An Optimization Framework for Planning of WAMS with a Heterogeneous Communication Network

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***Abstract*—In this work, we propose an optimization model for an integrated minimum-cost planning of a wide area measurement system (WAMS) and its underlying heterogeneous communication network. The integer linear program formulation of the proposed planning approach enables not only a simultaneous optimal placement of phasor data concentrators (PDC) and phasor measurement units (PMU) for the observability of the power system, but also the consideration of multiple communication technologies with their link parameters and costs. The novel network design approach is applied to test networks with different sizes and topological properties. The first results validate that the deployment cost of a WAMS can be reduced by identifying the possible use of low-cost communication technologies with the proposed integrated planning approach while satisfying network requirements.**

I. Introduction

The ambitious objectives of policy makers, to increase the share of renewable energy resources in the total energy consumption, have given rise to a paradigm shift in the planning, monitoring and operation of the power grid. Specifically, medium-voltage (MV) and low-voltage (LV) distribution networks, which have historically been passive elements for the system operators in the traditional power grid, have come into the focus of power system research, since the distributed energy resources (DER) are envisioned to feed their generation into the modern power grids. Thus, it becomes necessary to equip these networks with a wide area measurement system (WAMS), a sophisticated sensing and communication infrastructure, in a way that has not been done for traditional power grids. The WAMS plays a crucial role in the operation of a distribution network by measuring the system parameters with advanced sensing techniques and using these measurements with the aid of an underlying communication network to ensure a stable system operation and to optimize its operation for more efficiency. On the other hand, the deployment of such a sensing infrastructure with a communication network is associated with tremendous costs when the size of the distribution network is considered. In this context, the optimal planning of a WAMS is of crucial importance for the distribution system operators (DSO) to minimize the investment costs while taking the required system specifications into account.

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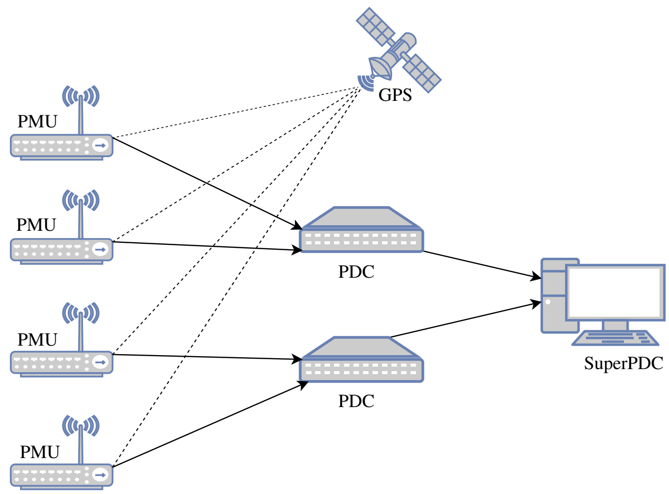

Fig. 1. Hierarchical network architecture of WAMS. PMUs send the phasor measurements, timestamped by the GPS signal, to SuperPDC over intermediate node PDCs

Until recently, the planning of the measurement infrastructures for power networks has been studied without taking into account the requirements for the underlying communication network. The main concern for the optimal measurement placement problem has been the minimization of the required number of phasor measurement units (PMU) ensuring a target observability criterion under various contingencies, cf. [1]. In majority of these studies, the communication network limitations are not considered, whereas some studies restrict the placement of the PMUs to the power system nodes with communication capabilities with assumed fiber links.

However, the necessity of system monitoring over a larger part of the power grid has brought the concerns for the communication aspects to the center of measurement design considerations. *IEEE Standard for Synchrophasor Data Transfer for Power Systems* [2] lays down the architecture for the communication network in WAMS as shown in Figure 1. This architecture postulates a hierarchical transmission of sensor data from PMUs over intermediate nodes, called phasor data concentrators (PDC), to a larger central unit, called SuperPDC (SPDC). Motivated by this development, few number of recent studies have proposed planning approaches for WAMS considering both power system constraints and communication network constraints. The authors of [3] introduce an optimization approach which ensures the observability of the power system while establishing a connected communication network between installed PMUs. This study, however, does not consider the accepted hierarchical and scalable structure of WAMS. In addition, only fiber links are considered in the planning. The authors of [4] approach the design problem from a different perspective by placing a PDC at the system bus which would minimize the communication link distances with the help of a shortest path algorithm, and then finding the optimum locations for PMUs. As in other mentioned studies, the communication links are also assumed to be fiber links. In another recent work, Wen *et al.* [5] proposes an integer linear program formulation of PDC placement problem with the objective of minimizing the total WAMS traffic when locations of PMUs are known.

