

Partial Distributed Beamforming Design in Passive Radar Sensor Networks

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Abstract—In this paper, we address the optimal power allocation problem in a distributed passive radar sensor network system, where closely located nodes are capable of distributed beamforming. In this setup, the network of sensor nodes is viewed as a combination of node clusters, where each cluster is capable of time synchronization, and coordinated transmission. The goal of the network is to provide a reliable estimation of a source signal, by collecting and combining the individual observations from the sensor nodes in a centralized node. In this regard, a minimum mean squared error (MMSE) problem is formulated for the class of unbiased estimators. As it is shown, the resulting problem can be formulated as a convex optimization problem, which is solvable with the standard numerical solvers. At the end, numerical simulations illustrate the effect of the different network parameters on the resulting performance, and a significant gain is observed by enabling the proposed distributed beamforming algorithm for multiple sensor clusters.

1. INTRODUCTION

Nowadays, many applications benefit from the idea of distributed sensor networks for purposes of observation and communication. The range of these applications especially covers health care, traffic monitoring, radio astronomy, particle physics [1], [2], as well as various use-cases in the area of space and extreme environments. In particular, goal of a distributed passive radar is to provide a reliable estimation from a source signal, by collecting and combining the individual passive observations from a network of sensor nodes (SN)s in a centralized node, i.e., fusion center (FC). As an interesting example, we can mention the ‘*IceCube Neutrino Observatory*’ at the south pole, where a sensor network with more than 5000 SNs is deployed to observe certain characteristics of subatomic particles [3]. Since the operation of the whole sensor network is mostly intended to consume minimum resources while keeping the individual cost and maintenance of SNs low, an energy efficient operation is highly desirable. Hence, the related problem of optimal power allocation and corresponding energy-aware system design has been addressed in many works, e.g., in [4], [5] and [6]. In particular, an optimal power allocation scheme in the context of a distributed passive radar is provided in [1], where an analytical framework is proposed, and afterwards extended regarding computational complexity [7], network lifetime and energy efficiency [8], [9], and occasional node failure and network data inaccuracy [10].

Other than the gainful approaches regarding smart power allocation and energy-aware design, it is known that the network energy efficiency can be significantly improved by

benefiting from spatially coordinated transmission schemes, e.g., distributed beamforming [11]. Nevertheless, due to the small and cheap nature of the SNs, an ideal coordination among all sensors in the network level is difficult to achieve. As a result, a joint operation of the SNs is mostly separated in different channels to avoid interference and reduce complexity, see, e.g., [1], [12], [13]. In the early works [14], [15] the idea of local coordination of SNs is studied, where a cluster-based distributed beamforming is enabled for communication purposes. The aforementioned works follow the idea that the closely located SNs can communicate and coordinate their transmission schemes at a minimal energy and complexity. These works have been then extended in various aspects addressing the applications of distributed passive radar, e.g., target localizations [16], distributed imaging [17], and airspace surveillance [18]. Nevertheless, an optimal power allocation, as well as an optimal information fusion among different sensor clusters for a partially-coordinated distributed passive radar system is still an open problem.

In this work, we extend the optimal power allocation algorithm in [1] in the context of a passive distributed radar system, to the scenario where the clusters of closely located sensors can perform distributed beamforming. In particular, we provide optimal solutions regarding two open problems. Firstly, for a given network with multiple clusters, how should be the network power allocated among different sensor clusters, and secondly, how should be the observations from different clusters optimally combined in order to build a single reliable observation. In Section II, we define the investigated system model. Section III presents the optimal design of the system parameters, including an optimal signal fusion rule, as well as an optimal distributed beamforming within each sensor cluster. The numerical simulations evaluate the performance of the proposed algorithms under different network conditions.

2. SYSTEM MODEL

In this work we investigate a network of amplify-and-forward (AF) passive sensor nodes, cooperating to achieve a single global observation via a fusion center (FC), see Fig 1. The sensors which are closely located are clustered in a group, and are capable of time synchronization and coordinated transmission, i.e., distributed beamforming. The index $k \in \mathbb{K}$ and $l \in \mathbb{L}_k$ are used to represent different clusters and different SNs, where the sets \mathbb{K} and \mathbb{L}_k respectively represent the index set of all clusters, and the index set of all SNs in the k -th cluster. Both communication and sensing channels (frequency-flat fading) are assumed to be wireless and static during the

observation process. The final goal of each observation is to classify (or detect) a source signal $r \in \mathbb{C}$. Each observation can be segmented into three parts: sensing, communication, and information fusion. The detailed function of each SN is discussed in the following, see also [1, Section II] for a detailed discussion on a non-clustered network.

