Power Allocation for Multi-Target Sensor Networks

Ehsan Zandi, Gholamreza Alirezaei, Rudolf Mathar
Institute for Theoretical Information Technology, RWTH Aachen University, Germany
Email: {zandi, alirezaei, mathar}@ti.rwth-aachen.de

***Abstract*—This paper regards a network of wireless passive sensors, suitable for radar applications. The network observes multiple number of target signals to be estimated. We propose an unbiased estimator whose error is minimized by optimizing both the power allocation at sensor nodes and the fusion coefficients at the fusion center. Since the underlying optimization problems are subject to both the individual power constraints of sensor nodes and a sum-power constraint of the sensor network, the corresponding solution describes an optimized policy for efficient power allocation to the sensor nodes and enables an energy-aware operation of the sensor network. The results are compared via numerical simulations.**

***Index Terms*—resource optimization, signal detection, classification, radar, best linear unbiased estimator**

I. Introduction

In this paper which is an extension to the work in [1], we consider a wireless sensor network to estimate the true values of L active targets. The network consists of K wireless passive sensors. This means that targets emit some signal from themselves, while sensors passively sense the emitted signals. Indeed, sensor nodes consume no power for sensing the targets, while they consume power for transmitting their observation towards a fusion center. We assume two different types of power constraint, i.e., an individual output power constraint and furthermore a sum-power constraint among all sensors. The latter is to manage the interference in the network, on the one hand, and to save worthwhile resources in order to prolong the network lifetime, on the other hand. Then, the fusion center multiplies each received signal by a so-called fusion coefficient (or fusion rule) to estimate the real value of the targets. The goal is to optimize the communication powers of the sensor nodes and the fusion coefficients in order to minimize the estimation error at the fusion center. The considered scenario is equivalent to a distributed multi-target radar system in which expensive radar units are replaced by cheap and weak sensor nodes. The system of consideration well suits the monitoring applications in extreme environments. The reason is that in such environments it is very hard to replace the batteries of the sensors, therefore, lifetime management of the network is of great significance.

Due to the rise of certain applications in the 5^{th} generation wireless systems (5G), sensor networks for sensing and monitoring are drastically gaining importance. However, for an accurately performed sensing and monitoring task, an optimal resource utilization is necessary, since the estimation performance is increasing over the energy and power consumption of the network. Thus, the optimization of energy and power resources for a required system performance is of high interest and investigated in many publications, especially for the single target case. Indeed, the power allocation problem subject to individual power limitations of the sensor nodes as well as a sum-power constraint is solved in closed-form in [1]. For the multi-target case the main topics for investigations address the tracking and coverage problems. In [2], the objective is to maximize the lifetime subject to power constraints and coverage regions. Instead of maximizing the lifetime, we minimize the estimation error in the present work. The GaussMarkov mobility model is used in [3] to formulate the tracking problem as a hierarchical Markov decision process, which in turn is solved by neurodynamic programming. We exploit an heuristic to achieve an accurate suboptimal solution instead of the usage of sophisticated programming methods. Since a centralized processing is difficult to handle in multi-target scenarios, the authors in [4] investigate the tracking problem by a distributed data processing. The approach in [5] considers a scenario in which sensor nodes can be put into a sleep mode with a timer that determines the sleep duration. They show that optimizing the sleep duration improves the tracking performance in sensor networks. In contrast to [4] and [5], we investigate the centralized scenario and determine the least reliable sensor nodes to keep them asleep for a uniform time duration, respectively. It is to mention, that our approach is more general and it can be used not only for tracking but also for detection and classification of targets, cf. [6].

The organization of this paper is as follows: the system model is described in Sec. II. We propose an unbiased estimator in Sec. III whose variance of error can be further minimized by optimizing the power allocation among the sensor nodes as well as optimizing the fusion rule at the fusion center. The resulting optimization problems are solved in Sec. IV and V. While the simulation results are presented in Sec. VI, Sec. VII concludes this paper.

Notations: In this paper, upper and lower case boldface symbols denote matrices and vectors, respectively. The symbol $\mathbb{N}$ shows the set of natural numbers and the set of all complex (real) matrices of size $k \times n$ is denoted by $\mathbb{C}^{k\times n}$ ($\mathbb{R}^{k\times n}$). The set of all real vectors of size k is also shown by $\mathbb{R}^k$. The trace of a matrix is shown by $\mathrm{tr}(\,\cdot\,)$. While $[\mathbf{x}]_m$ or x_m notifies the m^{th} element of vector $\mathbf{x}$, we refer to the entry ij of matrix $\mathbf{A}$ by $[\mathbf{A}]_{ij}$ or a_{ij}. We use $\mathbf{I}$ and $\mathbf{0}$ to denote identity matrix and all-zero vector, respectively, of proper sizes. While $\mathrm{diag}(\mathbf{A})$ refers to a vector consisting of diagonal entries of $\mathbf{A}$, symbol $\mathbf{\Lambda_x}$ represents a diagonal matrix whose diagonal entries are denoted by vector $\mathbf{x}$. Moreover, $(\,\cdot\,)^*$ and $(\,\cdot\,)'$ are Hermitian and transpose operators. Note that Hermitian of a scalar value is the same as its complex conjugate. Moreover, $\mathcal{E}(\,\cdot\,)$ is the expected value and the Kronecker delta function is shown by

$$\delta_{lm} = \begin{cases} 1\,, & l = m\,, \\ 0\,, & l \neq m\,. \end{cases}$$

The operator $\mathrm{vec}(\mathbf{A})$ stacks all the columns of the matrix $\mathbf{A}$ into one long vector. Besides, $|\cdot|$ and $\|\cdot\|$ denote absolute value and Euclidean. Finally, $\mathcal{O}$ stands for big O notation.