In all of these studies, there are two common modelling approaches: First, only optical communication links are assumed. However, it is questionable that installing optical links to each PMU location will be the most feasible solution when the size of distribution networks is considered. It is more likely that a WAMS requires the use of multiple communication technologies in order to benefit from their distinctive advantages and to minimize deployment costs [6]. For example, the authors of [7] propose a method to make use of the available power lines for power line communication (PLC) in WAMS. They show that the number of high bandwidth links can be decreased by 80%, which would reduce the costs significantly. In this context and in addition to PLC, currently available low-cost wireless technologies and, in the future, the ones which are developed for low-delay, massive machine-to-machine communications can be an option in WAMSs. Furthermore, due to the requirement of fiber link, the communication requirements such as delay and capacity are ignored. Therefore, the proposed models are not sufficient for the planning of WAMS with a heterogeneous communication infrastructure. Secondly, all of the mentioned studies treat the placement of PDCs and the placement of PMUs in two stages, which prevents an integrated optimal network planning. A planning approach with a simultaneous consideration of PMU-PDC locations has been introduced in [8], in which the communication network constraints are integrated only from the perspective of reliability without any other considerations. Similarly, [9] proposes an integer programming model for simultaneous PMU-PDC placement. This model, however, does not cover multiple technologies and any delay and capacity constraints. Furthermore, the model is non-linear, which hinders an optimal solution.

In the present work, we propose an optimization model for the integrated planning of WAMS with a heterogeneous communication network. We start with the elaboration of the network architecture. Next, we present our planning approach along with the proposed optimization model which is formulated as an integer linear program (ILP) with its details. Afterwards, the proposed model is applied to several test networks, and the results are discussed. We conclude the paper with a summary of our contributions along with future work.

TABLE I
SYMBOL NOTATION

Symbol, Domain	Description
$\mathcal{B}$	set of binary numbers
$\mathcal{Z}_+$, $\mathcal{R}_+$	sets of non-negative real and integer numbers
$\mathcal{V}, n_{\text{bus}} \in \mathcal{Z}_+$	set and number of power system nodes
$\mathcal{E}, n_{\text{branch}} \in \mathcal{Z}_+$	set and number of power system branches
$G(\mathcal{V}, \mathcal{E})$	graph of the power system
$\boldsymbol{A} = \mathcal{B}^{n_{\text{bus}} \times n_{\text{bus}}}$	connectivity matrix of the power system
$\mathcal{P}_{\text{PMU}}, n_{\text{bus}} \in \mathcal{Z}_+$	set and number of possible PDC locations
$\mathcal{P}_{\text{PDC}}, n_p \in \mathcal{Z}_+$	set and number of possible PDC locations
$\mathcal{S}_c, n_c \in \mathcal{Z}_+$	set and number of possible PMU-PDC links
$\mathcal{S}_t$	set of possible link-technology matchings
$\mathcal{T}_1, n_t \in \mathcal{Z}_+$	set and number of available technologies in layer 1
$\mathcal{T}_2, n_b \in \mathcal{Z}_+$	set and number of available technologies in layer 2
$\boldsymbol{S}_c \in \mathcal{B}^{n_p \times n_{\text{bus}}}$	PMU-PDC connectivity matrix
$\boldsymbol{S}_t \in \mathcal{B}^{n_c \times n_t}$	Layer 1 link technology availability matrix
$\boldsymbol{B}_t \in \mathcal{B}^{n_p \times n_t}$	Layer 2 link technology availability matrix
$\boldsymbol{\mu}_1, \boldsymbol{D}_1, \boldsymbol{C}_1 \in \mathcal{R}_+^{n_c \times n_t}$	capacity, delay and cost of layer 1 technologies
$\boldsymbol{\mu}_2, \boldsymbol{D}_2, \boldsymbol{C}_2 \in \mathcal{R}_+^{n_p \times n_b}$	capacity, delay and cost of layer 2 technologies
$\boldsymbol{g} \in \mathcal{R}_+^{n_{\text{bus}} \times 1}$	bandwidth requirements at system nodes
$\boldsymbol{x} \in \mathcal{B}^{n_{\text{bus}} \times 1}$	optimization variable for PMU locations
$\boldsymbol{p} \in \mathcal{B}^{n_{\text{p}} \times 1}$	optimization variable for PDC locations
$\boldsymbol{y} \in \mathcal{B}^{n_{\text{p}} \times n_{\text{bus}}}$	opt. variable for PMU and PDC matchings
$\boldsymbol{z} \in \mathcal{B}^{n_{\text{c}} \times n_t}$	optimization variable for layer 1 technologies
$\boldsymbol{b} \in \mathcal{B}^{n_{\text{c}} \times n_t}$	optimization variable for layer 2 technologies