A. Operation of SNs

If a target signal $r \in \mathbb{C}$ is present, each SN receives and amplifies the incoming signal using an amplification coefficient $u_{k,l} \in \mathbb{C}$, $k \in \mathbb{K}, l \in \mathbb{L}_k$. The communication with FC is performed by using orthogonal waveforms for each cluster so that the signal from different clusters can be separated and processed in FC. Please note that for the SNs belonging to the same cluster, a time-delay is imposed on the amplified signal prior to transmission, such that the signals from the SNs belonging to a same cluster simultaneously arrive at the FC. See [11] for a more detailed synchronization process for the distributed beamforming scenarios. The process of each SN can be hence described as ¹

$$x_{k,l} := u_{k,l}(g_{k,l}r + m_{k,l}), \quad k \in \mathbb{K}, l \in \mathbb{L}_k \quad (1)$$

and

$$X_{k,l} := \mathcal{E}\{|x_{k,l}|^2\}, \quad R := \mathcal{E}\{|r|^2\}, \quad (2)$$

where the sensing channel coefficient, the transmit signal from the SNs and its power are respectively denoted by $g_{k,l} \in \mathbb{C}$, $x_{k,l} \in \mathbb{C}$ and $X_{k,l}$. The additive white Gaussian noise (AWGN) on the sensing process and its variance are respectively denoted as $m_{k,l} \in \mathbb{C}$ and $M_{k,l}$.

Furthermore, it is assumed that the power consumption of k -th SN cluster is limited by $X_{\max,k}$, while the total power consumption of the network is limited by X_{tot} . This is formulated as

$$\sum_{l \in \mathbb{L}_k} X_{k,l} \leq X_{\max,k}, \quad k \in \mathbb{K}, \quad (3)$$

and

$$\sum_{k \in \mathbb{K}} \sum_{l \in \mathbb{L}_k} X_{k,l} \leq X_{\text{tot}}. \quad (4)$$

Please note that the per-cluster power constraint is to ensure that the resources of each cluster, as a united group of the network resources, will not be over-utilized for a single observation process and remains functional for the future tasks. On the other hand, the total network power constraint is to ensure that the network remains functional over a reasonable lifetime.

B. Fusion Center

The transmitted signal from the SNs belonging to the same cluster pass through the same communication channel, with coefficient $h_{k,l} \in \mathbb{C}$, and arrive at the FC combined with

¹The described synchronization process by introducing time-delay is assumed implicit in the following formulations, and eliminated due to the notational simplicity.

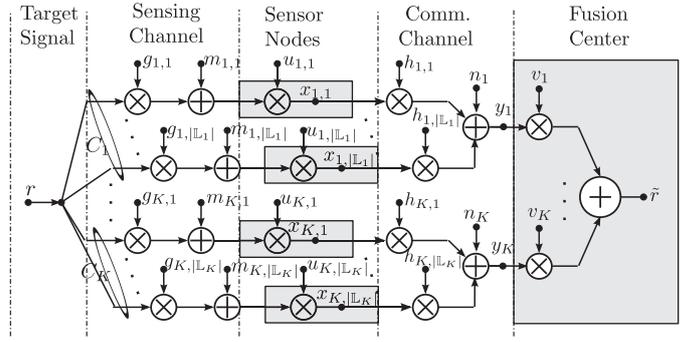


Figure 1. Signal flow at the defined sensor network consisting of a target signal, multiple sensor clusters ($C_1, \dots, C_{|\mathbb{K}|}$), multiple sensor nodes at each cluster, and a fusion center. See Section II for a detailed explanation.

an AWGN component $n_k \in \mathbb{C}$, with variance N_k . This is described as

$$y_k := n_k + \sum_{l \in \mathbb{L}_k} h_{k,l} x_{k,l}. \quad (5)$$

Please note that while the communication channel to the FC is shared among the SNs belonging to a same cluster, orthogonal communication channels are assigned to clusters. Hence, the received signals from different clusters can be separated and processed at the FC. In our system, a linear combination rule is then applied at the FC to achieve an estimation, \tilde{r} , from the observed target signal by the network. This is described as

$$\begin{aligned} \tilde{r} &:= \sum_{k \in \mathbb{K}} v_k y_k \\ &= \sum_{k \in \mathbb{K}} v_k \left(n_k + \sum_{l \in \mathbb{L}_k} h_{k,l} u_{k,l} g_{k,l} r + h_{k,l} u_{k,l} m_{k,l} \right). \end{aligned} \quad (6)$$

where $v_k \in \mathbb{C}$ represents the applied fusion weight at the received signal from the k -th sensor cluster, and \tilde{r} represents the estimated target signal at the FC.

