Efficient Set-Membership Filtering Algorithms for Wireless Sensor Networks

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***Abstract*—The paper discusses three main adaptive filtering algorithms with partial updates and low computational complexities that converge fast and have a significantly better mean square error (MSE) performance than their non selective-update versions when they are tuned well. The algorithms are set-membership normalized least mean squares (SM-NLMS), SM affine projection (SM-AP) and SM recursive least squares (SM-RLS, also known as BEACON). The lifetime of a wireless sensor network (WSN) is often governed by its power consumption. We show how the previous works for energy prediction, channel estimation, localization and data replication in WSNs can be improved in both accuracy and energy conservation by employing these algorithms. We derive two simplified versions of the SM-AP and BEACON algorithms to further minimize the computational load. The probable drawbacks of the algorithms and the alternative solutions are also investigated. To exhibit the improvements and compare the algorithms, computer simulations are conducted for different scenarios. The purpose is to show that many signal processing algorithms for WSNs can be replaced by one general low complexity algorithm which can perform different tasks by minor parameter adjustments.**

I. Introduction

A wireless sensor network (WSN) consists of many densely deployed sensor nodes that are often randomly distributed in an area. The sensor nodes are small in size with low computational capacity and limited power. Due to their wide range of applications in surveillance, military, habitat monitoring, etc., they have received considerable researcher attention through the last decade. The purpose of a WSN is to collect data about a phenomenon by many sensor nodes and forward them to the sink node or a fusion center for further processing. Based on the type of the sensor nodes, application and resource budget, this might be done through a multi-hop configuration which is often due to the high energy consumption or infeasibility of a long single hop. In case of a multi-hop network, a proper relaying scheme is responsible to forward the data to the sink node or receiver. Though the realization of multi-hop networks is difficult enough, most of the constraints on the WSNs are due to the stringent restrictions in power consumption and computational capacity of the sensor nodes. In fact, the lifespan extension is one of the main considerations in designing WSNs. The power management is also included in the protocol stack of WSNs. The protocols of WSNs are specifically designed to tackle the power and complexity constraints of such networks. To guarantee a theoretically infinite lifetime, energy harvesting systems have been incorporated into WSNs in which the required power is provided by the environment. The photovoltaic cells (PVs), thermal generators (TEGs), wind generators and vibration scavengers are examples of harvesters for such networks [1]. However, the solar energy is often preferred due to the high power density, relative predictability and ease of implementation [1], [2]. The relative predictability means the power can be predicted approximately and the sensor node activity including the computation load and data communication can be adapted accordingly so that the WSN sustains the periodic power shortages while the crucial applications of the network are not compromised. Figure 1 shows a multi-hop WSN with L relay groups between the transmitter nodes and the receiver. The receiver in this model is the final fusion center equipped with multiple antennas with unlimited power and complex processing capabilities. Based on the network configuration, the sensor nodes can cooperate in transmission, estimation and decision making or they can be individual data collectors with no data or resource sharing. The figure shows a general scheme where there are direct hops from the transmitter to the destination as well as the multi-hops through the relays. Similarly, the feedback links from the receiver to the transmitter and the relay nodes are shown. The direct transmitter-receiver hop has the advantage of halting the retransmissions by the relays when this direct connection can be established. The model in Figure 1 will be further discussed in the next sections. The purpose of this paper is to show that the proposed set-membership adaptive filtering methods can improve the current algorithms for channel estimation, data replication and energy prediction in terms of accuracy and computational complexity. The rest of the paper is organized as follows: In Section II, the previous works on energy prediction, channel estimation and data replication for WSNs are briefly introduced which are based on the adaptive filtering algorithms. In Section III, the proposed set-membership filtering algorithms are discussed in detail and the improved performance in terms of accuracy and complexity reduction is shown by the simulations in Section IV.

II. Previous works

As discussed before, the purpose of energy prediction in WSNs is often to make the system self-sustainable in terms of energy. The energy harvesters provide the required energy from the environment. The type of the energy source is very important in prediction. The PV cells are often preferred

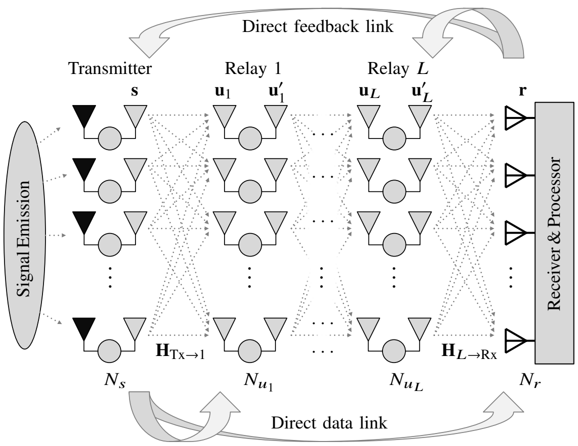

Fig. 1. A WSN with L relays. The number of sensor nodes in the transmitter, i^{th} relay and the receiver are denoted by N_s , N_{u_i} and N_r, respectively. $\mathbf{H}_{\text{Tx}\to 1}$ and $\mathbf{H}_{L\to \text{Rx}}$ denote the channel matrices of the corresponding hops.