II. Network Architecture

The communication network in WAMS is envisioned to have a hierarchical architecture in order to enable a scalable expansion of the network when the expected increase in the number of measurement units is considered. In the architecture shown in Figure 1, the system parameters are measured by PMUs. The measurement data are transmitted to an intermediate node PDC, where a pre-processing of the data received from multiple PMUs takes place, such as time alignment and quality check [2]. PDCs send required data to a higher level data aggregator, which is called SuperPDC, where the data from a larger part of the system are aggregated to execute energy management functions, such as state estimation and fault localization. In the following section, we introduce our system model, along with the notation used, for the integrated minimum cost planning of this network considering both power system and communication requirements.

III. System Model

We consider a power grid represented by a graph $G(\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of the system buses with $|\mathcal{V}| = n_{\text{bus}}$, and $\mathcal{E}$ is the set of the electrical links with $|\mathcal{E}| = n_{\text{branch}}$. The binary connectivity matrix $\boldsymbol{A} \in \mathcal{B}^{n_{\text{bus}} \times n_{\text{bus}}}$ is defined as

$$A_{ij} = \begin{cases} 1, & \text{if } i = j \text{ or node } i \text{ is connected to node } j, \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

where $\mathcal{B} = \{0, 1\}$. For the integrated planning of the WAMS, we assume that the SPDC is located at a predetermined and known power system node v_{SPDC}. In addition, we assume that a number of candidate PDC locations are available, denoted by the set $\mathcal{P}_{\text{PDC}} = \{p_i \mid i = 1, \ldots, n_p,\ p_i \neq v_{\text{SPDC}}\}$ with $|\mathcal{P}_{\text{PDC}}| = n_p$ and $|\mathcal{P}_{\text{PDC}}| \subset \mathcal{V}$. We denote the set of system nodes, where a PMU can be installed, by the set $\mathcal{P}_{\text{PMU}} = \{q_i \mid i = 1, \ldots, n_{\text{pmu}}\}$. Without loss of generality, we assume that PMUs can be placed at any node $v_i \in \mathcal{V}$ in the power network, i.e., $\mathcal{P}_{\text{PMU}} = \mathcal{V}$ and $n_{\text{pmu}} = n_{\text{bus}}$. In the following, we refer to all communication links between PMUs and PDCs as *layer 1*, and all communication links between PDCs and the SPDC as *layer 2* of the communication network. We denote the sets of available communication technologies in layer 1 and layer 2 by $\mathcal{T}_1$ and $\mathcal{T}_2$, respectively, where $|\mathcal{T}_1| = n_t$ and $|\mathcal{T}_2| = n_b$.

Furthermore, there are n_c possible matchings in layer 1 between PMUs q_j and possible PDC locations p_i, meaning that a communication link between q_j and p_i is possible at least by one technology. These possible links are denoted in the set $\mathcal{S}_c = \{(p_i, q_j) \mid p_i \in \mathcal{P}_{\text{PDC}},\ q_j \in \mathcal{P}_{\text{PMU}}\}$, where $|\mathcal{S}_c| = n_c$. The binary connectivity matrix $\boldsymbol{S}_c \in \mathcal{B}^{n_p \times n_{\text{bus}}}$ is defined for these possible matchings as

$$S_{c,ij} = \begin{cases} 1, & \text{if a communication link between} \\ & p_i \text{ and } q_j \text{ is available,} \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