II. System Model

The Fig. 1 depicts the block diagram of the considered system. As we see in the figure, the system of interest consists of $L \in \mathbb{N}$ target signals, i.e., $r_1, \cdots, r_L$, and $K \in \mathbb{N}$ sensors. Let $\mathbb{F}_K := \{1, \cdots, K\}$ and $\mathbb{F}_L := \{1, \cdots, L\}$ be the index sets of all sensors and targets, respectively.

The target signals are complex-valued unknowns whose power is assumed to be known, i.e., $R_l := \mathcal{E}(|r_l|^2), l \in \mathbb{F}_L$. Moreover, we assume that the targets slowly change so that we can assume they are constant over one round of estimation.

Each sensor receives a noisy copy of all signals each of which is multiplied by the sensing channel coefficient $g_{kl}[i] \in \mathbb{C}$. Indeed, $g_{kl}[i]$ corresponds to the sensing channel from l^{th} target towards the k^{th} sensor at discrete time instant i. Note that $g_{kl}[i]$ is a wide-sense stationary (WSS) random process with zero mean and variance of G_{kl}. Also, measurement noise $m_k[i] \in \mathbb{C}$ at time instant i is a zero-mean WSS random process whose variance is M_k. Both channel and noise terms are assumed to be identically and independently distributed (iid), which means

$$\mathcal{E}\Big(m_k[i] m_{k\prime}^*[i\prime]\Big) = \delta_{kk\prime}\delta_{ii\prime} M_k,\ \forall i, i\prime, k, k\prime \tag{1a}$$

$$\mathcal{E}\Big(g_{kl}[i] g_{k\prime l\prime}^*[i\prime]\Big) = \delta_{kk\prime}\delta_{ll\prime}\delta_{ii\prime} G_{kl},\ \forall i, i\prime, k, k\prime, l, l\prime. \tag{1b}$$

Furthermore, we assume that channel coefficients, noise terms and target signals are pairwise independent, i.e.,

$$\mathcal{E}\Big(m_k[i] r_l^*\Big) = \mathcal{E}\Big(m_k[i]\Big)\,\mathcal{E}(r_l^*) = 0,\ \forall i, k, l, \tag{2a}$$

$$\mathcal{E}\Big(g_{kl}[i] r_l^*\Big) = \mathcal{E}\Big(g_{kl}[i]\Big)\,\mathcal{E}(r_l^*) = 0,\ \forall i, k, l, \tag{2b}$$

$$\mathcal{E}\Big(g_{kl}[i] m_k^*[i]\Big) = \mathcal{E}\Big(g_{kl}[i]\Big)\,\mathcal{E}\Big(m_k^*[i]\Big) = 0,\ \forall i, k, l. \tag{2c}$$

Each sensor, e.g., sensor k, amplifies the noisy received signal by the non-negative coefficient u_k and outputs the signal $x_k[i]$ which is given by

$$x_k[i] = u_k\Big(m_k[i] + \sum_{l=1}^{L} g_{kl}[i] r_l\Big),\ k \in \mathbb{F}_K, \tag{3}$$

which corresponds to the following output power

$$X_k := \mathcal{E}\Big(\big|x_k[i]\big|^2\Big) = \Big(M_k + \sum_{l=1}^{L} G_{kl} R_l\Big) u_k^2. \tag{4}$$

Then, the transmitted signal from each sensor undergoes the impairment of the communication channel towards fusion center. Unlike the sensing channel $g_{kl}[i]$ which is time-variant, we assume that the communication channel h_k varies very slowly during the interval of estimation. Therefore, h_k can be seen as a time-invariant deterministic value. Each signal is further affected by noise $n_k[i]$ at the fusion center antenna. Similarly, we assume that $n_k[i]$ is complex-valued WSS random process with zero mean and variance of N_k. Both noise term and communication channel are also iid, which means

$$\mathcal{E}\Big(n_k[i] n_{k\prime}^*[i\prime]\Big) = \delta_{kk\prime}\delta_{ii\prime} N_k,\ \forall i, i\prime, k, k\prime, \tag{5a}$$

$$\mathcal{E}\Big(m_k[i] n_{k\prime}^*[i\prime]\Big) = \mathcal{E}\Big(m_k[i]\Big)\,\mathcal{E}\Big(n_{k\prime}^*[i\prime]\Big) = 0,\ \forall i, i\prime, k, k\prime. \tag{5b}$$