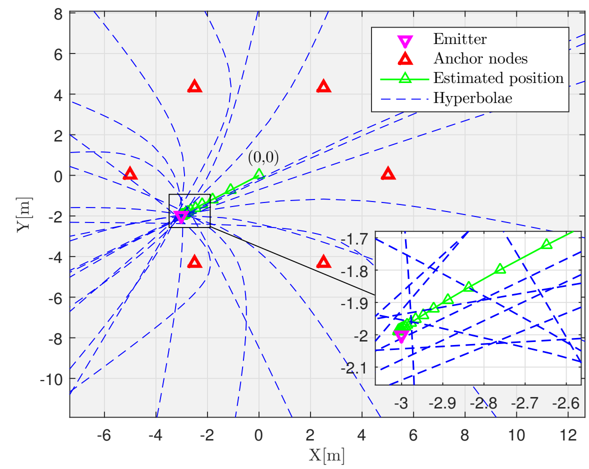

Fig. 2. The TDOA localization using six anchor nodes and NLMS with step-size $\mu = 0.4$. The noise power is $\sigma_n^2 = 0.49$. The magnified part shows how the NLMS algorithm converges to the position of the emitter.

due to their high energy density and relative predictability. In [3], an exponentially weighted moving average (EWMA) prediction was used based on [4]. For solar energy prediction, an algorithm called weather conditioned moving average (WCMA) was used in [5]. These two algorithms as well as the algorithm developed at the Swiss Federal Institute of Technology of Zurich (ETHZ) [6], [7] and a prediction based on neural networks [8] were simulated and compared in [2] in terms of complexity, memory and average error. The results of this comparison show that the WCMA algorithm has the minimum average error among the four algorithms, which is about one third of the next accurate algorithm EWMA, though it requires more memory and has a higher computational complexity. However, in [1], an adaptive filtering approach was employed for energy prediction using the normalized least mean squares algorithm (NLMS) and it was shown that in comparison to WCMA, the NLMS algorithm can remarkably reduce the complexity and memory while the average error is slightly higher than WCMA. Moreover, it also verifies that the average error of WCMA is about one third of the EWMA. As a consequence, this adaptive filtering algorithm seems to be the proper choice for energy prediction considering both of the constraints and accuracy. The adaptive filtering algorithms are also used for approximate data replication in WSNs. This technique is used in cases where the receiver can infer results from a fairly accurate estimate of the value at the source, rather than the exact value [9]. In fact, this technique reduces the rate of data transmission by transmitting only a subset of the sensed data and therefore preserves energy for the WSN. In [9], this technique has been employed using LMS, NLMS and recursive least squares (RLS) algorithms. Another approach for data reduction in WSNs was proposed by [10], where a hierarchical LMS (HLMS) algorithm was employed to accelerate the convergence speed. However, the HLMS algorithm is far more complex than the LMS and NLMS algorithms, as it combines several levels of independent LMS sub-filters. Note that the NLMS algorithm has a better MSE performance than the LMS algorithm, as it is tuned such that the a posteriori error is set to zero. In [11], a framework for combined data prediction, compression, and recovery was proposed where an optimal step size LMS algorithm (OSSLMS) was employed for prediction. The problem with the OSSLMS and the variable step size (VS) LMS algorithms is often the high complexity of the step size calculations, as derived in [12] and [13]. Moreover, some step size solutions require setting several proper constants to manage the performance of the algorithm. Therefore, these versions of the LMS algorithms often improve the MSE performance at the cost of significantly higher complexity and power consumption. Another application of the adaptive filtering in WSNs is the channel estimation, as proposed by [14] and [15]. In fact, when the a priori channel information is unavailable, the well known minimum MSE (MMSE) cannot be employed for channel estimation. Also due to the synchronization deficiency of the sensors at the transmitter of a hop or difficulty of implementation, the mutual pilot sequence orthogonality is not always possible. Therefore, the least squares (LS) estimator cannot be simplified by diagonal matrix inversion. Besides the channel estimation, the NLMS algorithm can be employed to perform channel equalization, as discussed in [16]. There are many applications where the data acquisition in the WSN should be accompanied by sensor location information. Adaptive filtering techniques can be also employed for localization. For example, the time difference of arrival (TDOA) localization has a nonlinear objective function which can be optimized using the NLMS algorithm after applying the first order Taylor series expansion, as discussed in [17]. Figure 2 illustrates this localization process, where six anchor nodes were used to locate an emitter at $(-3, -2)$. In this figure, the converging process of the NLMS algorithm from the initial guess $(0, 0)$ to the emitter's location is depicted, where the step size μ and the noise power σ_n^2 were set as $\mu = 0.4$ and $\sigma_n^2 = 0.49$, respectively. As a consequence of this section, the adaptive filtering algorithms, and in particular the NLMS algorithm, have remarkable applications in different fields of signal processing for WSNs. Therefore, if there is a general low complexity and fast converging technique in adaptive filtering with an MSE performance comparable to or better than that of the NLMS algorithm, one algorithm can be used to perform many tasks, with minor adjustments of constants for different applications.

