Power Allocation for Multi-Target Multi-Fusion-Rule Sensor Networks

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***Abstract*—In this paper we study a wireless passive sensor network. The sensors are deployed to estimate the true values of multiple active target signals. The sensors forward their observation to a fusion center, which processes the observation of each sensor by a set of fusion rules. One achievement of this paper is proposing an unbiased estimator with minimized variance of errors. To do so, we optimize both the power allocation to the sensor nodes and the fusion rules. The optimal solution to the fusion rules are attained analytically, while the power allocation is solved sub-optimally. Finally, our results are reinforced by numerical simulations.**

***Index Terms*—fusion rule optimization, target detection, classification, minimum variance unbiased estimator, passive radar**

I. Introduction

In this paper, we extend our previous work [1] and consider a wireless sensor network to observe L active targets by the aid of K wireless passive sensors. This means that sensors remain passive without making use of any sensing power and only receive the emitted signals of the targets. The sensors consume power for transmitting their observation towards a fusion center. The data communication to the fusion center is subject to individual output power constraints and a sum-power constraint among all sensors. The fusion center applies a linear fusion rule to combine the received signals and to estimate accurately the real value of the targets. A similar scenario is also considered in another work of ours [2]. The difference is, nevertheless, the number of fusion rules per target. While in the other work, estimation is performed by one scalar fusion coefficient per sensor, here each fusion rule of each sensor is a vector with number entries equal to number of targets. The system of consideration is appropriate for deployment with extreme environment conditions where sensor maintenance is very hard or even impossible. Obviously, in such environments it is crucial to have an a sensor network with increased lifespan. As an interesting example, we can mention the '*IceCube Neutrino Observatory*' at the south pole, where a sensor network with more than 5000 sensors is deployed to observe certain characteristics of sub-atomic particles [3].

The rise of certain applications in the 5$^{\text{th}}$ generation wireless systems (5G) increases drastically the importance of sensor networks for sensing and monitoring the environment. However, an optimal resource utilization is necessary for an accurately performed sensing and monitoring task, since the estimation performance increases with the energy and power consumption of the network. Hence, the optimization of power and energy resources for a required performance is of high interest and studied in many publications, especially for scenarios with a single target. In [1] we have solved in closed-form the power allocation problem subject to individual power limitations of the sensors as well as a given sum-power constraint. In multi-target scenarios the main topics for investigations address the tracking and coverage problem. For example, the focus in [4] is to maximize the lifetime subject to power constraints and coverage regions. In the present work we minimize the estimation error instead of maximizing the lifetime. The authors in [5] use the GaussMarkov mobility model to formulate the tracking problem as a hierarchical Markov decision process and is solved with the aid of neurodynamic programming. In contrast, we exploit an heuristic approach to obtain an accurate suboptimal solution instead of using sophisticated programming methods. Due to difficulties of a centralized processing to handle multi-target problems, the authors in [6] have studied the tracking problem by a distributed data processing approach. In [7] a special scenario is considered in which sensor nodes can be put into a sleep mode with a timer, that determines the sleep duration. By optimizing the sleep duration they show an improvement of the tracking performance in sensor networks. In contrast to [6] and [7], we investigate the centralized scenario and determine the least reliable sensor nodes to keep them asleep for a uniform time duration, respectively. It is to mention, that our approach is more general and it can be used not only for tracking but also for detection and classification of targets, cf. [8].

The organization of this paper is as follows: the system model is described in Sec. II. We propose an unbiased estimator in Sec. III whose variance of error can be further minimized by optimizing the power allocation among the sensor nodes as well as optimizing a set of fusion rules (per sensor node) at the fusion center. The resulting optimization problems are solved in Sec. IV and V. While the simulation results are presented in Sec. VI, Sec. VII concludes this paper.

Notations: In this paper, upper and lower case boldface symbols denote matrices and vectors, respectively. The symbol $\mathbb{N}$ shows the set of natural numbers and the set of all complex (real) matrices of size $k \times n$ is denoted by $\mathbb{C}^{k \times n}$ ($\mathbb{R}^{k \times n}$). The set of all real vectors of size k is also shown by $\mathbb{R}^k$. Trace of a matrix is shown by $\mathrm{tr}(\,\cdot\,)$. While $[\mathbf{x}]_m$ or $x_m$notifies the m^{th} element of vector $\mathbf{x}$, we refer to the entry ij of matrix $\mathbf{A}$

