{\text{tot}}$  is visualized. All curves show an increasing property in  $P_{\text{tot}}$ .

#### A. Simulation Setup

In order to fairly evaluate the performance of all optimization strategies with each other, we perform in each scenario five simulations with the same sensor network. For each scenario new realizations are thrown for the channels, noise signals, target signal, and the probability of healthiness  $\zeta_k$ . In contrast, in all five simulations all realizations remain the same to simplify subsequent comparisons of the five optimization methods. The realizations of the channels, noise, and target signal are drown randomly from independent Gaussian distributions. In each scenario, 6500 channel realizations are generated. The variances of the channel fluctuations are denoted by  $\sigma_q^2 := \mathcal{E}[|g_k|^2]$  and  $\sigma_h^2 := \mathcal{E}[|h_k|^2]$  with expectation over k. For each channel realization, 1000 random realizations are generated for each noise signal  $m_k$  and  $n_k$ , the target signal r, the probabilities  $\zeta_k$ , the different modes  $a_k$  and  $b_k$ , and the unwanted signal  $w_k$ . For the noise signals we always consider the same variances, i.e.,  $M_k = N_k$  for all k. The target signal r is randomly generated with zero-mean and variance R. The random values  $\zeta_k$  are drown from a uniform distribution over the range  $[\zeta_{\min}, 1]$ . The modes  $a_k$  and  $b_k$  are generated by a Bernoulli distribution with expectations  $\zeta_k$  and  $\gamma_k$ , where  $\gamma_k$  itself is uniformly distributed over the interval [0, 1]. The unwanted signal  $w_k$  is also zero-mean Gaussian distributed with variance W.

In each scenario we consider the influence of one of the four parameters  $P_{\text{tot}}$ , R,  $N_k = M_k$  and  $\zeta_{\min}$  on the SNR at the fusion center. For this task we vary the parameter under consideration while the other parameters are constant. Table II shows the standard values of all system parameters.

Hereinafter, the optimization techniques described in Sections III-A, III-B, III-C, III-D, and III-E are denoted in the legends by 'Optimal–Non-Robust', 'Optimal–Robust', 'Uniform– Non-Robust', 'Uniform–Robust', and 'Best–Selection', respectively. In this way, the effective number of iterations per simulation point is about  $6500 \times 1000$  in order to guaranty the convergence of the simulations and to obtain smooth curves. Because of the high number of iterations and the large number of SNs, it is necessary to hardly optimize the simulation framework in order to reduce the simulation time and to improve the accuracy of the results.



Fig. 4. The behavior of the SNR at the fusion center with respect to the target signal power R is visualized. All curves show an increasing property in R.

#### B. Simulation Results

In Fig. 3, we observe the simulation results of the first scenario. In this scenario, we are interested to compare the behavior of all five methods with respect to variation of the sum power  $P_{tot}$ . The first observation is that all curves are increasing in  $P_{\text{tot}}$ , which means that by increasing the sum power limitation the SNR at the fusion center will also increase. But, the 'Best-Selection' strategy shows that the increment of the SNR may also be very small, due to a potential selection of a brocken or defect SN as the most proper one. Furthermore, the results simply show that an optimal power allocation is always better than the corresponding uniform allocation. The optimal power allocation converges to the uniform strategy due to the individual power limitations given by  $P_{\text{max}}$ . On the other hand, robust optimization strategies achieve a better performance for larger values of  $P_{tot}$  than the non-robust strategies. The reason behind this behavior is the impact of the unbiasedness constraints (11) and (13). While for the high power region the unbiasedness can better be fulfilled since more SNs are involved, for the low power region the robust optimization methods must concentrate the available sum power on fewer SNs to satisfy the constraint (13). If the value of  $\zeta_k$  is small, the degeneration of the robust methods will further increase.

In Fig. 4, the increasing behavior of the SNR at the fusion center with respect to the power R of the target signal is depicted. The curves behave very similar to the ones achieved in the previous scenario with the exception of the 'Best–Selection' strategy. The curve again shows, that



Fig. 5. The behavior of the SNR at the fusion center with respect to the noise powers  $N_k = M_k$  for all k is visualized. All curves show an decreasing property in the noise powers.

the 'Best-Selection' strategy is very sensitive on failure of SNs which is similar to be highly sensitive with respect to the power of the target signal. Another difference in comparison to the previous scenario is shown between the 'Optimal-Robust' and the 'Uniform-Robust' strategies. The gap between both curves is increasing in R since this input power is always shared between all SNs by the 'Uniform-Robust' strategy and it hence cannot have much improvement on the SNR at the fusion center.

The simulation results of the third scenario are depicted in Fig. 5. We can observe a decreasing behavior of all curves with respect to the noise powers  $M_k$  and  $N_k$ . It is interesting to see that for the low noise power region the 'Optimal-Non-Robust' and the 'Optimal-Robust' strategies converge to 'Uniform-Non-Robust' and 'Uniform-Robust' strategies, respectively. But in contrast, for the high noise power region the 'Optimal-Non-Robust' and the 'Uniform-Non-Robust' strategies converge to 'Optimal-Robust' and 'Uniform-Robust' strategies, respectively. The behavior for the low noise power region is simple to explain since then the noise parts in (4) and (7) are of minor importance and the gap between both robust strategies and non-robust ones is due to the expression  $\sum_{k=1}^{K} b_k w_k (1-a_k) h_k v_k$  in (7). In the high noise power region this expression can be neglected and since the signals are noisy the both optimal solutions will behave similar and different from both uniform solutions. The 'Best-Selection' strategy achieves here also the worst performance.

The next scenario is for investigation of the impact caused by the probability  $\zeta_k$  on the system performance. We thus consider the curves in Fig. 6, where as usual the 'Best–Selection' strategy has the worst performance. All curves are increasing in  $\zeta_{min}$  since increasing  $\zeta_{min}$  will lead to a sensor network with less number of brocken and defect SNs. Fig. 6 highlights the importance of the robust optimization strategies. Both 'Optimal–Robust' and 'Uniform–Robust' strategies achieve a good performance in sensor networks in which the number of failure is very high.



Fig. 6. The behavior of the SNR at the fusion center with respect to the probability  $\zeta_{\min}$  is visualized. All curves show an increasing property in  $\zeta_{\min}$ .

# V. CONCLUSION

A comparison of different power optimization techniques is usually difficult, since different methods often need different frameworks and special scenarios to work properly. We have investigated five state-of-the-art optimization techniques based on the same system model, in order to achieve the same framework for further comparisons. In particular, we have discussed the optimal, the uniform, and the single-nodeselection power allocation strategies. In addition, we have extended both the optimal and the uniform power allocation strategies for the case in which some sensor nodes might be broken or defect. In practical applications, occasional node failures are usually the case, hence we have compared the robust optimization techniques against the non-robust methods within the same framework. We have seen that the singlenode-selection strategy achieves the worst performance in scenarios with occasional node failures. Optimal power allocation strategies achieve a better performance compared to the corresponding uniform allocation strategies. We have also shown that robust optimization techniques can only improve the system performance for specific ranges of parameters. For the complement range, the non-robust techniques achieve a better performance. Hence, it is important to know which ranges of parameters are given in particular scenarios in order to apply the most proper method and achieve the best performance.

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