III. Proposed Adaptive Filtering Algorithms

The algorithms proposed here are obtained by applying the set-membership filtering (SM) technique on the adaptive filtering algorithms. This filtering framework improves the MSE performance while it reduces the complexity by means of selective updates and matrix sparsity. We consider a general complex-valued matrix based estimation scheme here so that it encompasses a wide range of applications such as complex-valued channel estimation with arbitrary number of sensor nodes at the transmitter and receiver of each hop. Denoting the received vector at each hop as $\mathbf{r}$, the complex valued channel matrix as $\mathbf{H}$, the known pilot vectors as $\mathbf{s}$ and the additive white Gaussian noise (AWGN) as $\mathbf{n}$, the received vector can be expressed as $\mathbf{r}_k = \mathbf{H}_k\mathbf{s}_k + \mathbf{n}_k$, where the subscript k denotes the k^{th} time-step. For convenience, this can be considered as the transmission through the direct data link illustrated in Figure 1, where $\mathbf{r}_k, \mathbf{n}_k \in \mathbb{C}^{N_r}$, $\mathbf{s}_k \in \mathbb{C}^{N_s}$ and $\mathbf{H}_k \in \mathbb{C}^{N_r\times N_s}$. However, this is the general model for each hop where $\mathbf{r}_k$ and $\mathbf{s}_k$ can be replaced by the received vector $\mathbf{u}_i$ and the transmitted vector $\mathbf{u}'_{i-1}$, respectively, with i denoting the relay number. Depending on the relaying plan, vector $\mathbf{u}'_i$ can be an amplified-and-forward, compressed-and-forward or decoded-and-forward version of vector $\mathbf{u}_i$. Note that this is a general model based on the complex-valued matrix estimation and it can be used for the estimation of other parameters, whereas most of the adaptive filtering techniques are based on the real-valued vector estimations. Since the SM-NLMS algorithm is a special case of the SM-AP algorithm where the data reuse factor is set to 1, we discuss the latter here. The objective of the AP algorithm is to minimize $\|\mathbf{H}_{k+1} - \mathbf{H}_k\|_F^2$ subject to $\mathbf{r}_{k-i} - \mathbf{H}_{k+1}\mathbf{s}_{k-i} = \mathbf{0}\,\forall i \in \mathbb{F}_0^{P-1}$, where $P \in \mathbb{N}$ denotes the data reuse factor, F implies the Frobenius norm and $\mathbb{F}_i^j := \{i, i+1, \ldots, j\}\,\forall i,j \in \mathbb{Z}, i \le j$. This means that the next estimate, i.e., $\mathbf{H}_{k+1}$, is found such that all of the a posteriori errors are set to zero. Denoting the optimal $\mathbf{H}$ by $\mathbf{H}_o$, i.e., $\mathbf{r}_k = \mathbf{H}_o\mathbf{s}_k + \mathbf{n}_k\,\forall k$, it is clear that in a general case $\mathbf{r}_k - \mathbf{H}_o\mathbf{s}_k = \mathbf{n}_k \neq \mathbf{0}$, consequently, forcing the a posteriori errors to zero in the AP algorithm for P times in each step degrades the MSE performance, though improves the convergence speed. The SM framework resolves this issue by setting $\mathbf{r}_{k-i} - \mathbf{H}_{k+1}\mathbf{s}_{k-i} = \mathbf{g}_{k,i}\,\forall i \in \mathbb{F}_0^{P-1}$, where for a $\gamma \in \mathbb{R}^+$, $\|\mathbf{g}_{k,i}\| \le \gamma$. Defining the *feasibility set* $\mathbf{\Theta}$ as

$$\mathbf{\Theta} := \bigcap_{(\mathbf{r},\mathbf{s})\in\mathbb{S}} \{\mathbf{H} \in \mathbb{C}^{N_r\times N_s} : \|\mathbf{r} - \mathbf{H}\mathbf{s}\| \le \gamma\}$$

where $\mathbb{S}$ denotes the set of all possible data pairs $(\mathbf{r}, \mathbf{s})$, the *constraint set* at any time instant k as

$$\mathbb{H}_k := \{\mathbf{H} \in \mathbb{C}^{N_r\times N_s} : \|\mathbf{r}_k - \mathbf{H}\mathbf{s}_k\| \le \gamma\}$$

and the *membership set* at time instant k as $\boldsymbol{\psi}_k := \bigcap_{i=1}^{k} \mathbb{H}_i$, the latter can be expressed as follows

$$\boldsymbol{\psi}_k = \bigcap_{i=1}^{k-P}\mathbb{H}_i \bigcap_{j=k-P+1}^{k}\mathbb{H}_j = \boldsymbol{\psi}_k^{(k-P)} \bigcap \boldsymbol{\psi}_k^{(P)}$$

where $\boldsymbol{\psi}_k^{(k-P)}$ and $\boldsymbol{\psi}_k^{(P)}$ are the intersections of the first $k-P$ and the last P constraint sets, respectively. Thus, the P constraints of the problem are equivalent to $\mathbf{H}_{k+1} \in \boldsymbol{\psi}_k^{(P)}$. From the previous work [14], the solutions for the Lagrangian function are as follows