C. Remarks

In the present work:

- we assume the availability of perfect channel information for both sensing and communication channels. In reality, it is rather difficult to estimate the sensing channel in an accurate way unless the channel has a highly stationary nature, see, e.g., [3]. Hence, for scenarios where the sensing channel is not stationary, or the statistics regarding node failure is not tractable, the results of this paper can be treated as theoretical limits.
- we assume a perfect time synchronization between the SNs belonging to a same cluster, as well as perfect separation of the channels belonging to different SN clusters. A study regarding the effects of the time and frequency synchronization error will be a focus of our future research.

3. MMSE DESIGN OF NETWORK PARAMETERS WITH PARTIAL DISTRIBUTED BEAMFORMING

In this section we propose an MMSE design of the network parameters for unbiased class of estimators. In the first step, we observe that the unbiased estimation property can be written as

$$\mathcal{E}\{\tilde{r} - r\} = 0 \Leftrightarrow \sum_{k \in \mathbb{K}} v_k \sum_{l \in \mathbb{L}_k} h_{k,l} u_{k,l} g_{k,l} = 1, \quad (7)$$

following the identity (6) and the fact that all noise terms are zero mean. Furthermore, the mean squared error of the estimation can be written as

$$\begin{aligned} V &:= \mathcal{E}\{|\tilde{r} - r|^2\} \\ &= \sum_{k \in \mathbb{K}} |v_k|^2 N_k + \sum_{k \in \mathbb{K}} |v_k|^2 \sum_{l \in \mathbb{L}_k} |h_{k,l} u_{k,l}|^2 M_{k,l}, \end{aligned} \quad (8)$$

where (8) follows via the application of (7), and the fact that all the noise terms are mutually independent and zero-mean. As a result, the MSE minimization problem can be formulated for an unbiased class of estimates which satisfy the defined power constraints as

$$\underset{u_{k,l}, v_k, k \in \mathbb{K}, l \in \mathbb{L}_k}{\text{minimize}} \quad V \quad (9a)$$

$$\text{s.t.} \quad (7), (3), (4). \quad (9b)$$

Unfortunately, (9) is not a jointly convex optimization problem. Nevertheless it is separately convex over the fusion weights, i.e., v_k , $k \in \mathbb{K}$. In the following part we obtain a set of optimal fusion weights for a fixed set of amplification coefficients.

A. Optimal Linear Fusion

Please note that the defined power constraints in (3), (4) are invariant to the choice of fusion weights. The corresponding optimization problem over the fusion weights, when the amplification coefficients are fixed, is formulated as

$$\underset{v_k, k \in \mathbb{K}}{\text{minimize}} \quad V \quad (10a)$$

$$\text{s.t.} \quad \sum_{k \in \mathbb{K}} v_k \sum_{l \in \mathbb{L}_k} h_{k,l} u_{k,l} g_{k,l} = 1, \quad (10b)$$

which is a convex optimization problem. The corresponding Lagrangian function to (10) can be subsequently written as

$$\mathcal{L}(v_k, \lambda) = \sum_{k \in \mathbb{K}} \lambda \left(1 - \sum_{k \in \mathbb{K}} v_k \sum_{l \in \mathbb{L}_k} h_{k,l} u_{k,l} g_{k,l} \right) \quad (11a)$$

$$+ \sum_{k \in \mathbb{K}} |v_k|^2 \sum_{l \in \mathbb{L}_k} |h_{k,l} u_{k,l}|^2 M_{k,l} \quad (11b)$$

$$+ \sum_{k \in \mathbb{K}} |v_k|^2 N_k. \quad (11c)$$

For any optimal solution to (10), the derivative of the Lagrangian should vanish with respect to v_k . This is written as

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial v_k} = 0 &\Leftrightarrow \\ v_k^* \left(N_k + \sum_{l \in \mathbb{L}_k} M_{k,l} |h_{k,l} u_{k,l}|^2 \right) - \lambda \sum_{l \in \mathbb{L}_k} h_{k,l} u_{k,l} g_{k,l} &= 0. \end{aligned} \quad (12)$$