by $[\mathbf{A}]_{ij}$ or a_{ij}. We use $\mathbf{I}_n$ to show the identity matrix of size $n \times n$. Moreover, $(\,\cdot\,)^*$ and $(\,\cdot\,)^T$ are Hermitian and transpose operators, respectively. Note that Hermitian of a scalar is the same as its complex conjugate. Kronecker product is $\otimes$, while $\mathbf{1}_n$ corresponds to all all-one vector of size n. Moreover, $\mathcal{E}(\,\cdot\,)$ is the expected value, while the Kronecker delta function is shown by

$$\delta_{lm} = \begin{cases} 1\,, & l = m\,, \\ 0\,, & l \neq m. \end{cases}$$

Also, the operator $\mathrm{vec}(\mathbf{A})$ stacks all the columns of the matrix $\mathbf{A}$ into one long vector. While $\mathrm{diag}(\mathbf{A})$ refers to a vector consisting of diagonal entries of $\mathbf{A}$, symbol $\mathbf{\Lambda_x}$ represents a diagonal matrix whose diagonal entries are the elements of vector $\mathbf{x}$. Finally, $|\cdot|$ and $\|\cdot\|$ denote absolute value and Euclidean norm, respectively. Finally, $\mathcal{O}$ stands for big O notation.

II. System Model

We consider a wireless sensor network which consists of $K \in \mathbb{N}$ passive sensor nodes in order to estimate multiple target signals. The block diagram of such a system is shown in Fig. 1. We assume that there are $L \in \mathbb{N}$ targets, i.e., $r_1, \cdots, r_L$, whose true values are complex and unknown. The index sets $\mathbb{F}_K := \{1, \cdots, K\}$ and $\mathbb{F}_L := \{1, \cdots, L\}$ correspond to sensors and targets, respectively. The power of each target is assumed to be known, i.e., $R_l := \mathcal{E}(|r_l|^2), l \in \mathbb{F}_L$ and, furthermore, the targets change slowly. Thus, they are constant over one round of estimation.

The target signal r_l is observed at sensor node k upon multiplying by sensing channel coefficient $g_{kl} \in \mathbb{C}$ and also summing up with measurement noise $m_k \in \mathbb{C}$. The sensing channel is assumed to be nearly constant over one round of estimation, so one can consider it as a time-invariant deterministic value. The measurement noise is further assumed to be zero-mean, identically and independently distributed (iid) with variance of M_k, which is also independent from the target signals. So, it is correct to state

$$\mathcal{E}(m_k m_{k\prime}^*) = \delta_{kk\prime} M_k, \; \forall k, k\prime, \tag{1a}$$
$$\mathcal{E}(r_l m_k^*) = \mathcal{E}(r_l)\, \mathcal{E}(m_k^*) = 0, \; \forall k, l. \tag{1b}$$

Each sensor accordingly amplifies its received signal by the real coefficient $u_k, \; k \in \mathbb{F}_K$ and transmits it towards the fusion center. The output of sensor k, i.e., x_k is represented by

$$x_k = u_k \Big(m_k + \sum_{l=1}^{L} g_{kl} r_l \Big), \; k \in \mathbb{F}_K. \tag{2}$$

The output power of sensor K is derived below

$$X_k := \mathcal{E}\Big(|x_k|^2 \Big) = \Big(M_k + \sum_{l=1}^{L} |g_{kl}|^2 R_l \Big) u_k^2, \tag{3}$$

which is limited due to the physical constraint of the sensors,

$$X_k \leq P_k, \; k \in \mathbb{F}_K. \tag{4}$$

The transmitted signal from each sensor propagates through the communication channel and arrives at the fusion center. We denote this signal by y_k which can be derived by:

$$y_k := h_k x_k + n_k = h_k u_k \Big(m_k + \sum_{l=1}^{L} g_{kl} r_l \Big) + n_k, \tag{5}$$

where h_k is the communication channel coefficient between the sensor node k and the fusion center. Similarly, the communication channel h_k is almost constant during the interval of estimation, and thus deterministic and time-invariant. Also, n_k represents the additive noise at the fusion center antenna, which is assumed to be zero-mean and iid with variance N_k. Therefore, we can write

$$\mathcal{E}(n_k n_{k\prime}^*) = \delta_{kk\prime} N_k, \; \forall k, k\prime, \tag{6a}$$
$$\mathcal{E}(m_k n_{k\prime}^*) = \mathcal{E}(m_k)\, \mathcal{E}(n_{k\prime}^*) = 0, \; \forall k, k\prime. \tag{6b}$$

