Integrated Network Design for Measurement and Communication Infrastructures in Smart Grids

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***Abstract*—A large-scale roll-out of a communication and measurement infrastructure is an essential prerequisite for more efficient and robust power grids with a high number of renewable energy resources. In this work, we propose an integrated optimization model for the minimum cost design of a wide area measurement system in smart power grids. The planning approaches proposed so far in the literature mostly consider the optimal placement of measurement devices and the design of a communication network independently, and assume the existence of only one communication technology. In contrast, our proposed novel model enables an integrated planning with a minimum number of both data concentrator and measurement units for observability of the whole power system, and a hierarchical heterogeneous communication network design under data communication requirements of delay and capacity. The application of the proposed model on test networks validates the reduction of the deployment and operational costs as a result of the integrated modeling.**

I. Introduction

In the developed countries, the power grids are undergoing a fundamental change triggered by the ambitious targets of national and international institutions to increase the share of renewable energy resources and to decrease the CO_2 emission. For example, the energy laws in Germany set the target for renewable resources to account for 80% of the total electricity generation by 2050, with intermediate targets of 40-45% by 2025 and 55-60% by 2035 [1]. The achievement of these targets requires a roll-out and extension of measurement and communication infrastructures in all voltage levels in order to ensure a stable network operation under the variations of volatile renewable energy resources [2]. Particularly, the roll-out of a wide area measurement system (WAMS) in low-voltage (LV) and medium-voltage (MV) distribution networks is one of the significant challenges towards a more decentralized, widely monitored and controlled distribution network, since the decentralized renewable energy producers are envisioned to feed their produced power in these networks. In this context, the deployment of a WAMS is associated with tremendous costs due to the massive size of distribution networks. To illustrate an example, electrical lines in LV and MV networks in Germany have a link length of about 1.61 million km, and constitute 92% and 99.9% of all national power grid in terms of link length and number of network nodes, respectively [3]. For these reasons, the optimal design of a WAMS is a challenge of crucial significance not only from the technical but also from the economical point of view.

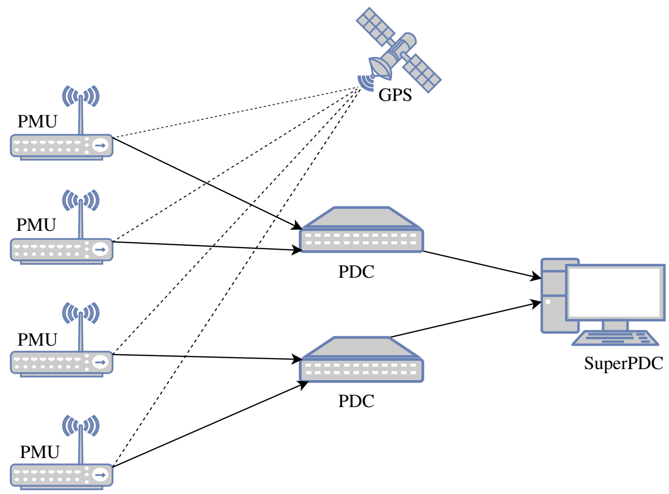

Fig. 1. Hierarchical network architecture of WAMS. PMUs send the phasor measurements, timestamped by the GPS signal, to a SuperPDC over intermediate PDCs.

A WAMS consists of *i)* many measurement devices, called phasor measurement units (PMU), which measure the voltage and current phasor values available at the system nodes where they are installed, *ii)* several data concentrator units, called phasor data concentrators (PDC), and *iii)* a data processing center, called SuperPDC (SPDC). *IEEE Standard for Synchrophasor Data Transfer for Power Systems* [4] lays down the architecture for the communication network in a WAMS as shown in Figure 1. This architecture postulates a hierarchical transmission of sensor data from PMUs to PDCs, where a preprocessing of the data takes place such as time alignment and consistency check. PDCs send the data to a larger central unit SPDC, where the measurement data from a larger part of the network are aggregated to execute energy management functions such as state estimation, cf. [5].