Following the identity

$$\sum_{k \in \mathbb{K}} v_k \frac{\partial \mathcal{L}}{\partial v_k} = 0 \quad (13)$$

we obtain $\lambda = V$. Furthermore, following

$$\sum_{k \in \mathbb{K}} \frac{(\sum_{l \in \mathbb{L}_k} h_{k,l} u_{k,l} g_{k,l})^*}{N_k + \sum_{l \in \mathbb{L}_k} M_{k,l} |h_{k,l} u_{k,l}|^2} \left(\frac{\partial \mathcal{L}}{\partial v_k} \right) = 0 \quad (14)$$

we obtain

$$V = \left(\sum_{k \in \mathbb{K}} \frac{|\sum_{l \in \mathbb{L}_k} h_{k,l} u_{k,l} g_{k,l}|^2}{N_k + \sum_{l \in \mathbb{L}_k} M_{k,l} |h_{k,l} u_{k,l}|^2} \right)^{-1} \quad (15)$$

which consequently from (12) results in

$$\begin{aligned} v_k^* &= \left(\sum_{k \in \mathbb{K}} \frac{|\sum_{l \in \mathbb{L}_k} h_{k,l} u_{k,l} g_{k,l}|^2}{N_k + \sum_{l \in \mathbb{L}_k} M_{k,l} |h_{k,l} u_{k,l}|^2} \right)^{-1} \\ &\quad \times \frac{\sum_{l \in \mathbb{L}_k} h_{k,l}^* u_{k,l}^* g_{k,l}^*}{N_k + \sum_{l \in \mathbb{L}_k} M_{k,l} |h_{k,l} u_{k,l}|^2}, \quad k \in \mathbb{K}, \end{aligned} \quad (16)$$

where v_k^* represents the optimal fusion weight, corresponding to the k -th sensor cluster.

B. Power Constraint Per-Cluster Distributed Beamforming

The obtained MSE in (15) corresponds to the optimal linear fusion at the FC for an unbiased class of estimators, via the utilization of the fixed set of $u_{k,l}$. In this part, we are looking for an optimal set of $u_{k,l}$ which result in the minimum estimation MSE in (15), while satisfying the total, and per-cluster power constraints. The corresponding optimization problem can be formulated as

$$\underset{u_{k,l}, k \in \mathbb{K}, l \in \mathbb{L}_k}{\text{maximize}} \quad \sum_{k \in \mathbb{K}} \frac{|\sum_{l \in \mathbb{L}_k} h_{k,l} u_{k,l} g_{k,l}|^2}{N_k + \sum_{l \in \mathbb{L}_k} M_{k,l} |h_{k,l} u_{k,l}|^2} \quad (17a)$$

$$\text{s.t.} \quad (3), (4), \quad (17b)$$

where the maximization objective reflects the inverse of the obtained minimum MSE in (15). As it can be observed, (17) is a coupled optimization problem over different clusters due to (4) and hence it is difficult to approach in the current form. In order to decouple the optimization of $u_{k,l}$ for each cluster, we equivalently reformulate (17) as a power allocation problem over different sensor clusters. This is written as

$$\underset{\mathbf{u}_k, \tilde{X}_k, k \in \mathbb{K}}{\text{maximize}} \quad \sum_{k \in \mathbb{K}} \frac{\mathbf{u}_k^H \mathbf{A}_k \mathbf{u}_k}{\mathbf{u}_k^H \mathbf{B}_k \mathbf{u}_k + N_k} \quad (18a)$$

$$\text{s.t.} \quad \mathbf{u}_k^H \mathbf{P}_k \mathbf{u}_k = \tilde{X}_k, \quad (18b)$$

$$\tilde{X}_k \leq X_{\max,k}, \quad \sum_{k \in \mathbb{K}} \tilde{X}_k \leq X_{\text{tot}}, \quad (18c)$$

where $\mathbf{u}_k \in \mathbb{C}^{|\mathbb{L}_k|}$ is a vector which is constructed by stacking the values of $u_{k,l}$, $l \in \mathbb{L}_k$ such that $[\mathbf{u}_k]_l = u_{k,l}$. The matrices \mathbf{B}_k and \mathbf{P}_k are diagonal matrices such that $[\mathbf{P}_k]_{ll} = R|g_{k,l}|^2 + M_{k,l}$ and $[\mathbf{B}_k]_{ll} = |h_{k,l}|^2 M_{k,l}$. Moreover we have $\mathbf{A}_k = \mathbf{a}_k \mathbf{a}_k^H$ such that $\mathbf{a}_k \in \mathbb{C}^{|\mathbb{L}_k|}$ and $[\mathbf{a}_k]_l = h_{k,l}^* g_{k,l}^*$, which also reveals the rank-1 nature of the matrix \mathbf{A}_k . In the above formulation, \tilde{X}_k holds the consumed power at the sensor cluster k , see (18b). Note that the objective in (18) is now a decoupled problem over $u_{k,l}$ for a fixed set of \tilde{X}_k ,