Let

$$\mathbf{r} = \big[r_1, \cdots, r_L \big]', \tag{7a}$$
$$\mathbf{x} = [x_1, \cdots, x_K]', \tag{7b}$$
$$\mathbf{m} = \big[m_1, \cdots, m_K \big]', \tag{7c}$$
$$\mathbf{y} = [y_1, \cdots, y_K]', \tag{7d}$$
$$\mathbf{u} = [u_1, \cdots, u_K]', \tag{7e}$$

then (2) can be recast into the vector form

$$\mathbf{x} = \mathbf{\Lambda_u} (\mathbf{G}\mathbf{r} + \mathbf{m}), \tag{8}$$

where $[\mathbf{G}]_{kl} = g_{kl}$ corresponds to entries of the sensing (channel) matrix $\mathbf{G}$ while $\mathbf{\Lambda_u} = \mathrm{diag}(\mathbf{u})$. Also, by using (8) and defining $\mathbf{\Lambda_h} = \mathrm{diag}(\mathbf{h})$, we can rewrite (5) into the vector form

$$\mathbf{y} = \mathbf{\Lambda_h} \mathbf{x} + \mathbf{n} = \mathbf{\Lambda_h} \mathbf{\Lambda_u} (\mathbf{G}\mathbf{r} + \mathbf{m}) + \mathbf{n}. \tag{9}$$

The fusion center then multiplies its input with the so-called fusion matrix $\mathbf{V} \in \mathbb{C}^{L \times K}$ which leads to the following observation vector

$$\tilde{\mathbf{r}} = \mathbf{V}\mathbf{y} = \mathbf{H}\mathbf{r} + \mathbf{w}, \tag{10}$$

where $\mathbf{H} := \mathbf{V}\mathbf{\Lambda_h}\mathbf{\Lambda_u}\mathbf{G}$ is the effective observation channel and $\mathbf{w} := \mathbf{V}\mathbf{\Lambda_h}\mathbf{\Lambda_u}\mathbf{m} + \mathbf{V}\mathbf{n}$ is the effective observation noise.

We further assume that channel state information is available at the fusion center. In order to increase the life time of our sensor network and also to manage the interference, we assume the existence of a sum power constraint, stated below

$$\sum_{k=1}^{K} X_k = \sum_{k=1}^{K} \Big(M_k + \sum_{l=1}^{L} |g_{kl}|^2 R_l \Big) u_k^2 \leq P_{\text{tot}}. \tag{11}$$

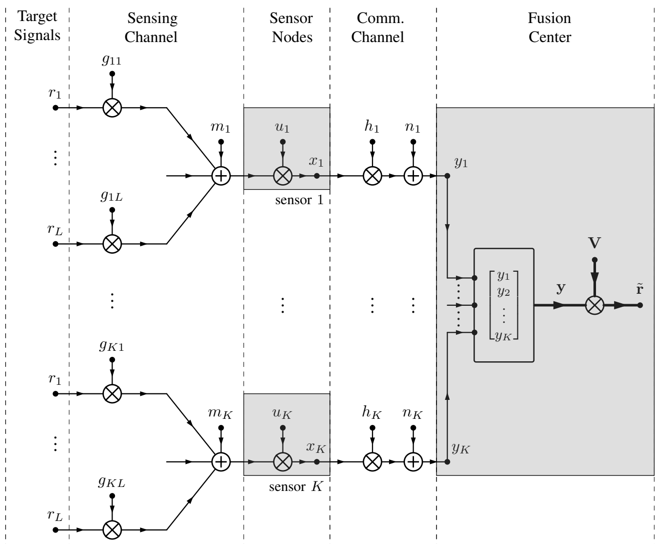


Fig. 1: Block diagram of the multi-target wireless sensor network with multiple fusion coefficients per sensor node