Finally, the fusion center multiplies the noisy received signal from sensor node k by the fusion coefficient $v_k \in \mathbb{C}$ which leads to

$$\begin{aligned} y_k[i] &:= v_k(h_k x_k[i] + n_k[i]) \\ &= v_k\Bigg(h_k u_k\Big(m_k[i] + \sum_{l=1}^{L} g_{kl}[i] r_l\Big) + n_k[i]\Bigg). \end{aligned} \tag{6}$$

Therefore, the total observation of the targets at fusion center is a (scalar and superimposed) signal described by

$$\begin{aligned} \tilde{r}[i] &:= \sum_{k=1}^{K} y_k[i] \\ &= \sum_{l=1}^{L} r_l\Big(\sum_{k=1}^{K} v_k h_k u_k g_{kl}[i]\Big) + \sum_{k=1}^{K} v_k(h_k u_k m_k[i] + n_k[i]). \end{aligned} \tag{7}$$

Obviously, we cannot estimate the true value of the target signals using (7) since we have L unknowns and only one equation. In order to have a feasible estimator, we need to build up a determined system of equations which means we need more than one time instant of observation. Let us define the following vectors:

$$\mathbf{r} = \big[r_1, \cdots, r_L\big]', \tag{8a}$$

$$\tilde{\mathbf{r}} = \big[\tilde{r}[1], \cdots, \tilde{r}[L]\big]', \tag{8b}$$

$$\mathbf{w} = \big[w[1], \cdots, w[L]\big]', \tag{8c}$$

where $w[i] = \sum_{k=1}^{K} v_k(h_k u_k m_k[i] + n_k[i])$ is the effective noise at time instant $i \in \mathbb{F}_L$. Based on the independence assumptions of the noise terms m_k and n_k, the noise covariance matrix $\mathbf{C} := \mathcal{E}(\mathbf{w}\mathbf{w}^*)$ can be written as

$$[\mathbf{C}]_{ij} = \begin{cases} \sum_{k=1}^{K} (|h_k|^2 u_k^2 M_k + N_k)|v_k|^2\,, & i = j\,, \\ 0\,, & i \neq j\,. \end{cases} \tag{9}$$

Also, we define $\mathbf{H}$ as the (effective) sensing matrix whose entries are represented by

$$[\mathbf{H}]_{il} = \sum_{k=1}^{K} v_k h_k u_k g_{kl}[i],\ i, l \in \mathbb{F}_L. \tag{10}$$

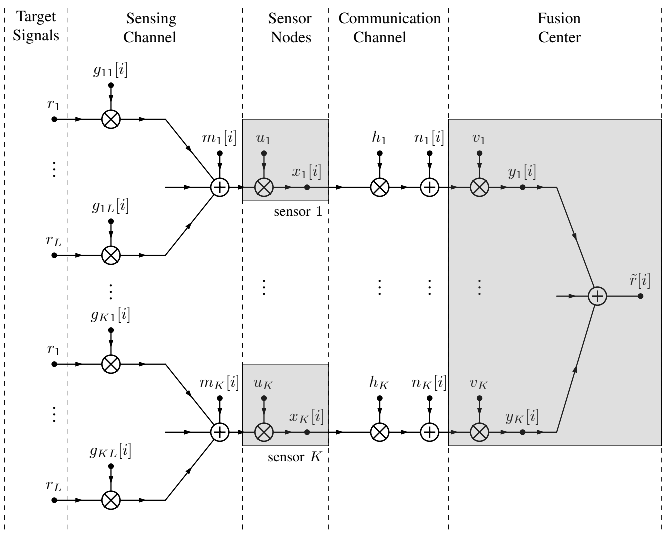


Fig. 1: Block diagram of the multi-target wireless sensor network

Then, the following equation describes the observation vector for the fusion center at L different time instants:

$$\tilde{\mathbf{r}} = \mathbf{H}\mathbf{r} + \mathbf{w}. \tag{11}$$

We assume that the sensor network under consideration deploys accurate methods of channel estimation, therefore, all the channel coefficients are known at the fusion center.

Furthermore, we assume that there are individual power constraints given by

$$X_k \leq P_k,\ k \in \mathbb{F}_K, \tag{12}$$

and also a sum-power constraint

$$\sum_{k=1}^{K} X_k = \sum_{k=1}^{K}\left(M_k + \sum_{l=1}^{L} G_{kl}R_l\right)u_k^2 \leq P_{\text{tot}}. \tag{13}$$

III. Proposed Estimator

Since the distribution of $[\mathbf{H}]_{il}$ is unknown, deriving a minimum variance unbiased estimator (MVUE) is not possible [7]. Instead, we are interested in the best linear unbiased estimator (BLUE), i.e., $\hat{\mathbf{r}}$, such that $\mathcal{E}(\hat{\mathbf{r}} - \mathbf{r}) = \mathbf{0}$ and has the minimum error variance. According to the *Gauss-Markov theorem* the BLUE of the linear equation (11) is given by

$$\begin{aligned}\hat{\mathbf{r}} = \mathbf{\Sigma}\tilde{\mathbf{r}} &= \left(\mathbf{H}^*\mathbf{C}^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^*\mathbf{C}^{-1}\tilde{\mathbf{r}} \\ &= \mathbf{r} + \left(\mathbf{H}^*\mathbf{C}^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^*\mathbf{C}^{-1}\mathbf{w},\end{aligned} \tag{14}$$

which leads to the following total estimation error

$$\text{tr}\Big(\left(\mathbf{H}^*\mathbf{C}^{-1}\mathbf{H}\right)^{-1}\Big). \tag{15}$$