while the constraint (18c) is coupled over different clusters. This decomposes (18) into a maximization of the objective for each cluster, for a fixed set of \tilde{X}_k as

$$\underset{u_{k,l}, l \in \mathbb{L}_k}{\text{maximize}} \quad \tau_k := \frac{\mathbf{u}_k^H \mathbf{A}_k \mathbf{u}_k}{\mathbf{u}_k^H \mathbf{B}_k \mathbf{u}_k + N_k} \quad (19a)$$

$$\text{s.t.} \quad \mathbf{u}_k^H \mathbf{P}_k \mathbf{u}_k = \tilde{X}_k. \quad (19b)$$

By defining $\tilde{\mathbf{u}}_k := \frac{\mathbf{P}_k^{-\frac{1}{2}} \mathbf{u}_k}{\left| \mathbf{P}_k^{-\frac{1}{2}} \mathbf{u}_k \right|}$, we can reformulate (19) as

$$\underset{\tilde{\mathbf{u}}_k}{\text{maximize}} \quad \tau_k = \frac{\tilde{X}_k \tilde{\mathbf{u}}_k^H \mathbf{P}_k^{-\frac{1}{2}} \mathbf{A}_k \mathbf{P}_k^{-\frac{1}{2}} \tilde{\mathbf{u}}_k}{\tilde{\mathbf{u}}_k^H \left(\tilde{X}_k \mathbf{P}_k^{-\frac{1}{2}} \mathbf{B}_k \mathbf{P}_k^{-\frac{1}{2}} + N_k \mathbf{I}_{|\mathbb{L}_k|} \right) \tilde{\mathbf{u}}_k} \quad (20a)$$

$$\text{s.t.} \quad \tilde{\mathbf{u}}_k^H \tilde{\mathbf{u}}_k = 1. \quad (20b)$$

The resulting problem follows the well-known Rayleigh-quotient structure, where the optimal $\tilde{\mathbf{u}}_k$ is obtained as

$$\begin{aligned} \tilde{\mathbf{u}}_k^* &= \mathcal{P}_{\max} \{ \mathbf{Y}_k \}, \quad \tau_k^* = \bar{\mathcal{P}}_{\max} \{ \mathbf{Y}_k \}, \\ \mathbf{Y}_k &:= \left(\tilde{X}_k \mathbf{P}_k^{-\frac{1}{2}} \mathbf{B}_k \mathbf{P}_k^{-\frac{1}{2}} + N_k \mathbf{I}_{|\mathbb{L}_k|} \right)^{-1} \tilde{X}_k \mathbf{P}_k^{-\frac{1}{2}} \mathbf{A}_k \mathbf{P}_k^{-\frac{1}{2}} \end{aligned} \quad (21)$$

where $\mathcal{P}_{\max} \{ \cdot \}$ and $\bar{\mathcal{P}}_{\max} \{ \cdot \}$ respectively obtain the dominant eigenvector and eigenvalue. Please note that τ_k^* represents the optimal value for τ_k , i.e., the objective value in (19). It is interesting to observe that the solution to the decomposed problem for each cluster follows a similar structure compared to a signal-to-noise maximization beamforming problem in a synchronized relay network, see [19, Section IV]. Nevertheless, the obtained structure is not sufficient to achieve an optimal inter-cluster resource allocation, which appear as a result of the coupled constraint in (18c). In the following, we aim at obtaining an explicit relation between the value of the \tilde{X}_k , and the resulting optimal objective value in (19), i.e., τ_k^* . It is known that the maximum eigenvalue operator is a convex function. In this way, τ_k^* can be presented as a convex and non-decreasing composition of a concave and non-decreasing function over \tilde{X}_k . Unfortunately, such a composition does not lead to a concave (or convex) function in general. Nevertheless, by taking advantage of the rank-1 nature of \mathbf{A}_k , we provide a concave representation of τ_k^* with respect to \tilde{X}_k in the following Lemma.