We further assume that for a possible pair $(p_i, q_j) \in \mathcal{S}_c$, there can be n_t different technologies. These possible matchings are described in the set $\mathcal{S}_t = \{((p_i, q_j), t_k) \mid (p_i, q_j) \in \mathcal{S}_c,\ t_k \in \mathcal{T}_1\}$. The binary connectivity matrix $\boldsymbol{S}_t \in \mathcal{B}^{n_c \times n_t}$ is defined for these possible matchings as

$$S_{t,(i,j)k} = \begin{cases} 1, & \text{if the technology } t_k \in \mathcal{T}_1 \text{ is available} \\ & \text{for the link } (p_i, q_j) \in \mathcal{S}_c, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

For each communication link-technology matching available in $\mathcal{S}_t$, we define the link capacity, delay, and cost matrices as $\boldsymbol{\mu}_1, \boldsymbol{D}_1$, and $\boldsymbol{C}_1 \in \mathcal{R}_+^{n_c \times n_t}$, respectively, where $\mathcal{R}_+$ denotes the set of non-negative real numbers.

In a similar manner to the matchings between PMU and PDC locations, we assume that there are n_b available technologies for the connection of possible PDC locations to the SPDC in layer 2. These possible matchings are denoted by the matrix $\boldsymbol{B}_t \in \mathcal{B}^{n_p \times n_b}$, whose entries are given as

$$B_{(i,j)k} = \begin{cases} 1, & \text{if the technology } t_k \in \mathcal{T}_2 \text{ is available} \\ & \text{for the PDC located at } p_i \in \mathcal{P}_{\text{PDC}}, \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

The capacity, the delay, and the cost of technologies in layer 2 are denoted by the matrices $\boldsymbol{\mu}_2$, $\boldsymbol{D}_2$, and $\boldsymbol{C}_2 \in \mathcal{R}_+^{n_p \times n_b}$, respectively. For the capacity considerations, we denote the required bandwidth at system nodes with the vector $\boldsymbol{g} = \{g_i \in \mathcal{R}_+ \mid i = 1, \ldots, n_{\text{bus}}\}$ where g_i is the bandwidth requirement at bus v_i. Note that g_i depends on the number of phasor signals which can be measured at node v_i, i.e., on its nodal degree in the power system graph.

The goal of the planning task is to find the optimal selection of PMU locations and PDC locations along with the communication links out of the given possibilities in $\boldsymbol{S}_c$, $\boldsymbol{S}_t$, and $\boldsymbol{B}_t$ for a certain objective. Therefore, we define the optimization variables $\boldsymbol{x} \in \mathcal{B}^{n_{\text{bus}}}$ for the selection of PMU locations, $\boldsymbol{p} \in \mathcal{B}^{n_{\text{p}}}$ for the selection of PDC locations, $\boldsymbol{y} \in \mathcal{B}^{n_p \times n_{\text{bus}}}$ for the selection of matchings between PMUs and PDCs, $\boldsymbol{z} \in \mathcal{B}^{n_c \times n_t}$ for the selection of technologies between PMUs and PDCs, and $\boldsymbol{b} \in \mathcal{B}^{n_p \times n_b}$ for the selection of technologies between PDCs and the SPDC. All of the vectors in this work are defined as column vectors. An overview of the used notation is given in Table I.

IV. Optimization Model

In the following, we formulate the integrated optimization problem with power system and communication network constraints for a WAMS.

A. Power System Observability

With the assumption that a PMU can measure the voltage phasor at its node and the current phasors to all its neighbors, the power system is observable if the PMU locations $\boldsymbol{x}$ satisfy

$$\boldsymbol{A}\boldsymbol{x} \succeq \mathbf{1}, \tag{5}$$

where $\succeq$ denotes the element-wise inequality operator for matrices of same size, and $\mathbf{1}$ is a vector of all-ones of size n_{bus}. The constraint in (5) means that a node is observable if a PMU is placed at that node or at any of its neighbors [1].