The design of a WAMS involves the determination of both the optimum number and locations of PMUs and PDCs in addition to the optimum design of the communication network, where the requirements of both power and communication networks must be taken into account. Until recently, the main concern of the network planners in power system research has been to find the minimum number of PMUs and their locations for observability under different redundancy requirements, where the communication network costs and requirements were not taken into account, cf. [6]. The main reason for this simplification was the necessity of monitoring only in a limited part of the power grid, where the communication costs played a minor role. Bearing in mind the above mentioned developments in renewable integration, it has become clear that the communication network design is significant for both the system performance and the reduction of investment costs. For example, the analysis in [7] shows that the use of power line communication (PLC) technology can reduce the number of high-bandwidth links by 80%. Thus, several researchers have attempted to propose methods for an optimal design of a WAMS including the design of a communication network. In [8], an evolutionary optimization approach is introduced which ensures the observability of the power system while establishing a connected communication network between installed PMUs. Similarly, [9] suggests a communication network design which minimizes the total length of communication links based on the predetermined PMU locations. These studies, however, do not consider the hierarchical and scalable structure of WAMS. In addition, only fiber links are considered in the planning. The authors of [10] approach the hierarchical design problem by placing a PDC at the system bus which would minimize the communication link distances with the help of a shortest path algorithm, and then finding the optimum locations for PMUs. As in other mentioned studies, the optimization is based on the shortest paths in the power network, which makes a consideration of multiple communication technologies with specific constraints impossible. In another recent work, Wen *et al.* [11] proposes an integer linear program formulation of PDC placement problem with the objective of minimizing the total WAMS traffic when locations of PMUs are known.

As this review shows, the planning approaches proposed so far are not flexible enough to enable an optimal planning in consideration of a heterogenous communication network. However, distribution network operators can lower the deployment and operational costs by benefiting from the distinctive advantages of available communication technologies, such as PLC, currently available low-cost wireless technologies, and in the future, the ones which are developed for low-delay, massive machine-to-machine communications [7], [12]. Furthermore, all of the mentioned studies treat the placement of PDCs and the placement of PMUs in two stages, which prevents an integrated optimal network planning. A planning approach with a simultaneous optimization of PMU and PDC locations is introduced in [13], in which the communication network constraints are integrated only from the perspective of reliability without any other considerations. Similarly, [14] proposes an integer programming model for simultaneous PMU-PDC placement. This model, however, does not cover multiple technologies and any data communication requirements such as latency and capacity. Furthermore, the model is non-linear, which massively increases the computational complexity to find an optimal solution.

In our previous work [15], we have introduced an optimization approach for an integrated design of WAMS with an heterogeneous communication network. This work, however,

TABLE I
SYMBOL NOTATION, SET DEFINITIONS AND INPUT PARAMETERS

Symbol, Domain	Description
$\mathcal{B}$	the binary set
$\mathcal{Z}_+$, $\mathcal{R}_+$	sets of nonnegative integer and real numbers
$\mathcal{V}_{\text{pow}}, n_{\text{bus}} \in \mathcal{Z}_+$	set of power system nodes and its cardinality
$\mathcal{E}_{\text{pow}}, n_{\text{branch}} \in \mathcal{Z}_+$	set of power system branches and its cardinality
$G_{\text{pow}}(\mathcal{V}_{\text{pow}}, \mathcal{E}_{\text{pow}})$	graph of the power system
$\mathcal{V}_{\text{ext}}, n_{\text{ext}} \in \mathcal{Z}_+$	set of communication nodes and its cardinality
$\mathcal{E}_{\text{ext}}, n_{\text{link}} \in \mathcal{Z}_+$	set of communication links and its cardinality
$G_{\text{ext}}(\mathcal{V}_{\text{ext}}, \mathcal{E}_{\text{ext}})$	extended graph of the communication network
$\mathcal{P}_{\text{PMU}}, n_{\text{bus}} \in \mathcal{Z}_+$	set of possible PMU locations and its cardinality
$\mathcal{P}_{\text{PDC}}, n_{\text{p}} \in \mathcal{Z}_+$	set of possible PDC locations and its cardinality
$\mathcal{T}, n_t \in \mathcal{Z}_+$	set of available technologies and its cardinality
$\boldsymbol{A} = \mathcal{B}^{n_{\text{bus}} \times n_{\text{bus}}}$	connectivity matrix of the power system
$\boldsymbol{S} \in \mathcal{B}^{n_{\text{ext}} \times n_{\text{ext}}}$	link availability matrix
$\boldsymbol{T} \in \mathcal{B}^{n_{\text{ext}} \times n_{\text{ext}} \times n_t}$	technology availability array
$\boldsymbol{M}, \boldsymbol{D} \in \mathcal{R}_+^{n_{\text{ext}} \times n_{\text{ext}} \times n_t}$	capacity and delay arrays
$\boldsymbol{C} \in \mathcal{R}_+^{n_{\text{link}} \times n_t}$	cost matrix
$\boldsymbol{g} \in \mathcal{R}_+^{n_{\text{bus}} \times 1}$	bandwidth requirements at system nodes
$c_{\text{WiMAX}} \in \mathcal{R}_+$	fixed cost for WiMaX