So, our objective is to solve the following optimization problem in order to find the best power allocation and fusion strategy, i.e., u_k and v_k, which jointly minimize the total estimation error of the BLUE:

$$\min_{\substack{v_k, u_k \\ k\in\mathbb{F}_K}} \ \text{tr}\Big(\left(\mathbf{H}^*\mathbf{C}^{-1}\mathbf{H}\right)^{-1}\Big) \tag{16a}$$

$$\text{s.t.} \ \left(M_k + \sum_{l=1}^{L} G_{kl}R_l\right)u_k^2 \leq P_k,\ k \in \mathbb{F}_K, \tag{16b}$$

$$\sum_{k=1}^{K}\left(M_k + \sum_{l=1}^{L} G_{kl}R_l\right)u_k^2 \leq P_{\text{tot}}. \tag{16c}$$

Unfortunately, the problem (16) is hardly tractable due to its very complicated objective function. Therefore, we set out to obtain a suboptimal estimator by comparing the optimal solution (14) of the BLUE with the observation (11). Since both $\mathbf{H}$ and $\mathbf{C}$ are functions over all u_k and v_k, it seems to be natural to optimize all u_k and v_k such that the observation $\tilde{\mathbf{r}}$ achieves the same structure as is shown by (14). This means that $\mathbf{H}$ must approach $\mathbf{I}$ for optimal $u_k^\star$ and $v_k^\star$. Replacing $\mathbf{H} = \mathbf{I}$ into the objective (16a) yields the new objective function which can be easily written, as below, using (9) and (11)

$$\mathcal{E}(\|\tilde{\mathbf{r}} - \mathbf{r}\|^2) = L\sum_{k=1}^{K}(|h_k|^2 u_k^2 M_k + N_k)|v_k|^2 \,. \tag{17}$$

By following this heuristic we first make the observation at the fusion center unbiased and second minimize the variance of the difference $|\tilde{\mathbf{r}} - \mathbf{r}|$. In this way, the new optimization problem reads

$$\min_{\substack{v_k, u_k \\ k\in\mathbb{F}_K}} \ \sum_{k=1}^{K}(|h_k|^2 u_k^2 M_k + N_k)|v_k|^2 \tag{18a}$$

$$\text{s.t.} \ \sum_{k=1}^{K} v_k h_k u_k g_{kl}[i] = \delta_{il},\ i, l \in \mathbb{F}_L, \tag{18b}$$

as well as (16b) and (16c). The constraint given by (18b) guarantees the unbiasedness of the proposed estimator. The feasibility of this problem necessitates $K > L^2$. The reason is as follows: since the number of entries of $\mathbf{H} \in \mathbb{C}^{L\times L}$ is L^2, we need at least L^2 variables to prevent the underlying system of equations from being overdetermined. In summary, the feasibility of the system under consideration imposes some requirements which are summarized in the Table I.

TABLE I: Facts and figures of the proposed estimator

Number of channel estimations per target estimation	$K + L^2K$
Minimum number of sensors	L^2
Number of system variables	$2K$
Minimum number of system variables	$2L^2$

IV. Optimizing Fusion Rule

Let us rewrite the objective function in (18) into the vector form below

$$f(\mathbf{u},\mathbf{v}) \coloneqq \sum_{k=1}^{K}(|h_k|^2 u_k^2 M_k + N_k)|v_k|^2 = \mathbf{v}^*\mathbf{\Lambda_d}\mathbf{v}, \quad (19)$$

where $[\mathbf{v}]_k \coloneqq v_k$, $[\mathbf{u}]_k \coloneqq u_k$, $[\mathbf{d}]_k \coloneqq |h_k|^2 u_k^2 M_k + N_k$ for all $k \in \mathbb{F}_K$. Also the constraint (18b) can be easily written in the vector form

$$(\mathbf{R} + \jmath\mathbf{Q})\mathbf{\Lambda_u}\mathbf{v} = \mathbf{e}, \quad (20)$$

where $\mathbf{R},\mathbf{Q} \in \mathbb{R}^{L^2\times K}$ are, respectively, the real and imaginary part of the matrix $\mathbf{S} \in \mathbb{C}^{L^2\times K}$ defined as

$$\mathbf{S} \coloneqq \begin{bmatrix}\mathbf{S}_1\\ \vdots \\ \mathbf{S}_L\end{bmatrix}, \; [\mathbf{S}_l]_{ik} \coloneqq h_k g_{kl}[i],\; i,l \in \mathbb{F}_L, k \in \mathbb{F}_K. \quad (21)$$