Lemma 1. *Let $\mathbf{W}_k \Sigma_k \mathbf{W}_k^H$ be the singular-value decomposition of the matrix $\mathbf{P}_k^{-\frac{1}{2}} \mathbf{B}_k \mathbf{P}_k^{-\frac{1}{2}}$, where $\mathbf{w}_{k,l}$ represent the l -th column of \mathbf{W}_k and $\sigma_{k,l} = [\Sigma_k]_{ll}$. Then we have*

$$\tau_k^* = \sum_{l \in \mathbb{L}_k} \frac{\tilde{X}_k \mathbf{w}_{k,l}^H \mathbf{P}_k^{-\frac{1}{2}} \mathbf{A}_k \mathbf{P}_k^{-\frac{1}{2}} \mathbf{w}_{k,l}}{\tilde{X}_k \sigma_{k,l} + N_k}. \quad (22)$$

Proof: See Appendix. \blacksquare

Please note that (22) is a concave function over \tilde{X}_k . Consequently, the optimization problem for power allocation over different clusters can be formulated as

$$\underset{\tilde{X}_k, k \in \mathbb{K}}{\text{maximize}} \quad \sum_{k \in \mathbb{K}} \sum_{l \in \mathbb{L}_k} \frac{\tilde{X}_k \eta_{k,l}}{\tilde{X}_k \sigma_{k,l} + N_k}. \quad (23a)$$

$$\text{s.t.} \quad \tilde{X}_k \leq X_{\max,k}, \quad \sum_{k \in \mathbb{K}} \tilde{X}_k \leq X_{\text{tot}}, \quad (23b)$$

which is a maximization of a jointly concave function over \tilde{X}_k , with affine constraint, and $\eta_{k,l} := \mathbf{w}_{k,l}^H \mathbf{P}_k^{-\frac{1}{2}} \mathbf{A}_k \mathbf{P}_k^{-\frac{1}{2}} \mathbf{w}_{k,l}$. As a result, (23) holds a convex optimization problem structure, can be solved using the famous numerical solvers, e.g., SeDuMi, SDPT3, see [20]. The optimal amplification coefficients, and the optimal fusion weights can be obtained via the utilization of (19), and subsequently (16).

4. SIMULATION RESULTS

In this part we investigate the performance of the defined system via numerical simulations. We simulate a network including a source, a FC, and 256 SNs, see Fig. 2. All sensing and communication channel coefficients are flat-fading, zero-mean and follow a Gaussian distribution. The variance of the channel between two nodes, is determined as the $d^{-\zeta}$, where d represents the distance between two nodes, and ζ is the path loss exponent. Unless stated otherwise, the given values in Table I are used as the default network parameters. In order to identify the sensor clusters, we assume that a cluster can be made by the neighboring SNs in a squared-shape area. In this respect we consider 5 different scenarios, where the sensor clusters are defined as square-shaped groups including $|\mathbb{L}_k| \in \{1, 4, 16, 64, 256\}$ sensors, see Table II. For each set of channel realizations, i.e., $h_{k,l}, g_{k,l}$, 10000 realizations of $r, n_{k,l}, m_{k,l}$ are generated, following the defined statistics, to evaluate the network performance. The resulting network performance is then averaged over 100 channel realizations.

In Fig. 3 the resulting estimation accuracy, in terms of RV^{-1} [dB], is depicted for different noise intensity on the sensing and communication channels. It is observed that the higher noise intensity results in the reduced estimation accuracy. Furthermore, a significant beamforming gain is observed for different noise intensities, in the comparison between Scenario 1, i.e., where all SNs contribute to the distributed beamforming setup within a single cluster, and Scenario 5 with no coordinated sensor group. Furthermore, it is observed that the gain of distributed beamforming on the estimation accuracy is monotonically increasing with the sensor cluster size, i.e., $|\mathbb{L}_k|$, see Scenario 1 to Scenario 5. In the other words, more coordination among SNs, and hence larger sensor clusters, result in higher estimation accuracy.