III. Proposed Estimator

It is only possible to find an efficient estimator, which is unbiased and further attains the Cramer-Rao lower bound, when the probability distribution function of the observation is known [9]. This does not apply to our problem, since we know only the first and the second moments of the observation. In this case even finding a minimum variance unbiased estimator (MVUE) is impossible. Under such circumstances, it is ideal to come up with the best linear unbiased estimator (BLUE), $\hat{\mathbf{r}}$, which delivers $\mathcal{E}(\hat{\mathbf{r}}-\mathbf{r}) = \mathbf{0}$ and minimizes the error variance. It is known from the *Gauss-Markov theorem* that the BLUE of the linear observation (10) is given by

$$\begin{aligned}\hat{\mathbf{r}} = \boldsymbol{\Sigma}\tilde{\mathbf{r}} &= \left(\mathbf{H}^*\mathbf{C}^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^*\mathbf{C}^{-1}\tilde{\mathbf{r}} \\ &= \mathbf{r} + \left(\mathbf{H}^*\mathbf{C}^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^*\mathbf{C}^{-1}\mathbf{w}\,, \end{aligned} \tag{12}$$

where $\mathbf{C} := \mathcal{E}(\mathbf{w}\mathbf{w}^*)$ is the covariance matrix of the effective noise, i.e.,

$$[\mathbf{C}]_{ij} = \sum_{k=1}^{K}(|h_k|^2 u_k^2 M_k + N_k) v_{ik} v_{jk}^*\,. \tag{13}$$

This results in the total estimation error

$$\operatorname{tr}\Big(\big(\mathbf{H}^*\mathbf{C}^{-1}\mathbf{H}\big)^{-1}\Big). \tag{14}$$

It is, thus, reasonable to further reduce the estimation error by solving the following optimization problem

$$\min_{\mathbf{V},\mathbf{u}} \ \operatorname{tr}\Big(\big(\mathbf{H}^*\mathbf{C}^{-1}\mathbf{H}\big)^{-1}\Big) \tag{15a}$$

$$\text{s.t.} \ \left(M_k + \sum_{l=1}^{L}|g_{kl}|^2 R_l\right)u_k^2 \leq P_k,\ k \in \mathbb{F}_K, \tag{15b}$$

$$\sum_{k=1}^{K}\left(M_k + \sum_{l=1}^{L}|g_{kl}|^2 R_l\right)u_k^2 \leq P_{\text{tot}}. \tag{15c}$$

Unfortunately, the problem (15) is very hard to solve, because of its very complicated objective function in (15a). We alternatively resort to an easier, but more tractable estimator. Comparing the observation (10) with the optimal estimator (12) reveals that forcing $\mathbf{H}$ to $\mathbf{I}$ by optimizing $\mathbf{V}$ will result in an unbiased observation. In turn, the objective (14) approaches $\operatorname{tr}(\mathbf{C})$ which can further be minimized by the aid of $\mathbf{u}$. Moreover, by replacing $\mathbf{H} = \mathbf{V}\boldsymbol{\Lambda}_{\mathbf{h}}\boldsymbol{\Lambda}_{\mathbf{u}}\mathbf{G} = \mathbf{I}$ into (10) and using (13) it is easy to show that

$$f(\mathbf{u},\mathbf{V}) =: \sum_{k=1}^{K}\left((|h_k|^2 u_k^2 M_k + N_k)\cdot\sum_{l=1}^{L}|v_{lk}|^2\right), \tag{16}$$

or equivalently,

$$f(\mathbf{u},\mathbf{V}) = \operatorname{tr}(\mathbf{V}\boldsymbol{\Lambda}_{\mathbf{d}}\mathbf{V}^*), \tag{17}$$

where $[\mathbf{d}]_k := |h_k|^2 u_k^2 M_k + N_k$ for all $k \in \mathbb{F}_K$. Therefore, by solving the following proposed optimization problem, we first make the observation unbiased and second minimize the variance of the error:

$$\min_{\mathbf{u},\mathbf{V}} \ \operatorname{tr}(\mathbf{V}\boldsymbol{\Lambda}_{\mathbf{d}}\mathbf{V}^*) \tag{18a}$$

$$\text{s.t.} \ \mathbf{V}\boldsymbol{\Lambda}_{\mathbf{h}}\boldsymbol{\Lambda}_{\mathbf{u}}\mathbf{G} = \mathbf{I}, \tag{18b}$$

as well as (15b) and (15c). The unbiasedness is provided by the constraint in (18b). Note that $K \geq L$ is required by the feasibility of the problem (18). This simply means that at least L sensors must be active for an unbiased estimation in order to detect L targets. In summary, the feasibility of the system under consideration, imposes some requirements which are summarized in the Table I.