B. Communication Network Connectivity

To begin with, the optimization variables $\boldsymbol{y}$, $\boldsymbol{z}$, and $\boldsymbol{b}$ are restricted by the available links on the one hand and the available technologies on the other hand. Therefore, we have the constraints for $\boldsymbol{y}$, $\boldsymbol{z}$, and $\boldsymbol{b}$ as

$$\boldsymbol{y} \preceq \boldsymbol{S}_c, \tag{6}$$

$$\boldsymbol{z} \preceq \boldsymbol{S}_t, \text{ and} \tag{7}$$

$$\boldsymbol{b} \preceq \boldsymbol{B}_t. \tag{8}$$

It has to be ensured that a node, at which a PMU is installed, has a communication link to one PDC location. This constraint can be formulated as

$$\sum_{i=1}^{n_{\text{p}}} y_{ij} = x_j, \quad \forall j = 1, \ldots, n_{\text{bus}}, \tag{9}$$

which, together with (6), also excludes the power system buses without a communication connection from PMU placement. Furthermore, the constraints for the selection of exactly one technology for a selected link $(p_i, q_j) \in \mathcal{S}_c$ is written as

$$y_{ij} = \sum_{k=1}^{n_t} z_{(i,j)k}, \quad \forall (p_i, q_j) \in \mathcal{S}_c. \tag{10}$$

We write the constraint for the installation of a PDC at i^{th} bus as

$$p_i = \max\{y_{ij} \mid j = 1, \ldots, n_{\text{bus}}\}, \quad \forall i = 1, \ldots, n_p, \tag{11}$$

which ensures the selection of a PDC location, if it serves an installed PMU. Similarly, we have the constraints for the selection of one and only one technology in the second layer for a selected PDC location as

$$\sum_{j=1}^{n_b} b_{ij} = p_i, \quad \forall i = 1, \ldots, n_p. \tag{12}$$

C. Data Communication Requirements

In addition to the observability and the connectivity requirements discussed so far, the delays on the communication links and their capacities play a vital role as stated in [2]. The delay and capacity requirements are dependent on the power system application, and these constraints have to be taken into consideration in the planning phase.

1) End-to-end Delay: We consider an upper limit for the end-to-end delay over the communication link from an installed PMU to the SPDC. If the maximum allowable delay for the system is δ_{th}, the total delay constraint for possible layer 1 links $(p_i, q_j) \in \mathcal{S}_c$ and layer 2 links can be written as

$$\sum_{k=1}^{n_t} z_{(i,j)k} D_{1,(i,j)k} + \sum_{k=1}^{n_b} b_{ik} D_{2,ik} \le \delta_{th}, \quad \forall (p_i, q_j) \in \mathcal{S}_c. \tag{13}$$

2) Capacity: Regarding the capacity limitations, we differentiate between the wired technologies, such as PLC and fiber networks whose topologies can be assumed to match the power network topology, and the technologies which can be modeled abstractly with a general capacity constraint. We denote the set of the former technologies with $\mathcal{T}_1^w$ in the first layer and $\mathcal{T}_2^w$ in the second layer, and the set of the latter ones with $\mathcal{T}_1^g$ and $\mathcal{T}_2^g$. The reason for this differentiation is that in wired links on power systems, traffic flows from different sources are aggregated at certain nodes along their path to a higher layer in the network. Therefore, each branch in the power system graph should be constrained by its capacity. For this purpose, we use the following approach: With the reasonable assumption that a wired link of a technology in $\mathcal{T}_1^w$ will use the shortest possible path from a PMU to PDC in the power system graph, we find the path of each link $(p_i, q_j) \in \mathcal{S}_c$ by calculating the shortest paths from q_j to p_i in the first layer. We define the binary matrix $\boldsymbol{E}_1 = \mathcal{B}^{n_c \times n_{\text{branch}}}$ whose nonzero entries in its corresponding row denote the branches on the shortest path from q_j to p_i. Thus, the total capacity constraint for each branch of the power system graph can be formulated for wired technologies as

$$\sum_{(p_i,q_j)\in\mathcal{S}_c} z_{(i,j)k} E_{1,(i,j)e} g_j \le \mu_{1,ek}, \forall t_k \in \mathcal{T}_1^w, \forall e \in \mathcal{E}, \tag{14}$$

where g_j stands for the traffic generated by the PMU q_j, and $\mu_{1,ek}$ stands for the capacity of technology t_k over branch e. Similarly, the capacity constraint in the second layer can be written as

$$\sum_{r=1}^{n_p} E_{2,re} b_{rs} \sum_{(p_r,q_j)\in\mathcal{S}_c} y_{rj} g_j \le \mu_{2,es}, \forall t_s \in \mathcal{T}_2^w, \forall e \in \mathcal{E}, \tag{15}$$