Also, $\mathbf{e} \in \{0,1\}^{L^2}$ is constructed by stacking columns of $\mathbf{I}$ into one vector. Let $\mathbf{v}_r,\mathbf{v}_q \in \mathbb{R}^K$ be the real and imaginary parts of $\mathbf{v}$, then the optimal fusion strategy is described by the solution of the following optimization problem

$$f(\mathbf{u},\mathbf{v}^\star) = \min_{\mathbf{v}_r,\mathbf{v}_q\in\mathbb{R}^K} \; \mathbf{v}_r'\mathbf{\Lambda_d}\mathbf{v}_r + \mathbf{v}_q'\mathbf{\Lambda_d}\mathbf{v}_q \quad (22a)$$

$$\text{s.t. } \; \mathbf{B}_r\mathbf{v}_r - \mathbf{B}_q\mathbf{v}_q = \mathbf{e}, \quad (22b)$$

$$\mathbf{B}_r\mathbf{v}_q + \mathbf{B}_q\mathbf{v}_r = \mathbf{0}, \quad (22c)$$

where $\mathbf{B}_r \coloneqq \mathbf{R}\mathbf{\Lambda_u}$ and $\mathbf{B}_q \coloneqq \mathbf{Q}\mathbf{\Lambda_u}$. Before finding the solution of this problem, let us define $\mathbf{B}_1,\mathbf{B}_2 \in \mathbb{R}^{L^2\times L^2}$ as

$$\mathbf{B}_1 \coloneqq \mathbf{B}_r\mathbf{\Lambda_d}^{-1}\mathbf{B}_r' + \mathbf{B}_q\mathbf{\Lambda_d}^{-1}\mathbf{B}_q', \quad (23a)$$

$$\mathbf{B}_2 \coloneqq \mathbf{B}_r\mathbf{\Lambda_d}^{-1}\mathbf{B}_q' - \mathbf{B}_q\mathbf{\Lambda_d}^{-1}\mathbf{B}_r'. \quad (23b)$$

Let

$$\begin{aligned}\tilde{L}(\mathbf{v}_r,\mathbf{v}_q;\boldsymbol{\lambda},\boldsymbol{\eta}) \coloneqq& \mathbf{v}_r'\mathbf{\Lambda_d}\mathbf{v}_r + \mathbf{v}_q'\mathbf{\Lambda_d}\mathbf{v}_q - \\ & 2\boldsymbol{\lambda}'(\mathbf{B}_r\mathbf{v}_r - \mathbf{B}_q\mathbf{v}_q - \mathbf{e}) - \\ & 2\boldsymbol{\eta}'(\mathbf{B}_q\mathbf{v}_r + \mathbf{B}_r\mathbf{v}_q),\end{aligned} \quad (24)$$

be the Lagrange dual function of the problem (22), where $\boldsymbol{\lambda}$, $\boldsymbol{\eta}$ are Lagrangian multipliers. By using the KKT conditions, one can easily find the solution of (22). The corresponding solution, given below, is optimal as the problem itself is convex, therefore, the duality gap between primal and dual problem is zero [8]. The optimal value of the problem (22) and the Lagrangian multipliers are given by

$$\mathbf{v}_r^\star = \mathbf{\Lambda_d}^{-1}\big(\mathbf{B}_r'\boldsymbol{\lambda}^\star + \mathbf{B}_q'\boldsymbol{\eta}^\star\big), \quad (25a)$$

$$\mathbf{v}_q^\star = \mathbf{\Lambda_d}^{-1}\big(\mathbf{B}_r'\boldsymbol{\eta}^\star - \mathbf{B}_q'\boldsymbol{\lambda}^\star\big), \quad (25b)$$

$$\boldsymbol{\lambda}^\star = \Big(\mathbf{B}_1 + \mathbf{B}_2\mathbf{B}_1^{-1}\mathbf{B}_2\Big)^{-1}\mathbf{e}, \quad (25c)$$

$$\boldsymbol{\eta}^\star = \mathbf{B}_1^{-1}\mathbf{B}_2\boldsymbol{\lambda}^\star, \quad (25d)$$

$$f(\mathbf{u},\mathbf{v}^\star) = \mathbf{e}'\Big(\mathbf{B}_1 + \mathbf{B}_2\mathbf{B}_1^{-1}\mathbf{B}_2\Big)^{-1}\mathbf{e}. \quad (25e)$$

Note that the fusion rules are independent from the power constraint (16b) and (16c).

V. Power Allocation

In this section we try to find the best power allocation strategy for our sensor network for two different setups, i.e., with and without sum-power constraint. In both cases, an individual power constraint is assumed for each sensor node.