TABLE I: Facts and figures of the proposed estimator

Number of channel estimations per target estimation	$(L+1)K$
Minimum number of sensors	L
Number of system variables	$(L+1)K$
Minimum number of system variables	$(L+1)L$

IV. Optimizing Fusion Rule

It is worthwhile mentioning that power constraints (15b) and (15c) are independent from fusion rules, so the optimization of fusion rules are not constrained by power consumption of the sensors. The objective function in (18) can be rewritten as

$$f(\mathbf{u},\boldsymbol{v}) = \boldsymbol{v}^*(\boldsymbol{\Lambda}_{\mathbf{d}} \otimes \mathbf{I}_L)\boldsymbol{v}, \tag{19}$$

where $\boldsymbol{v} := \operatorname{vec}(\mathbf{V})$. Also using the identity

$$\operatorname{vec}(\mathbf{V}\boldsymbol{\Lambda}_{\mathbf{h}}\boldsymbol{\Lambda}_{\mathbf{u}}\mathbf{G}) = (\mathbf{G}'\boldsymbol{\Lambda}_{\mathbf{h}}\boldsymbol{\Lambda}_{\mathbf{u}} \otimes \mathbf{I}_L)\boldsymbol{v} \tag{20}$$

the constraint (18b) can be easily written in a vector form

$$\mathbf{B}\boldsymbol{v} = \big((\mathbf{R} + \jmath\mathbf{Q})\boldsymbol{\Lambda}_{\mathbf{u}} \otimes \mathbf{I}_L\big)\boldsymbol{v} = \mathbf{e}, \tag{21}$$

where $\mathbf{R},\mathbf{Q} \in \mathbb{R}^{L\times K}$ are the real and imaginary part of the matrix $\mathbf{G}'\boldsymbol{\Lambda}_{\mathbf{h}}$ and $\mathbf{e} = \operatorname{vec}(\mathbf{I}_L)$. Let us denote real and imaginary parts of $\boldsymbol{v}$ by $\boldsymbol{v}_r, \boldsymbol{v}_q \in \mathbb{R}^{LK}$, then the $\boldsymbol{v}^\star$ is the solution of the optimization problem

$$\min_{\boldsymbol{v}_r, \boldsymbol{v}_q \in \mathbb{R}^K} \quad \boldsymbol{v}_r'(\boldsymbol{\Lambda}_{\mathbf{d}} \otimes \mathbf{I}_L)\boldsymbol{v}_r + \boldsymbol{v}_q'(\boldsymbol{\Lambda}_{\mathbf{d}} \otimes \mathbf{I}_L)\boldsymbol{v}_q \tag{22a}$$

$$\text{s.t.} \quad (\mathbf{B}_r \otimes \mathbf{I}_L)\boldsymbol{v}_r - (\mathbf{B}_q \otimes \mathbf{I}_L)\boldsymbol{v}_q = \mathbf{e}, \tag{22b}$$

$$(\mathbf{B}_r \otimes \mathbf{I}_L)\boldsymbol{v}_q + (\mathbf{B}_q \otimes \mathbf{I}_L)\boldsymbol{v}_r = \mathbf{0}, \tag{22c}$$

where $\mathbf{B}_r \coloneqq \mathbf{R}\boldsymbol{\Lambda}_{\mathbf{u}}$ and $\mathbf{B}_q \coloneqq \mathbf{Q}\boldsymbol{\Lambda}_{\mathbf{u}}$. Let $\mathbf{B}_1, \mathbf{B}_2 \in \mathbb{R}^{L \times L}$ be defined by

$$\mathbf{B}_1 \coloneqq \mathbf{B}_r\boldsymbol{\Lambda}_{\mathbf{d}}^{-1}\mathbf{B}_r' + \mathbf{B}_q\boldsymbol{\Lambda}_{\mathbf{d}}^{-1}\mathbf{B}_q', \tag{23a}$$

$$\mathbf{B}_2 \coloneqq \mathbf{B}_r\boldsymbol{\Lambda}_{\mathbf{d}}^{-1}\mathbf{B}_q' - \mathbf{B}_q\boldsymbol{\Lambda}_{\mathbf{d}}^{-1}\mathbf{B}_r'. \tag{23b}$$