$$\mathbf{H}_{k+1} = \begin{cases} \mathbf{H}_k + \mu(\mathbf{E}_k - \mathbf{G}_k)(\mathbf{S}_k^H\mathbf{S}_k)^{-1}\mathbf{S}_k^H & \text{if } \|\mathbf{e}_k\| > \gamma; \\ \mathbf{H}_k & \text{else,} \end{cases}$$

where $\mathbf{e}_k := \mathbf{r}_k - \mathbf{H}_k\mathbf{s}_k\,\forall k \in \mathbb{Z}$, $\boldsymbol{\epsilon}_{k,i} := \mathbf{r}_{k-i} - \mathbf{H}_k\mathbf{s}_{k-i}\,\forall i \in \mathbb{F}_1^{P-1}$, $\mathbf{E}_k := [\boldsymbol{\epsilon}_{k,P-1}\,\boldsymbol{\epsilon}_{k,P-2}\,\ldots\,\boldsymbol{\epsilon}_{k,1}\,\mathbf{e}_k] \in \mathbb{C}^{N_r\times P}$, $\mathbf{G}_k := [\mathbf{g}_{k,P-1}\,\mathbf{g}_{k,P-2}\,\ldots\,\mathbf{g}_{k,1}\,\mathbf{g}_{k,0}] \in \mathbb{C}^{N_r\times P}$, $\mathbf{S}_k := [\mathbf{s}_{k-P+1}\,\mathbf{s}_{k-P+2}\,\ldots\,\mathbf{s}_k] \in \mathbb{C}^{N_s\times P}$ and $0 < \mu \le 1$ is the step size. Different choices for matrix $\mathbf{G}_k$ were discussed in [14]. Also for the case $N_r = 1, \mathbf{s}_k, \mathbf{H}_k^T \in \mathbb{R}^{N_s}, \mathbf{r}_k, \mathbf{n}_k \in \mathbb{R}^{N_r}$, the choice $\tilde{\mathbf{G}} := [\gamma sign(\mathbf{E})]$ was employed in [18], where $sign(\mathbf{E})$ returns the element-wise sign of the matrix. Here, we show that the choice $\mathbf{G}_k^* = [\boldsymbol{\epsilon}_{k,P-1}\,\boldsymbol{\epsilon}_{k,P-2}\,\ldots\,\boldsymbol{\epsilon}_{k,1}\,\gamma\frac{\mathbf{e}_k}{\|\mathbf{e}_k\|}]$ has the best performance. First of all, from the previous updates, we have $\mathbf{H}_k \in \mathbb{H}_{k-j}\,\forall j \in \mathbb{F}_1^{P-1}$, therefore with the $\mathbf{G}_k$ defined above, we have $\|\mathbf{g}_{k,i}\| = \|\boldsymbol{\epsilon}_{k,i}\| = \|\mathbf{r}_{k-i} - \mathbf{H}_k\mathbf{s}_{k-i}\| \le \gamma\,\forall i \in \mathbb{F}_1^{P-1}$. So the first $P-1$ vectors of $\mathbf{G}_k$ are bounded by γ. Secondly, the last vector $\gamma\frac{\mathbf{e}_k}{\|\mathbf{e}_k\|}$ is obtained such that it has the minimum distance to the error vector $\mathbf{e}_k$ while it has a norm not greater than γ. Thus, by simply expanding $\|\mathbf{E}_k - \mathbf{G}_k\|_F^2$ as

$$\|\mathbf{E}_k - \mathbf{G}_k\|_F^2 = \sum_{i=1}^{P-1}\|\boldsymbol{\epsilon}_{k,i} - \mathbf{g}_{k,i}\|^2 + \|\mathbf{e}_k - \mathbf{g}_{k,0}\|^2,$$

it can be seen that $\mathbf{G}_k^*$ is the unique solution for minimizing $\|\mathbf{E}_k - \mathbf{G}_k\|_F^2$ subject to $\|\mathbf{g}_{k,i}\| \le \gamma\,\forall i \in \mathbf{F}_0^{P-1}$. Finally, the update equation above can be simplified as follows

$$\mathbf{H}_{k+1} = \begin{cases} \mathbf{H}_k + \mu(1 - \frac{\gamma}{\|\mathbf{e}_k\|})\mathbf{e}_k\mathbf{q}_k^H\mathbf{S}_k^H & \text{if } \|\mathbf{e}_k\| > \gamma; \\ \mathbf{H}_k & \text{else,} \end{cases} \quad (1)$$

where $\mathbf{q}_k$ is the last column vector of the matrix $(\mathbf{S}_k^H\mathbf{S}_k)^{-1}$. Thus, the calculation of other entries of the inverse matrix is not required. As discussed, the SM-AP algorithm derives the solution for $\mathbf{H}_{k+1}$ with respect to the bounding performance in the P most recent time steps. Applying the SM filtering framework on the RLS algorithm, the obtained SM-RLS algorithm (also known as BEACON) takes into account the error from all previous steps with a varying forgetting factor. In other words, matrix $\mathbf{H}_k$ is found by setting $\nabla\mathcal{L} = \mathbf{0}$, where ∇ denotes the gradient operator and the Lagrangian function $\mathcal{L}$ is defined as

$$\mathcal{L}_k := \sum_{i=1}^{k-1}\lambda_k^{k-i}\|\mathbf{r}_i - \mathbf{H}_k\mathbf{s}_i\|^2 + \lambda_k\left(\|\mathbf{r}_k - \mathbf{H}_k\mathbf{s}_k\|^2 - \gamma^2\right)$$

with λ_k playing the role of both the Lagrange multiplier and the forgetting factor [15]. The solution for the BEACON algorithm can be found in [15]. This algorithm is generally more complex than the SM-AP algorithm but it has a better MSE performance and does not require any explicit matrix inversion. For convenience, this algorithm is shown here.