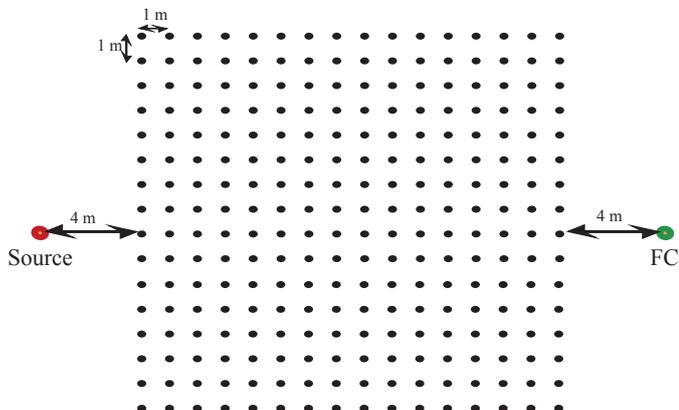
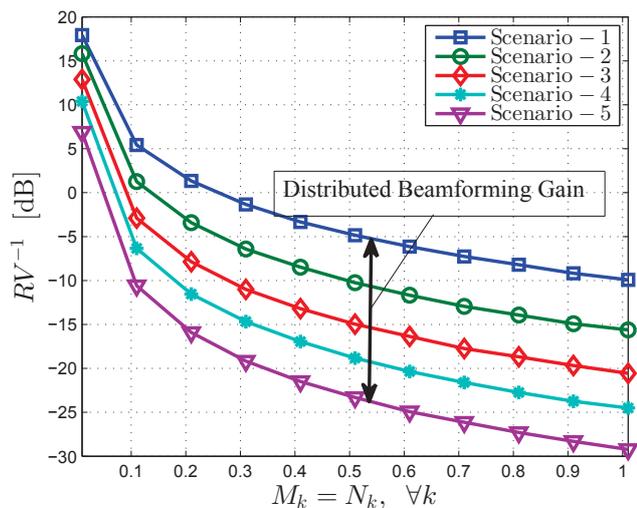
In Fig. 4 and Fig. 5 the resulting estimation accuracy, in terms of RV^{-1} [dB], is depicted for different noise intensity on the sensing channel, i.e., $M_{k,l}$ and on the communication channel N_k , respectively. Similar to Fig. 3, it is observed that the higher noise intensity results in the reduced estimation accuracy. Moreover, it is observed that the coordination among the SNs, and performing distributed beamforming increases the resulting estimation accuracy. Nevertheless, while the aforementioned gain increases as the N_k increases (and vice versa), it remains rather constant for different values of $M_{k,l}$. This is expected since, while the distributed beamforming among sensors significantly enhances the communication process by aligning the useful signal components at the FC, the performance is still limited by the quality of the sensing process. Furthermore, similar to Fig. 3, the comparison among Scenario 1 to Scenario 5 reveals that the more coordination capability among SNs, i.e., larger cluster sizes $|\mathbb{L}_k|$, result in a higher estimation accuracy.

Table II. SCENARIOS FOR SN CLUSTERS

Scenario	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
Set size	$ \mathbb{K} = 1, \mathbb{L}_k = 256$	$ \mathbb{K} = 4, \mathbb{L}_k = 64$	$ \mathbb{K} = 16, \mathbb{L}_k = 16$	$ \mathbb{K} = 64, \mathbb{L}_k = 4$	$ \mathbb{K} = 256, \mathbb{L}_k = 1$

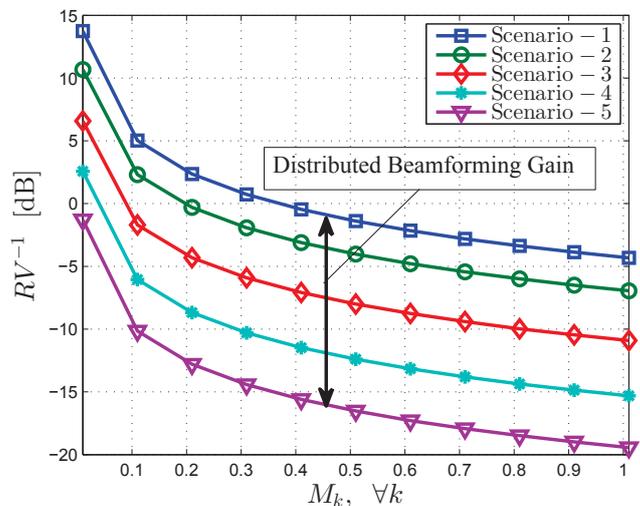
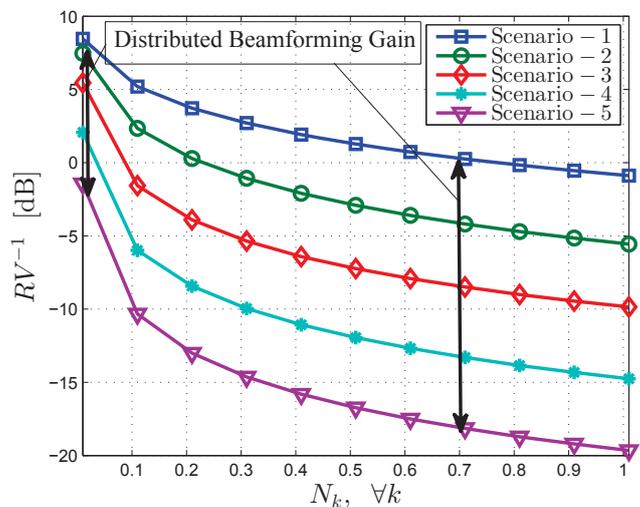
Table I. REFERENCE SIMULATION PARAMETERS

R	$X_{\max,k}$	X_{tot}	ζ	$M_{k,l}$	N_k
1	$ \mathbb{L}_k $	100	2.5	0.1	0.1

Figure 2. The simulated network, including a source, a network of 256 SNs (16×16 setup), and a FC.Figure 3. The resulting estimation accuracy in terms of RV^{-1} [dB] vs. Noise variance at both sensing and communication channels $N_k = M_{k,l}, \forall k \in \mathbb{K}, l \in \mathbb{L}_k$. Significant distributed beamforming gain is observable for different noise intensities.