where $\boldsymbol{E}_2 = \mathcal{B}^{n_p \times n_{\text{branch}}}$ stores the shortest path from p_i to SPDC in its i^{th} row. The capacity constraints for generic communication technologies, which we model with a general capacity limit, can be written in the first and the second layers as

$$\sum_{(p_i,q_j)\in\mathcal{S}_c} z_{(i,j)k} g_j \le \mu_{1,k}, \quad \forall t_k \in \mathcal{T}_1^g, \tag{16}$$

$$\sum_{r=1}^{n_p} b_{rs} \sum_{(p_r,q_j)\in\mathcal{S}_c} y_{rj} g_j \le \mu_{2,s}, \quad \forall t_s \in \mathcal{T}_2^g. \tag{17}$$

It should be noted that, if a technology is considered for both layers, constraints (14-15) and (16-17) must be considered in joint constraints, respectively. Furthermore, if a point-to-point wireless communication, such as a microwave link, is used in the second layer at PDC location p_i, its capacity constraints can be written as

$$\sum_{(p_i,q_j)\in\mathcal{S}_c} y_{ij} g_j \le \mu_{2,ik}, \quad \forall t_k \in \mathcal{T}_{\text{p2p}}, \tag{18}$$

where $\mathcal{T}_{\text{p2p}}$ denotes the set of point-to-point wireless links.

Note that the constraints (15) and (17) involve nonlinear terms due to the multiplication of binary variables b_{rs} and y_{rj} for all $(p_r, q_j) \in \mathcal{S}_c$, and $t_s \in \mathcal{T}_2$. In order to linearize these constraints, we define the binary optimization variables $\rho_{rsj} = b_{rs} y_{rj}$ with following additional linear constraints as

$$\rho_{rsj} \le b_{rs}, \quad \forall (p_r, q_j) \in \mathcal{S}_c, \quad \forall t_s \in \mathcal{T}_2, \tag{19}$$

$$\rho_{rsj} \le y_{rj}, \quad \forall (p_r, q_j) \in \mathcal{S}_c, \quad \forall t_s \in \mathcal{T}_2, \tag{20}$$

$$\rho_{rsj} \ge b_{rs} + y_{rj} - 1, \quad \forall (p_r, q_j) \in \mathcal{S}_c, \quad \forall t_s \in \mathcal{T}_2. \tag{21}$$

Then, we can reformulate (15) and (17) as

$$\sum_{r=1}^{n_p} E_{2,re} \sum_{(p_r,q_j)\in\mathcal{S}_c} \rho_{rsj} g_j \le \mu_{2,es}, \forall t_s \in \mathcal{T}_2^w, \forall e \in \mathcal{E}, \tag{22}$$

$$\sum_{r=1}^{n_p} \sum_{(p_r,q_j)\in\mathcal{S}_c} \rho_{rsj} g_j \le \mu_{2,s}, \quad \forall t_s \in \mathcal{T}_2^g, \tag{23}$$

in linear form, respectively [10].

D. Objective Function

In the present work, we define our objective function $F(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{z}, \boldsymbol{b})$ as the total cost of the WAMS deployment which we formulate as

$$F(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{z}, \boldsymbol{b}) = \underbrace{\boldsymbol{c}_x^T \boldsymbol{x}}_{\text{PMU costs}} + \underbrace{\boldsymbol{c}_p^T \boldsymbol{p}}_{\text{PDC costs}} + F_{\text{comm}}(\boldsymbol{z}, \boldsymbol{b}) \tag{24}$$

where $F_{\text{comm}}(\boldsymbol{z}, \boldsymbol{b})$ is the cost function for all communication links. When the case is considered, where a wired communication technology is candidate for both layers, we can write $F_{\text{comm}}(\boldsymbol{z}, \boldsymbol{b})$ as

$$F_{\text{comm}}(\boldsymbol{z}, \boldsymbol{b}) = \sum_{i=1}^{n_c} \sum_{\substack{t_j \in \mathcal{T}_1 \\ t_j \notin \mathcal{T}_c^w}} C_{1,ij} z_{ij} + \sum_{i=1}^{n_p} \sum_{\substack{t_j \in \mathcal{T}_2 \\ t_j \notin \mathcal{T}_c^w}} C_{2,ij} b_{ij}$$
$$+ \sum_{e\in\mathcal{E}} \sum_{t\in\mathcal{T}_c^w} C_{e,t} \max\{z_{lt} E_{1,le}, b_{rt} E_{2,re} \mid l \in \mathcal{S}_c, p_r \in \mathcal{P}_{\text{PDC}}\} \tag{25}$$

where $C_{e,t}$ is the cost of technology t over branch $e \in \mathcal{E}$, and $\mathcal{T}_c^w$ denotes the set of wired communication technologies considered for both layers.