Similar to [2], the optimal value of the problem (22) is

$$\boldsymbol{v}_r^\star = \left(\boldsymbol{\Lambda}_{\mathbf{d}}^{-1}\mathbf{B}_r' \otimes \mathbf{I}_L\right)\boldsymbol{\lambda}^\star + \left(\boldsymbol{\Lambda}_{\mathbf{d}}^{-1}\mathbf{B}_q' \otimes \mathbf{I}_L\right)\boldsymbol{\eta}^\star, \tag{24a}$$

$$\boldsymbol{v}_q^\star = \left(\boldsymbol{\Lambda}_{\mathbf{d}}^{-1}\mathbf{B}_r' \otimes \mathbf{I}_L\right)\boldsymbol{\eta}^\star - \left(\boldsymbol{\Lambda}_{\mathbf{d}}^{-1}\mathbf{B}_q' \otimes \mathbf{I}_L\right)\boldsymbol{\lambda}^\star, \tag{24b}$$

$$\boldsymbol{\lambda}^\star = \left(\left(\mathbf{B}_1 + \mathbf{B}_2\mathbf{B}_1^{-1}\mathbf{B}_2\right)^{-1} \otimes \mathbf{I}_L\right)\mathbf{e}, \tag{24c}$$

$$\boldsymbol{\eta}^\star = \left(\mathbf{B}_1^{-1}\mathbf{B}_2 \otimes \mathbf{I}_L\right)\boldsymbol{\lambda}^\star, \tag{24d}$$

$$f(\mathbf{u}, \boldsymbol{v}^\star) = \mathbf{e}'\left(\left(\mathbf{B}_1 + \mathbf{B}_2\mathbf{B}_1^{-1}\mathbf{B}_2\right)^{-1} \otimes \mathbf{I}_L\right)\mathbf{e}, \tag{24e}$$

where $\boldsymbol{\lambda}$ and $\boldsymbol{\eta}$ are the corresponding Lagrangian multipliers.

V. Power Allocation

At this point our is goal is performing power allocation in an optimized fashion for two different cases, one with and one without sum-power constraint, where both cases consider an individual power constraint.

A. Individual Power Constraint

As $\mathbf{B}_1$ and $\mathbf{B}_2$ depend on u_k, then, they depend on output power X_k of the sensor nodes k given in (3). Then, $\mathbf{B}_1$ and $\mathbf{B}_2$ can be rewritten as

$$\mathbf{B}_1 = \sum_{k=1}^{K} \frac{X_k}{X_k + \beta_k^2} \frac{1}{|h_k|^2 M_k} \left(\mathbf{r}_k\mathbf{r}_k' + \mathbf{q}_k\mathbf{q}_k'\right), \tag{25a}$$

$$\mathbf{B}_2 = \sum_{k=1}^{K} \frac{X_k}{X_k + \beta_k^2} \frac{1}{|h_k|^2 M_k} \left(\mathbf{r}_k\mathbf{q}_k' - \mathbf{q}_k\mathbf{r}_k'\right), \tag{25b}$$

where $\mathbf{r}_k$ and $\mathbf{q}_k$ are k^{th} column of $\mathbf{R}$ and $\mathbf{Q}$ and

$$\beta_k \coloneqq \sqrt{\frac{N_k\left(M_k + \sum_{l=1}^{L} G_{kl}R_l\right)}{|h_k|^2 M_k}}. \tag{26}$$

By replacing the objective function of (18) by (24e), and using (3), we achieve the following optimizing problem

$$\min_{\substack{X_k \\ k \in \mathbb{F}_K}} \quad \mathbf{e}'\left(\left(\mathbf{B}_1 + \mathbf{B}_2\mathbf{B}_1^{-1}\mathbf{B}_2\right)^{-1} \otimes \mathbf{I}_L\right)\mathbf{e} \tag{27a}$$

$$\text{s.t.} \quad 0 \le X_k \le P_k, \ k \in \mathbb{F}_K. \tag{27b}$$

Due to difficulty of the objective function (27a) finding the optimum is not straightforward. Therefore, we try to find a suboptimal solution. Similar to what we have done in [2], it is easy to see that the objective function (27a) can be well approximated by

$$\sum_{l=1}^{L} \frac{1}{b_l}, \tag{28}$$

which leads us to the relaxed optimization problem

$$\min_{\substack{X_k, \ k \in \mathbb{F}_K \\ b_l, \ l \in \mathbb{F}_L}} \quad \sum_{l=1}^{L} \frac{1}{b_l} \tag{29a}$$

$$\text{s.t.} \quad 0 \le X_k \le P_k, \ k \in \mathbb{F}_K, \tag{29b}$$

$$\operatorname{diag}(\mathbf{B}_1) = \mathbf{b}. \tag{29c}$$

This problem is convex and can be solved easily by relevant numerical tools. We use CVX [10] to solve this problem. In order to compare the solution of the relaxed problem with the original one, we solve the original problem (27) by two numerical solvers of MATLAB, i.e., *fmincon* and *patternsearch*, whose solutions in our case are very close to each other. Even though it cannot be claimed that they achieve the global optimum, but the similarity of solutions gives us the impression of global/near optimality. Nevertheless, providing any proof is very hard. In addition, as wee see in the simulation results, the solution of the relaxed problem is very similar to the one of the fmincon and patternsearch. In all three solutions, we observe that each sensor consumes the whole available individual power, i.e., $X_k = P_k$, which is in compliance with the single target case [1] and also with [2].