A. Individual Power Constraint

We know that $\mathbf{B}_1$ and $\mathbf{B}_2$ depend on the u_k, or accordingly, on the output power X_k of sensor nodes k, defined by (4). Let us introduce β_k as

$$\beta_k \coloneqq \sqrt{\frac{N_k\Big(M_k + \sum\limits_{l=1}^{L} G_{kl}R_l\Big)}{|h_k|^2 M_k}}, \quad (26)$$

such that $\mathbf{B}1$ and $\mathbf{B}_2$ can be rewritten as

$$\mathbf{B}_1 = \sum_{k=1}^{K}\frac{X_k}{X_k+\beta_k^2}\frac{1}{|h_k|^2 M_k}\big(\mathbf{r}_k\mathbf{r}_k' + \mathbf{q}_k\mathbf{q}_k'\big), \quad (27a)$$

$$\mathbf{B}_2 = \sum_{k=1}^{K}\frac{X_k}{X_k+\beta_k^2}\frac{1}{|h_k|^2 M_k}\big(\mathbf{r}_k\mathbf{q}_k' - \mathbf{q}_k\mathbf{r}_k'\big), \quad (27b)$$

where $\mathbf{r}_k$ and $\mathbf{q}_k$ are k^{th} column of $\mathbf{R}$ and $\mathbf{Q}$. By replacing the objective function of (18) by (25e), and using (4), we achieve the following optimizing problem

$$\min_{\substack{X_k\\ k\in\mathbb{F}_K}} \; \mathbf{e}'\Big(\mathbf{B}_1 + \mathbf{B}_2\mathbf{B}_1^{-1}\mathbf{B}_2\Big)^{-1}\mathbf{e} \quad (28a)$$

$$\text{s.t. } \; 0 \le X_k \le P_k,\; k \in \mathbb{F}_K. \quad (28b)$$

One can easily see that the objective function (28a) is not only non-convex but also very complicated due to multiplication and more importantly, twice matrix inversions. Therefore, finding the optimum is not an easy task. We thus resort to a suboptimal solution as follows.

From (27a) and (27b) it is evidenced that the diagonal entries of $\mathbf{B}_1$ are summation of only positive values (unlike the off-diagonal entries of $\mathbf{B}_1$ and $\mathbf{B}_2$ which are summation of positive and negative values), thus they tend to be much larger than the others. Note that the diagonal entries of $\mathbf{B}_2$ are always zero. Consequently, we assume that at the optimum $\mathbf{B}_1$ has dominant diagonal entries, i.e., $\mathbf{B}_1 = \mathbf{\Lambda_b} + \bar{\mathbf{B}}_1$ where the entries of $\bar{\mathbf{B}}_1$ and also $\mathbf{B}_2$ are negligible in comparison with diagonal entries $\mathbf{b}$ of $\mathbf{B}_1$. Then, using Taylor expansion [9, eq. (191)] we infer

$$\mathbf{B}_1^{-1} \approx \mathbf{\Lambda_b}^{-1} - \mathbf{\Lambda_b}^{-1}\bar{\mathbf{B}}_1\mathbf{\Lambda_b}^{-1} \approx \mathbf{\Lambda_b}^{-1}, \quad (29)$$

$$\mathbf{B}_1 + \mathbf{B}_2\mathbf{B}_1^{-1}\mathbf{B}_2 \approx \mathbf{B}1. \quad (30)$$

Then the objective function (28a) approximates to

$$\sum_{l\in\bar{\mathbb{F}}_L}\frac{1}{b_l}, \quad (31)$$

where $\bar{\mathbb{F}}_L$ is the set of indices such that $e_l = 1,\, l \in \bar{\mathbb{F}}_L$. It is obvious that cardinality of $\bar{\mathbb{F}}_L$ is L. For example in case of 2 targets $\mathbf{e} = [1,0,0,1]'$ and then $\bar{\mathbb{F}}_2 = \{1,4\}$. Therefore, instead of solving (28) we try to solve the relaxed optimization problem as follows

$$\min_{\substack{X_k,\ k\in\mathbb{F}_K \\ b_l,\ l\in\{1,\cdots,L^2\}}} \quad \sum_{l\in\bar{\mathbb{F}}_L} \frac{1}{b_l} \tag{32a}$$

$$\text{s.t.} \quad 0 \le X_k \le P_k,\ k \in \mathbb{F}_K, \tag{32b}$$

$$\operatorname{diag}(\mathbf{B}_1) = \mathbf{b}. \tag{32c}$$

This problem is convex and can be solved easily by relevant numerical tools. We use CVX [10] to solve this problem. In order to compare the solution of the relaxed problem (32) with the original one, we solve the original problem (28) by two numerical solvers of MATLAB, i.e., *fmincon* and *patternsearch*, whose solutions in our case are very close to each other. Even though it cannot be claimed that they achieve the global optimum, but the similarity of solutions gives us the impression of global/near optimality. Nevertheless, providing any proof is very hard. In addition, as we see in the simulation results, the solution of the relaxed problem is very similar to the one of the fmincon and patternsearch. In all three solutions, we observe that each sensor consumes the whole available individual power, i.e., $X_k = P_k$, which is in compliance with the single target case [1].