Note that the constraints in (11) and $F_{\text{comm}}(\boldsymbol{z}, \boldsymbol{b})$ in (25) are not linear due to the maximum over sets of binary variables. However, these constraints can be linearized by additional linear constraints and optimization variables [10]. For example, a term $\max\{b_i \mid i = 1, \dots, n\}$ can be replaced by a new binary optimization variable b' along with the linear constraints

$$b' \geq b_i, \quad \forall i = 1, \dots, n \tag{26a}$$

$$b' \leq \sum_{i=1}^{n} b_i. \tag{26b}$$

Having applied these linearization steps, the optimization problem can be formulated in linear form as

$$\begin{aligned} \text{minimize} \quad & F(\boldsymbol{x}, \boldsymbol{p}, \boldsymbol{z}, \boldsymbol{b}) \\ \text{subject to} \quad & \text{(5)-(10),(12)-(14),(16),(18)-(23)} \end{aligned} \tag{27}$$

along with the binary constraints for all optimization variables. This ILP can be solved optimally by solvers, such as Gurobi [11], which we use in this work.

V. Test Cases

In order to assess the proposed integrated optimization model, we apply it with representative technology and cost assumptions to MV test networks which are stochastically generated by the tool presented in [12]. The power system graphs that are considered have two topological properties: *i*) uniform or clustered distribution of the nodes over the considered area, and *ii*) radial or ring characteristics.

To validate the advantage of the proposed model, we compare its optimal cost value with a multi-stage approach where the communication network design and PDC placement are carried out after the placement of the minimum number of PMUs required for observability. In addition, we include the cost reduction against the integrated approach with fiber links only.

In the following, we briefly mention the details of the parameter assumptions and the optimization process for a given network. The traffic generated by a PMU is calculated according to the frame sizes of the IEEE C37.118 standard as described in [2] with the communication overhead of UDP/IP protocol layers. We consider PLC, broadband PLC (BPLC), fiber and WiMaX links in layer 1, and BPLC, fiber and microwave links in layer 2 as available technologies, which are used in real world applications [6]. The assumptions for the cost and parameters of the communication technologies are based on available standards and studies in the literature [6], [13]–[15]. The summary of link parameter and cost assumptions is given in Table II. We assume the costs for a PMU and PDC as €7500 and €15000 based on the prices of commercially available metering products [16].

Note that in these test cases, we do not consider the delay constraints introduced in Section IV.C. A thorough modeling and analysis of link delays in WAMS for specific applications are planned in the future work.

In the optimization procedure for a given network, we select the SPDC location as the node with the largest nodal degree in the power system graph $\mathcal{V}$, whereas n_p PDC locations are randomly selected over 6 equally divided subregions of the total area to ensure a uniform distribution of the PDC locations over the whole area. Next, we identify all possible communication links and technologies along with their parameters and costs, by using the power system graph properties and communication technology limitations, namely branch distances and maximum communication ranges. As a result of this discovery process, the matrices $\boldsymbol{S}_c, \boldsymbol{S}_t$, and $\boldsymbol{B}_t$ are generated, as well as the problem data $\boldsymbol{A}$, $\boldsymbol{E}_1$, $\boldsymbol{E}_2$, and $\boldsymbol{g}$. Then, the problem is solved by the ILP solver Gurobi [11].