5. CONCLUSION

In this paper we have studied a system of passive distributed radar sensor network, where the sensors in each cluster are capable of distributed beamforming. In this respect, we have provided an optimal linear fusion strategy for the observation of different sensor clusters, together with an optimal power allocation among different clusters under a total network power constraint. Numerical simulations show that a significant gain

Figure 4. The resulting estimation accuracy in terms of RV^{-1} [dB] vs. Noise variance at the sensing channel $M_k = M_{k,l}, \forall l \in \mathbb{L}_k$. Significant distributed beamforming gain is observable for different noise intensities.Figure 5. The resulting estimation accuracy in terms of RV^{-1} [dB] vs. Noise variance at the communication channel $N_k, \forall k \in \mathbb{K}$. Larger distributed beamforming gain is observable as N_k increases.

is achieved in terms of the resulting estimation accuracy, by enabling optimal distributed beamforming within sensor clusters. The aforementioned gain is increased for a network with higher noise intensity on the communication channel.

APPENDIX

Firstly, let $\mathbf{U}_d \boldsymbol{\Sigma}_d \mathbf{U}_d^{-1}$ be the eigenvalue decomposition for a general rank-1 square matrix \mathbf{D} . We can easily observe that

$$\begin{aligned} \bar{\rho}_{\max} \{\mathbf{D}\} &= \text{tr}(\boldsymbol{\Sigma}_d) = \text{tr}(\boldsymbol{\Sigma}_d \mathbf{U}_d^{-1} \mathbf{U}_d) \\ &= \text{tr}(\mathbf{U}_d \boldsymbol{\Sigma}_d \mathbf{U}_d^{-1}) = \text{tr}(\mathbf{D}). \end{aligned} \quad (24)$$

By applying the result from (24) into (21) we have

$$\begin{aligned} \tau_k^* &= \text{tr}(\mathbf{Y}_k) \\ &= \text{tr} \left(\left(\tilde{X}_k \mathbf{W}_k \boldsymbol{\Sigma}_k \mathbf{W}_k^H + N_k \mathbf{I}_{|\mathbb{L}_k|} \right)^{-1} \tilde{X}_k \mathbf{P}_k^{-\frac{1}{2}} \mathbf{A}_k \mathbf{P}_k^{-\frac{1}{2}} \right) \\ &= \text{tr} \left(\left(\mathbf{W}_k \left(\tilde{X}_k \boldsymbol{\Sigma}_k + N_k \mathbf{I}_{|\mathbb{L}_k|} \right) \mathbf{W}_k^H \right)^{-1} \tilde{X}_k \mathbf{P}_k^{-\frac{1}{2}} \mathbf{A}_k \mathbf{P}_k^{-\frac{1}{2}} \right) \\ &= \text{tr} \left(\mathbf{W}_k \left(\tilde{X}_k \boldsymbol{\Sigma}_k + N_k \mathbf{I}_{|\mathbb{L}_k|} \right)^{-1} \tilde{X}_k \mathbf{W}_k^H \mathbf{P}_k^{-\frac{1}{2}} \mathbf{A}_k \mathbf{P}_k^{-\frac{1}{2}} \right) \\ &= \text{tr} \left(\left(\tilde{X}_k \boldsymbol{\Sigma}_k + N_k \mathbf{I}_{|\mathbb{L}_k|} \right)^{-1} \tilde{X}_k \mathbf{W}_k^H \mathbf{P}_k^{-\frac{1}{2}} \mathbf{A}_k \mathbf{P}_k^{-\frac{1}{2}} \mathbf{W}_k \right) \\ &= \sum_{l \in \mathbb{L}_k} \frac{\tilde{X}_k \mathbf{w}_{k,l}^H \mathbf{P}_k^{-\frac{1}{2}} \mathbf{A}_k \mathbf{P}_k^{-\frac{1}{2}} \mathbf{w}_{k,l}}{\tilde{X}_k \sigma_{k,l} + N_k}. \end{aligned} \quad (25)$$

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