B. Individual and Sum Power Constraint

It is a common practice to limit the total power consumed in a communication system by imposing a sum-power constraint. This increases lifetime of the network, while the interference in the network is reduced. Therefore, we consider the optimization problem

$$\min_{\substack{X_k \\ k\in\mathbb{F}_K}} \ \mathbf{e}'\Big(\mathbf{B}_1 + \mathbf{B}_2\mathbf{B}_1^{-1}\mathbf{B}_2\Big)^{-1}\mathbf{e} \tag{33a}$$

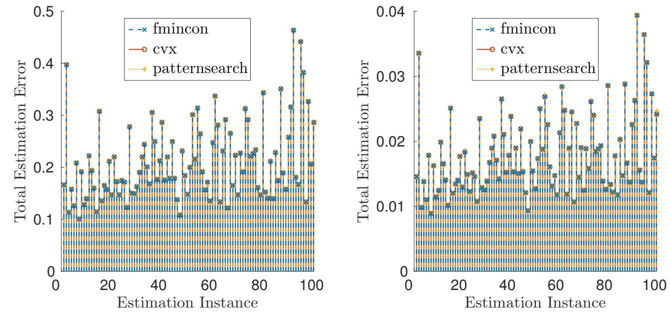
$$\text{s.t.} \quad 0 \le X_k \le P_k,\ k \in \mathbb{F}_K, \tag{33b}$$

$$\sum_{k=1}^{K} X_k \le P_{\text{tot}}. \tag{33c}$$

The solution of this problem is not straightforward, but at this point we suffice to solve the problem by means of numerical methods, i.e., *fmincon* and *patternsearch* and leave any further simplification, relaxation or analytical solution to a later time.

VI. Simulations

In this section, we present simulation results to evaluate the system performance and also to validate our proposed methods. In our simulations we perform the estimation of the targets several times, each time with a different observation due to different realizations of channel and noise. The channel coefficients and noise terms are complex-valued, iid and normal distributed. The power of each target signal is also assumed to be one. The variance of each sensing and communication channel is one, while the variance of noise terms defines the signal-to-noise ration (SNR). More precisely, $\text{SNR} := -20\log\sigma$, where σ^2 is the variance of noise terms m_k and n_k.



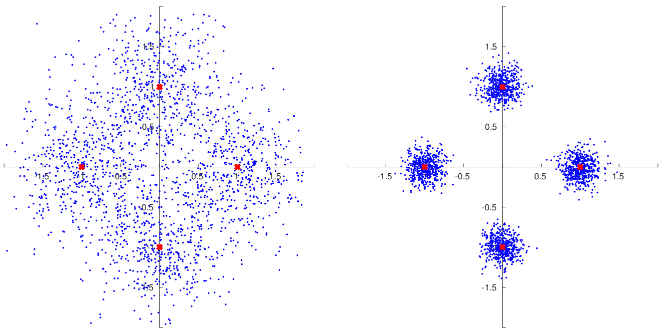
(a) SNR = 0 dB (b) SNR = 10 dB

Fig. 2: Total estimation error of the proposed estimator ($L = 2$, $K = 16$, $P_k = 2$, $P_{\text{tot}} \to \infty$) over different observations. The power allocation is performed by solving (28) using *fmincon*, *patternsearch* and also by solving the relaxed optimization problem (32) using CVX. All tools achieve the same solution except for numerical inaccuracies.

First, we present the simulation result for the case with only individual power constraint. The individual power constraints are chosen such that the total injected power into the system is independent of the number of sensors, so we can have a fair comparison between networks with different number of sensors, i.e., $P_k = \frac{P_0}{K}$ with $P_0 = 32$. Fig. 2 depicts the total estimation error of the proposed power allocation in (28) and also the relaxed proposed method in (32). As we see in the figure their performances are very close to each other. This does not provide any proof of optimality, but rather an evidence that the solvers do not get stuck in local optima. More importantly, it highlights that the proposed relaxation of (32) does not result in a noticeable performance loss.

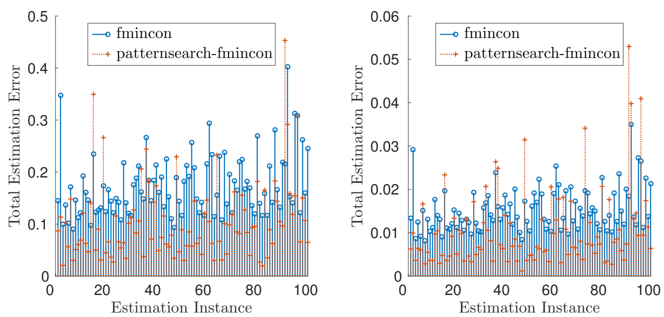
Also, Fig. 3 plots the estimated signal targets over 500 different observations corresponding to different channel, i.e., $g_{kl}[i],\ i \in \{1,\cdots,500\}$ and noise realizations. In the figure the red crosses show the constellation points of 4 different targets. There are several facts regarding the figure which must be considered: the estimates are symmetrically distributed around each constellation point, i.e., within balls centered at each of the 4 points with almost equal radii. The centers of these balls, w.r.t targets, are neither rotated, nor shifted, nor scaled. All these together emphasize the unbiasedness of the proposed estimator. The radii of the balls depend on the noise power, and as we see by increase of the SNR the balls shrink and, thus, total estimation error reduces.