TABLE II
LINK PARAMETERS AND COST ASSUMPTIONS [6], [13]–[15]

Technology	Range	Capacity	Cost(€)
PLC	5 km	250 Kbps	200 / hop
BPLC	2 km	10 Mbps	500 / hop
WiMaX	3 km	30 Mbps	1000/connection
Fiber	100 km	10 Gbps	1000 / km
Microwave	5 km	10 Mbps	10000/connection

TABLE III
COST COMPARISON RESULTS - REGION OF 10 km × 10 km

n_{bus}	n_p	Node Distribution	Topology	Reduction(%) vs. Multi Stage		Reduction(%) vs. Fiber Only	
				Com.	Tot.	Com.	Tot.
50	8	Uniform	Radial	3.1	0.5	45.9	10.7
50	**8**	**Uniform**	**Ring**	**6.9**	**0.9**	**59.6**	**13.8**
50	8	Cluster	Radial	2.9	0.3	51.9	9.7
50	8	Cluster	Ring	4.8	0.5	50.8	10.2
100	8	Uniform	Radial	5.9	0.8	36.3	6.4
100	8	Uniform	Ring	4.8	0.7	31.7	5.9
100	8	Cluster	Radial	1.9	0.2	22.6	3.4
100	8	Cluster	Ring	2.4	0.3	21.5	3.2

VI. Results & Discussion

We present the results of the optimization procedure described in Section V applied to MV networks with 50 and 100 buses, for all possible topological parameter sets and over region sizes of $10\,\text{km} \times 10\,\text{km}$ and $20\,\text{km} \times 20\,\text{km}$. For each case, we use 5 test networks with the same topological properties. Tables III and IV show the average percentage reduction in total WAMS deployment cost for each case *i*) against a multi-stage planning and *ii*) against an integrated planning with fiber

TABLE IV
COST COMPARISON RESULTS - REGION OF 20 km × 20 km

n_{bus}	n_p	Node Distribution	Topology	Reduction(%) vs. Multi Stage		Reduction(%) vs. Fiber Only	
				Com.	Tot.	Com.	Tot.
50	8	Uniform	Radial	7.4	2.0	32.8	8.8
50	8	Uniform	Ring	2.5	2.2	30.3	9.9
50	8	Cluster	Radial	3.4	2.6	35	9.6
50	8	Cluster	Ring	1.8	1.4	26	8.8
100	12	Uniform	Radial	6.7	1.5	32.1	6.2
100	12	Uniform	Ring	2.8	2.2	25.2	5.2
100	12	Cluster	Radial	5.1	1.2	25.1	5.7
100	12	Cluster	Ring	9.5	1.4	30.2	7.2

links only. Furthermore, the average percentage reduction in communication link costs and total costs including PMU and PDC costs are given separately. As an example, the third row of Table III, which is highlighted, shows the average percentage reductions in both aforementioned comparisons over 5 power networks with 50 uniformly distributed nodes in ring topology over an area of $10\,\mathrm{km} \times 10\,\mathrm{km}$. In this case, we see a reduction of 6.9% in communication costs and 0.9% in total costs against a multistage planning. Similarly, cost savings of 59.6% and 13.8%, respectively in communication and total costs, are achieved by our model against a planning with fiber only.

The results show that the use of multiple technologies enables a significant reduction of the communication and total deployment costs. Depending on the power system branch lengths, and the ranges and capacities of PLC and BPLC communication links, the proposed optimization model is able to detect certain PMU and PDC locations which are more advantageous for a minimum-cost planning. Note that as the costs for the required PMU and PDC do not change considerably with the compared optimization strategies due to the requirement of observability, the reduction of total cost deployment costs are lower in percentage.

We observe, furthermore, that the simultaneous optimization of PMU and PDC locations reduces the costs, as well. However, the reduction in percentage is modest with a maximum of 9.5% in communication costs, and it is less than 3% in total cost in all cases. This relatively modest improvement can be attributed to the requirement of full observability, which requires the installation of PMUs scattered almost uniformly over all regions of the power system graph. As both simultaneous and multistage planning try to make use of the most cost-efficient communication links, the improvement remains moderate in percentage. Nevertheless, as the total cost is high in general, this modest improvement will lead to significant cost savings in a deployment. Regarding the results for different topological properties, we observe diverse reduction values which do not correlate with certain topologies.

VII. Conclusion

In this work, we have proposed a novel optimization model and approach for the integrated planning of WAMS for power distribution systems. The ILP formulation of the optimization model enables a joint optimal network design under the constraints of power system observability, communication network connectivity and data communication requirements with multiple communication technologies. The application of the proposed model on test networks has shown that the model is able to decrease the total deployment costs due to the joint consideration of network requirements. Future work will involve the analysis of latencies on the planning for specific smart grid applications and consideration of further objectives.

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