Algorithm 1. BEACON (as derived in [15])

(1) Initialization: $\mathbf{H}_0 = \mathbf{0}\,, \mathbf{P}_0 = \mathbf{I}$
(2) For $k \geq 1$, compute $\boldsymbol{\epsilon}_k = \mathbf{r}_k - \mathbf{H}_{k-1}\mathbf{s}_k$

$$\lambda_k = \begin{cases} \frac{1}{G_k}\left(\frac{\|\boldsymbol{\epsilon}_k\|}{\gamma} - 1\right) & \text{if } \|\boldsymbol{\epsilon}_k\| > \gamma; \\ 0 & \text{else,} \end{cases} \quad \text{where} \quad G_k = \mathbf{s}_k^H \mathbf{P}_{k-1}\mathbf{s}_k$$

$$\mathbf{k}_k = \frac{\mathbf{s}_k^H \mathbf{P}_{k-1}}{1+\lambda_k G_k}\,,\ \text{update} \begin{cases} \mathbf{H}_k &= \mathbf{H}_{k-1} + \lambda_k \boldsymbol{\epsilon}_k \mathbf{k}_k \\ \mathbf{P}_k &= \mathbf{P}_{k-1} - \lambda_k \mathbf{P}_{k-1}\mathbf{s}_k \mathbf{k}_k \end{cases}$$

Since the number of multiplications is generally the most dominant factor for complexity and power consumption evaluation, we aim at slightly reducing this number here. From Algorithm 1, the following relation is deduced

$$\mathbf{P}_{k-1}\mathbf{s}_k\mathbf{k}_k = \frac{\mathbf{P}_{k-1}\mathbf{s}_k\mathbf{s}_k^H\mathbf{P}_{k-1}}{1+\lambda_k G_k}.$$

By substitution, the relation below holds for every k

$$\mathbf{P}_k = \begin{cases} \mathbf{P}_{k-1} - \frac{\|\boldsymbol{\epsilon}_k\|/\gamma - 1}{\mathbf{s}_k^H\mathbf{P}_{k-1}\mathbf{s}_k}\frac{\mathbf{P}_{k-1}\mathbf{s}_k\mathbf{s}_k^H\mathbf{P}_{k-1}}{1+\lambda_k G_k} & \text{if } \|\boldsymbol{\epsilon}_k\| > \gamma \\ \mathbf{P}_{k-1} & \text{otherwise.} \end{cases}$$

Considering $1+\lambda_k G_k \in \mathbb{R}\,\forall k \in \mathbb{Z}$, it can be easily observed that if $\mathbf{P}_{k-1}$ is Hermitian, then $\mathbf{P}_k$ is also Hermitian, and since $\mathbf{P}_0 = \mathbf{I}$, by mathematical induction it is concluded that for every k, $\mathbf{P}_k^H = \mathbf{P}_k$. In other words, it is not necessary to compute the lower triangular part of $\mathbf{P}_k$ as it can be reconstructed from the upper triangular part by a complex conjugation. So in Algorithm 1, $\mathbf{P}_k$ can be updated as follows

$$\mathbf{vech}(\mathbf{P}_k) = \mathbf{vech}(\mathbf{P}_{k-1}) - \lambda_k \mathbf{vech}(\mathbf{P}_{k-1}\mathbf{s}_k\mathbf{k}_k)$$

where $\mathbf{vech}(.)$ denotes the half vectorization. Also using this Hermitian property, it is clear that $[\mathbf{P}_{k-1}\mathbf{s}_k]^H = \mathbf{s}_k^H\mathbf{P}_{k-1}$, therefore, $\mathbf{k}_k$ and $\mathbf{P}_k$ can be obtained from $\mathbf{z}_k := \mathbf{P}_{k-1}\mathbf{s}_k$ without any multiplication or addition to calculate $\mathbf{z}_k$ again. The modified BEACON algorithm is shown in Algorithm 2. Defining $\Delta\mathbf{H}_k$ as $\Delta\mathbf{H}_k := \mathbf{H}_o - \mathbf{H}_k$, one can simply prove that $\mathbf{e}_k = \mathbf{n}_k + \Delta\mathbf{H}_k\mathbf{s}_k$. Therefore, J_k can be defined and expressed as follows