For simulating power allocation with sum-power constraint, i.e., problem (33), we have chosen $P_{\text{tot}} = 32$ and $P_k \to \infty$. It means the performance of the sensor network is not limited by individual power constraint, but by sum-power constraint. As we see in Fig. 4 *fmincon* always outperforms *patternsearch*, since the total estimation error of the latter minus the one of the former is always positive. Also, Fig. 5 depicts the estimation of 2 target signals with *fmincon*. Here, the same arguments of unbiasedness about Fig. 3 are valid.



(a) SNR = 0 dB (b) SNR = 10 dB

Fig. 3: Estimation of target signals with proposed estimator ($L = 4$, $K = 256$, $P_k = 0.125$, $P_{\text{tot}} \to \infty$). Estimation is performed 500 times from different observations (different realizations). Power allocation is done by solving (32) using CVX. The constellation of targets are shown by red, while the estimates are shown by blue dots.

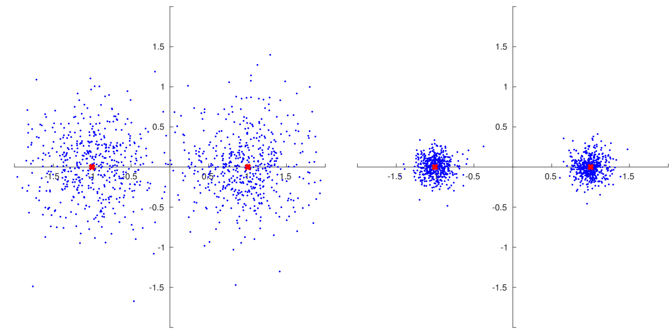


(a) SNR = 0 dB (b) SNR = 10 dB

Fig. 4: Total estimation error of the proposed estimator ($L = 2$, $K = 16$, $P_k \to \infty$, $P_{\text{tot}} = 32$) over different observations. The power allocation is performed by solving (33) using *fmincon* and *patternsearch*.

VII. Conclusion

In this paper we have proposed an unbiased estimator to estimate the values of multiple target signals within a wireless network of sensors. The estimator does not impose any assumption on the distribution of channel or noise. We further have minimized the total estimation error of the estimator by optimizing the fusion rules of the fusion center and also by doing power allocation subject to individual and sum-power constraints. The optimal fusion rule is provided in closed-form, while for doing power allocation we have solved the proposed optimization problem by means of numerical solvers. Moreover, in case of only individual power constraint, we have proposed a low complexity algorithm for power allocation which does not suffer a big loss of performance in comparison with the former solution. The solutions are not yet proved to be optimal, but they seem promising.



(a) SNR = 0 dB (b) SNR = 10 dB

Fig. 5: Estimation of target signals with proposed estimator ($L = 2$, $K = 16$, $P_k \to \infty$, $P_{\text{tot}} = 32$). Estimation is performed 500 times from different observations (different realizations). Power allocation is performed by solving (33) using *fmincon*. The constellation of targets are shown by red and the estimates are shown by blue dots.

Acknowledgment

This work was supported by the German Federal Ministry of Education and Research (BMBF) in the context of the "SwarmGrid" project (grant 03EK3568A).

Also, authors would like to thank MSc. candidate Peter Martin Vieting for his great support with numerical analysis of the current work.

References

[1] G. Alirezaei, M. Reyer, and R. Mathar, "Optimum power allocation in sensor networks for passive radar applications," *IEEE Transactions on Wireless Communications*, vol. 13, no. 6, pp. 3222–3231, Jun. 2014.

[2] M. Cardei, M. T. Thai, Y. Li, and W. Wu, "Energy-efficient target coverage in wireless sensor networks," in *Proceedings IEEE 24th Annual Joint Conference of the IEEE Computer and Communications Societies.*, vol. 3, March 2005, pp. 1976–1984.

[3] W. L. Yeow, C. K. Tham, and W. C. Wong, "Energy efficient multiple target tracking in wireless sensor networks," *IEEE Transactions on Vehicular Technology*, vol. 56, no. 2, pp. 918–928, March 2007.

[4] J. Liu, M. Chu, and J. E. Reich, "Multitarget tracking in distributed sensor networks," *IEEE Signal Processing Magazine*, vol. 24, no. 3, pp. 36–46, May 2007.

[5] J. A. Fuemmeler and V. V. Veeravalli, "Energy efficient multi-object tracking in sensor networks," *IEEE Transactions on Signal Processing*, vol. 58, no. 7, pp. 3742–3750, July 2010.

[6] G. Alirezaei, O. Taghizadeh, and R. Mathar, "Optimum power allocation with sensitivity analysis for passive radar applications," *IEEE Sensors Journal*, vol. 14, no. 11, pp. 3800–3809, Nov. 2014.

[7] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Upper Saddle River, NJ, USA: Prentice-Hall, Inc., 1993.

[8] S. Boyd and L. Vandenberghe, *Convex Optimization*. New York, NY, USA: Cambridge University Press, 2004.

[9] K. B. Petersen and M. S. Pedersen, "The matrix cookbook," Nov. 2012, version 20121115.

[10] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 2.1," http://cvxr.com/cvx, Mar. 2014.