B. Individual and Sum Power Constraint

It is known that sum-power constraint help a network to increase its lifespan and also to minimize the overall interference. Having such constraint incorporated into our problem, we achieve

$$\min_{\substack{X_k \\ k \in \mathbb{F}_K}} \quad \mathbf{e}'\left(\left(\mathbf{B}_1 + \mathbf{B}_2\mathbf{B}_1^{-1}\mathbf{B}_2\right)^{-1} \otimes \mathbf{I}_L\right)\mathbf{e} \tag{30a}$$

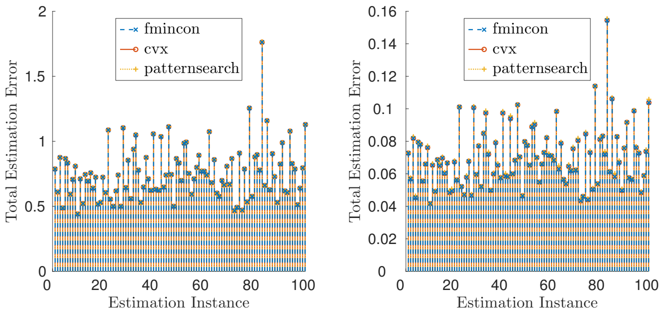
$$\text{s.t.} \quad 0 \le X_k \le P_k, \ k \in \mathbb{F}_K, \tag{30b}$$

$$\sum_{k=1}^{K} X_k \le P_{\text{tot}}. \tag{30c}$$

The solution of this problem is not straightforward, but at this point we suffice to solve the problem by means of numerical methods, i.e., *fmincon* and *patternsearch* and leave any further simplification, relaxation or analytical solution to a later time.

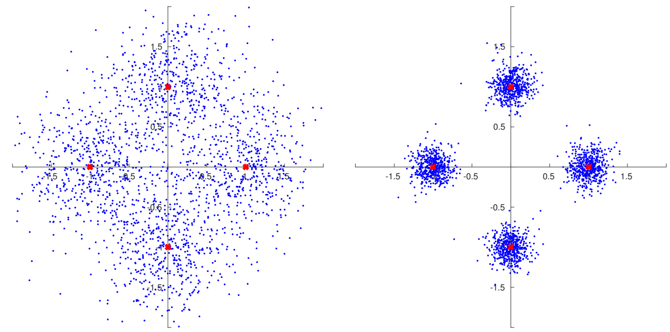
VI. Simulations

At this point, simulation results are presented to evaluate the performance of proposed estimators. In our simulations we perform the estimation of the targets several times, each time with a different observation due to different realizations of channel and noise. The channel coefficients and noise terms are complex-valued, iid with Gaussian distribution. We define the signal-to-noise ration (SNR) by $-20\log\sigma$, where σ^2 is the variance of the noise terms. The variances of both sensing and



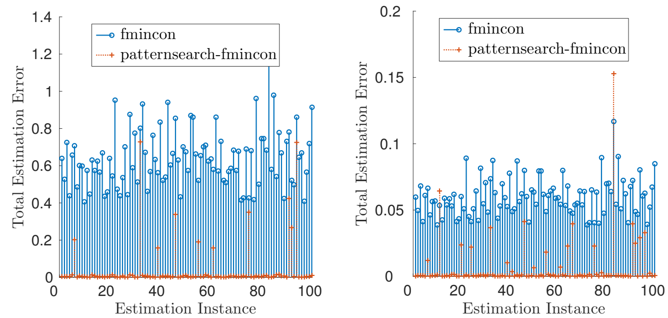
(a) SNR = 0 dB (b) SNR = 10 dB

Fig. 2: Total estimation error of the proposed estimator ($L = 4$ $K = 16$, $P_k = 2$, $P_{\text{tot}} \to \infty$) over different observations. The power allocation is performed by solving (27) using *fmincon*, *patternsearch* and also by solving the relaxed optimization problem (29) using CVX. All three solutions are identical up to a numerical precision.