$$\begin{aligned} J_k := E\left[\|\mathbf{e}_k\|^2\right] =& E\left[\mathbf{e}_k^{\mathrm{H}}\mathbf{e}_k\right] \\ =& E\left[\left(\mathbf{n}_k^H + \mathbf{s}_k^H\Delta\mathbf{H}_k^H\right)\left(\mathbf{n}_k + \Delta\mathbf{H}_k\mathbf{s}_k\right)\right] \\ =& E\left[\|\mathbf{n}_k\|^2\right] + E\left[\mathbf{s}_k^H\Delta\mathbf{H}_k^H\Delta\mathbf{H}_k\mathbf{s}_k\right] \\ =& N_r\sigma_n^2 + E\left[\|\Delta\mathbf{H}_k\mathbf{s}_k\|^2\right] \end{aligned} \tag{2}$$

where the property $E[\mathbf{n}_k] = \mathbf{0}$ was used to simplify the relation assuming a zero-mean white Gaussian noise. Excluding the additive noise term in Equation (2), the excess MSE (EMSE) is defined as $EMSE := \lim_{k\to\infty} E\left[\|\Delta\mathbf{H}_k\mathbf{s}_k\|^2\right]$. The optimum γ, denoted by γ_o, can be found by minimizing the EMSE, i.e.,

$$\gamma_o = \arg\min_{\gamma} \lim_{k\to\infty} E\left[\|\Delta\mathbf{H}_k\mathbf{s}_k\|^2\right].$$

The major drawback of the SM-AP and BEACON algorithms is the fact that finding a convenient closed-form solution for EMSE, and subsequently γ_o, is very difficult. However, even a suboptimal performance can be significantly better than the NLMS algorithm if γ is close enough to the optimum value. One simple solution is to set γ for different estimations based on the statistical performance.

Algorithm 2. Modified BEACON

(1) Initialization: $\mathbf{H}_0 = \mathbf{0}\,, \mathbf{P}_0 = \mathbf{I}$
(2) For $k \geq 1$, compute $\boldsymbol{\epsilon}_k = \mathbf{r}_k - \mathbf{H}_{k-1}\mathbf{s}_k$

$$\lambda_k = \begin{cases} \frac{1}{\mathbf{s}_k^H\mathbf{z}_k}\left(\frac{\|\boldsymbol{\epsilon}_k\|}{\gamma} - 1\right) & \text{if } \|\boldsymbol{\epsilon}_k\| > \gamma; \\ 0 & \text{else,} \end{cases} \quad \text{where} \quad \mathbf{z}_k = \mathbf{P}_{k-1}\mathbf{s}_k$$

$$\mathbf{k}_k = \frac{\gamma\mathbf{z}_k^H}{\|\boldsymbol{\epsilon}_k\|}\,,\ \text{update} \begin{cases} \mathbf{H}_k &= \mathbf{H}_{k-1} + \lambda_k \boldsymbol{\epsilon}_k \mathbf{k}_k \\ \mathbf{vech}(\mathbf{P}_k) &= \mathbf{vech}(\mathbf{P}_{k-1}) - \lambda_k \mathbf{vech}(\mathbf{z}_k \mathbf{k}_k) \end{cases}$$

The performance of the SM-AP algorithm with respect to different γ values will be discussed in Section IV. Another approach to prevent overbounding or underbounding is to employ time-varying bounds. This method slightly increases the computational load and often involves setting some extra parameters for tuning. For our complex- valued BEACON matrix estimation problem, the time varying bound can be expressed as $\gamma_{k+1} = (1-\beta)\gamma_k + \beta\sqrt{\alpha}\|\mathbf{H}_k\|_F\sigma_n$, where β is the forgetting factor, α is the tuning parameter and σ_n^2 is the noise power [15], [19].

IV. Simulations

In this section, we show how the proposed low complexity algorithms can improve the MSE performance in terms of steady state error (SSE) and convergence speed, while reducing the power consumption by selective updates. We consider a hop with $N_s = 5$, $N_r = 6$ where SNR is fixed at 30dB and the actual channel matrices $\mathbf{H}_o$ to generate $\mathbf{r}_k$ are drawn from uniform distributions $U(0,1)$ and $U(-\pi,\pi)$ for amplitude and phase, respectively. Also the pilots $\mathbf{s}_k$ are generated by $exp(j\lambda\pi\theta)$ where $\theta \sim U(0,1)$ and $0 < \lambda \leq 2$ is a constant to manage the pilot correlation, i.e., for smaller λ values, the pilots are more correlated. Figure 3 shows the performance of the SM-AP algorithm for different γ values, where $P = 3$, $\mu = 1$, $\lambda = 2$ and $\mathbf{G}_k = \mathbf{G}_k^*$. In the figures of this section, $\bar{\beta}$ denotes the average update rate for each algorithm. As shown in Figure 3, in steps around 200, where the algorithms have approximately converged, the performance of the algorithm with $\gamma = 0.1$ is superior. Also as the magnified part shows, from $\gamma = 0.05$ to 0.1, the MSE performance improves steadily, while from $\gamma = 0.1$ to 0.13, the performance degrades in a similar manner. This behaviour implies $\gamma_o = 0.1$, which is validated by the EMSE performance shown in this figure. As expected, the update rate decreases with γ. Figure 4 shows the performance of the SM-AP, SM-NLMS, BEACON and