(a) SNR = 0 dB (b) SNR = 10 dB

Fig. 3: Estimation of target signals with proposed estimator ($L = 4$ $K = 16$, $P_k = 2$, $P_{\text{tot}} \to \infty$). Estimation is performed 500 times from different observations (different realizations). Power allocation is done by solving (29) using CVX. The constellation of targets are shown by red, while the estimates are shown by blue dots.



(a) SNR = 0 dB (b) SNR = 10 dB

Fig. 4: Total estimation error of the proposed estimator ($L = 4$, $K = 16$, $P_k \to \infty$, $P_{\text{tot}} = 32$) over different observations. The power allocation is performed by solving (30) using *fmincon* and *patternsearch*.

communication channels are set to one. Also, the power of all targets are chosen to be one along the simulations.

First of all, we show the simulations for the case with individual power constraints, i.e., $P_k = \frac{32}{K}$, and without sum-power constraint, i.e., $P_{\text{tot}} \to \infty$. The individual power constraints are chosen such that the total available power is independent of the number of sensors, so one can fairly examine the influence caused by varying the number of sensors.

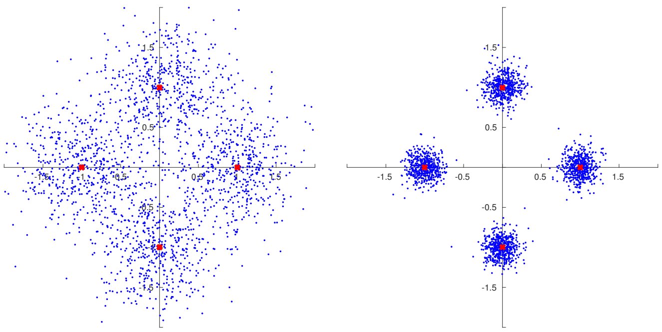
Fig. 2 depicts the optimized total estimation error of the proposed power allocation in (27) and also the relaxed proposed method in (29). As we see in the figure the performances of all estimators are very close to one another. It does not prove optimality obviously, though provides an insight that the solvers do not get stuck in local optima. Moreover, it sheds light on the fact that the relaxation in (29) barely changes the performance in one hand, and reduces the complexity on the other hand.

In Fig. 3 the estimations (blue) of signal targets (red) are plotted, where each estimation is based on a different observation related to a different channel and noise realization, i.e., $g_{kl}[i], m_k[i], n_k[i],\ i \in \{1, \cdots, 500\}$. It needs to be mentioned that the estimated points are symmetrically distributed around each constellation point, i.e., within equally spread balls co-centered with constellation points. Their centers, with respect to constellation points, are neither rotated, nor shifted, nor scaled. All the aforementioned facts are direct results of unbiasedness of the proposed estimator. The SNR variations surely change the radii of the balls, i.e., in higher SNR the total estimation error is less and equivalently the balls are smaller.

It is also interesting to observe the simulation results when power allocation is performed subject to sum-power constraint, i.e., problem (30), with $P_{\text{tot}} = 32$ and $P_k \to \infty$. The total estimation error of the given scenario is depicted in Fig. 4 where *fmincon* and *patternsearch* are deployed for doing the optimization. As we can easily see the total estimation error of the second solver subtracted by the one of the first solver is positive for all observations, therefore we conclude *patternsearch* is outperformed by *fmincon* whose estimated points are shown by blue dots in Fig. 5. One can bring up the same arguments of unbiasedness that we just did for Fig. 3.

VII. Conclusion

In this paper we have considered a sensor network for estimating multiple targets and have proposed a estimator which is unbiased without enforcing any assumptions on the channel and noise distributions. We have additionally improved the performance of the estimator by optimizing the power allocation and fusion strategy which in turn result in minimized total estimation error. As per power allocation we have considered two different types of constraints, i.e., individual and sum-power constraints. A closed-form fusion strategy is one of our contributions along with numerical solutions for power allocation. Additionally, in case of only individual power constraint, we have proposed a low complexity algorithm based on convex relation which reveals a performance negligibly different from the not-relaxed problem, yet much less complex.



(a) SNR = 0 dB (b) SNR = 10 dB

Fig. 5: Estimation of target signals with proposed estimator ($L = 4$, $K = 16$, $P_k \to \infty$, $P_{\text{tot}} = 32$). Estimation is performed 500 times from different observations (different realizations). Power allocation is performed by solving (30) using *fmincon*. The constellation of targets are shown by red and estimates by blue.

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