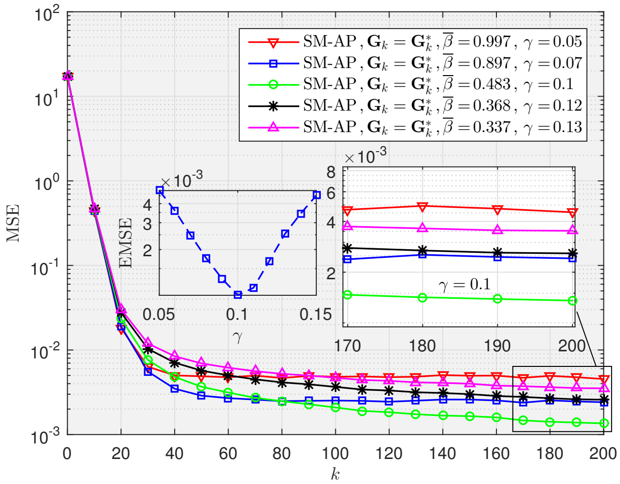

Fig. 3. The MSE performance of the proposed SM-AP algorithm for different error bounds. The performance in the magnified part shows that $\gamma_o = 0.1$, which is also validated by the EMSE curve.

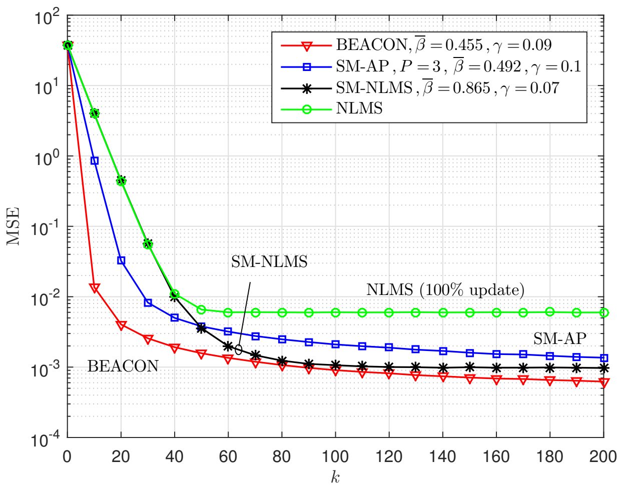

Fig. 4. The MSE performance of the SM-AP, SM-NLMS, BEACON and NLMS algorithms in comparison to each other for a Rayleigh fading channel. Compared to NLMS, SM-NLMS is superior.

NLMS algorithms in comparison to each other for a Rayleigh fading channel, where, the independent real and imaginary components are distributed by $\mathcal{N}(0, 0.5)$. In this figure, $\lambda = 2$, $\mu = 1$ and $P = 3$. It is evident that the proposed SM-AP and BEACON algorithms outperform the NLMS algorithm in terms of convergence speed and SSE. Note that SM-NLMS is obtained from SM-AP with $P = 1$. As shown in this figure, the SM-NLMS algorithm is clearly superior compared to the NLMS algorithm in terms of SSE (about 7.9dB lower) and update rate $\bar{\beta} = 86.5\%$. This rate for BEACON and SM-AP with $P > 1$ decreases to 45.5% and 49.2%, respectively, which is due to the data reuse property in these algorithms. Figure 5 shows the performance of SM-AP and SM-NLMS for different input signal correlations $\lambda = 0.7, 1$ and 2, where other simulation parameters are the same as in Figure 4. As the curves

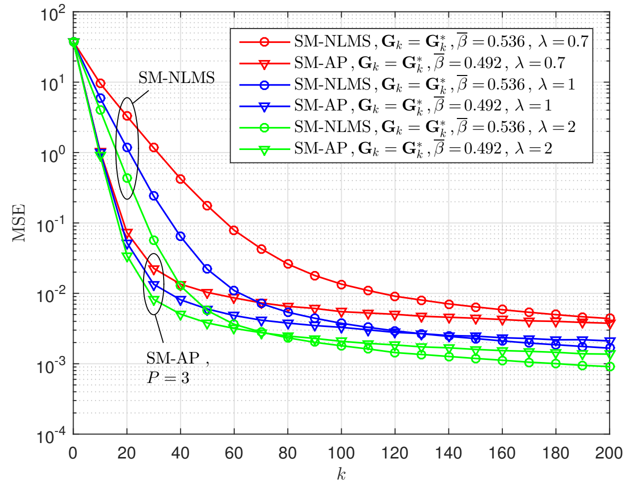

Fig. 5. Comparison of convergence speed between SM-NLMS and SM-AP for different correlated inputs. For highly correlated inputs, i.e., small λ, SM-AP converges significantly faster than SM-NLMS.

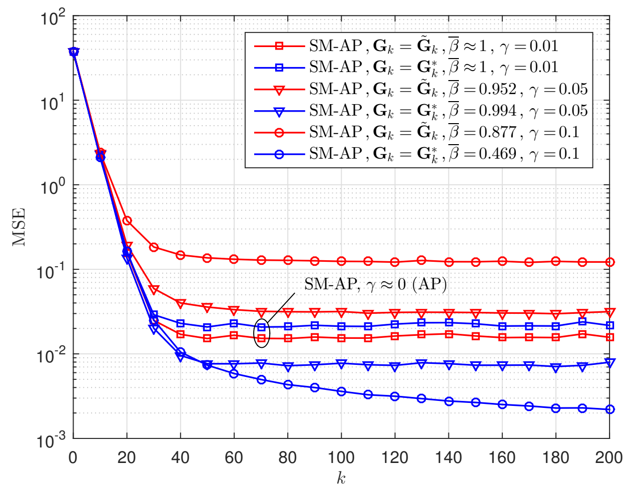

Fig. 6. MSE performance of the SM-AP algorithm for $\mathbf{G}_k = \mathbf{G}_k^*$ and $\mathbf{G}_k = \tilde{\mathbf{G}}_k$ with different values of γ. Except for $\gamma = 0.01$, the performance of $\mathbf{G}_k^*$ is conspicuously better.

exhibit, the SM-AP algorithm converges faster than the SM-NLMS algorithm, and this convergence is more pronounced when the signals are highly correlated. In fact, compared to SM-NLMS, the SM-AP algorithm with $P > 1$ improves the convergence speed at the cost of a slight increase in the SSE value. As a result, for highly correlated input signals, the SM-AP with $P > 1$ is more appropriate than SM-NLMS, as it requires less training symbols to converge. Figure 6 shows the MSE performance of SM-AP for $\mathbf{G}_k = \tilde{\mathbf{G}}_k$ and $\mathbf{G}_k = \mathbf{G}_k$ where $P = 2$, $\mu = 1$, $\lambda = 2$. For this simulation, the pilot vectors are generated by $\Re\{exp(j\lambda\pi\theta)\}$, i.e., the real component of $exp(j\lambda\pi\theta)$, and consequently SNR is less than 30 dB for this figure. The reason for choosing real pilots while other parameters are the same as before is the fact that the main relation $\mathbf{r}_k = \mathbf{H}_k\mathbf{s}_k + \mathbf{n}_k$ with $\mathbf{s}_k \in \mathbb{R}^{Ns}$ and a circularly symmetric complex Gaussian noise can be decomposed as two real-valued equations $\Re\{\mathbf{r}_k\} = \Re\{\mathbf{H}_k\}\mathbf{s}_k + \Re\{\mathbf{n}_k\}$ and $\Im\{\mathbf{r}_k\} = \Im\{\mathbf{H}_k\}\mathbf{s}_k + \Im\{\mathbf{n}_k\}$, where each of the recent relations can be further decomposed as Nr separate real-valued vector-based estimations. Therefore, if we define $\boldsymbol{\Theta}$ and $\mathbb{H}_k$ as

$$\boldsymbol{\Theta} := \bigcap_{(\mathbf{r},\mathbf{s})\in\mathbb{S}} \left\{\mathbf{H} \in \mathbb{C}^{N_r \times N_s} : \|\mathbf{r} - \mathbf{H}\mathbf{s}\|_{\max}^{\Re\wedge\Im} \leq \gamma\right\}$$

and

$$\mathbb{H}_k := \left\{\mathbf{H} \in \mathbb{C}^{N_r \times N_s} : \|\mathbf{r}_k - \mathbf{H}\mathbf{s}_k\|_{\max}^{\Re\wedge\Im} \leq \gamma\right\}$$

where for any vector like $\mathbf{x}$, $\|\mathbf{x}\|_{\max}^{\Re\wedge\Im} \leq \gamma$ means $\|\Re\{\mathbf{x}\}\|_{\max} \leq \gamma$ and $\|\Im\{\mathbf{x}\}\|_{\max} \leq \gamma$, then $\tilde{\mathbf{G}}_k$ is applicable and we can make a fair comparison of the MSE performance for $\tilde{\mathbf{G}}_k$ and $\mathbf{G}_k^*$. As shown in Figure 6, the proposed $\mathbf{G}_k^*$ completely outperforms $\tilde{\mathbf{G}}_k$ for different values of γ (except for $\gamma = 0.01$), while, due to the removal of the a posteriori errors and major parts of the matrix inversion (Equation (1)), its computational complexity is significantly less than using $\tilde{\mathbf{G}}_k$. This figure verifies the mathematical proof presented in Section III. Note that in both cases, when $\gamma = 0$, the SM-AP algorithm becomes the regular AP algorithm. Therefore, as shown in this figure, the performance in both cases are similar for $\gamma = 0.01$. As a consequence, similar to the superiority of SM-NLMS over NLMS in Figure 4, SM-AP clearly outperforms the regular AP algorithm in terms of error performance and complexity.

V. Conclusions

The diverse applications of adaptive filtering techniques, and in particular, the NLMS algorithm, for signal processing in WSNs were investigated and two general adaptive algorithms based on the set-membership filtering were proposed, which can estimate complex-valued matrices in the presence of AWGN. The set-membership filtering reduces the complexity and consequently the energy consumption by means of selective updates and sparse matrix calculations, while it improves the error performance as well. The purpose was to show that many signal processing algorithms for WSNs can be replaced by one general low complexity algorithm which can perform different tasks by minor parameter adjustments. In this way, we eliminate the need to implement many energy and memory consuming algorithms for different purposes and replace them with one energy efficient and accurate structure. In the simulations section, we showed that the MSE performance of the proposed algorithms are better than that of the NLMS algorithm, where, the latter technique was one of the main algorithms used in the previous works.

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