TABLE II
OPTIMIZATION VARIABLES

Symbol, Domain	Description
$\boldsymbol{x} \in \mathcal{B}^{n_{\text{bus}}}$	optimization variable for PMU locations
$\boldsymbol{y} \in \mathcal{B}^{n_P}$	optimization variable for PDC locations
$f_{1,ij}^{(r,k)} \in \mathcal{B}$	optimization variable for layer 1 flow from PMU q_r to PDC p_k on $(v_i, v_j) \in \mathcal{E}_{\text{ext}}$
$f_{2,ij}^{k} \in \mathcal{B}$	optimization variable for layer 2 flow from PDC p_k to SPDC on $(v_i, v_j) \in \mathcal{E}_{\text{ext}}$
$t_{ij\phi} \in \mathcal{B}$	optimization variable for the selection of technology ϕ on $(v_i, v_j) \in \mathcal{E}_{\text{ext}}$
$l_{e\phi} \in \mathcal{B}$	optimization variable for the deployment of link with technology ϕ on $e \in \hat{\mathcal{E}}_{\text{ext}}$
$t_w \in \mathcal{B}$	optimization variable for the use of WiMaX

considered the connectivity of the communication network as a matching problem between PMUs and PDCs, and enabled only links consisting of one technology for the connectivity between PMUs and PDCs, as well as PDCs and the SPDC. In this paper, we present a more flexible and thorough mathematical model which fully exploits the network data for the heterogenous communication network design. Therefore, our main contribution in this work is a novel optimization model for a minimum cost planning of a WAMS under power system observability and data communication requirements, where a heterogeneous communication network is considered. We start with the elaboration of our system and optimization model in Section II. Next, we apply the proposed model to a test network, whose details are presented in Section III. The results are discussed in Section IV. Finally, we conclude the paper in Section V with a summary of achievements and planned future work.

II. SYSTEM AND OPTIMIZATION MODEL

In the following, we use the following mathematical notations: Vectors are defined as column vectors and denoted by boldfaced lowercase letters, whereas the matrices and arrays are denoted by boldfaced uppercase letters. An element of a vector or a matrix is denoted by the same letter but not in boldface and with the relevant index as a subscript. The sets are denoted by calligraphic uppercase letters. We consider a power system denoted by an undirected graph $G_{\text{pow}}(\mathcal{V}_{\text{pow}}, \mathcal{E}_{\text{pow}})$, where $\mathcal{V}_{\text{pow}}$ is the set of power system nodes with $|\mathcal{V}_{\text{pow}}| = n_{\text{bus}}$, and $\mathcal{E}_{\text{pow}}$ is the set of power system branches with $|\mathcal{E}_{\text{pow}}| = n_{\text{branch}}$. Note that due to the wired communication technologies such as PLC and optical communication, whose topology is assumed to follow the power system topology, each power system branch can be modeled as a candidate link in the communication network design. In addition to the power system nodes, we assume that there are n_{com} communication network nodes denoted by the set $\mathcal{V}_{\text{com}}$, which can be used for the transmission of measurement data, for example as a wireless relay or a data concentrator. Furthermore, $\mathcal{E}_{\text{com}}$, the symmetric set of ordered pairs, denotes the possible communication links from the nodes in $\mathcal{V}_{\text{pow}}$ to the nodes in $\mathcal{V}_{\text{com}}$, as well as the possible links between the nodes in $\mathcal{V}_{\text{com}}$. We define the extended directed communication graph $G_{\text{ext}}(\mathcal{V}_{\text{ext}}, \mathcal{E}_{\text{ext}})$, where $\mathcal{V}_{\text{ext}} = \mathcal{V}_{\text{pow}} \cup \mathcal{V}_{\text{com}}$ with $|\mathcal{V}_{\text{ext}}| = n_{\text{ext}} = n_{\text{bus}} + n_{\text{com}}$, and $\mathcal{E}_{\text{ext}} = \tilde{\mathcal{E}}_{\text{pow}} \cup \mathcal{E}_{\text{com}}$, where $\tilde{\mathcal{E}}_{\text{pow}}$, extension of $\mathcal{E}_{\text{pow}}$, is the directed set of power system branches. We denote all the possible communication links by the set $\hat{\mathcal{E}}_{\text{ext}}$, where $|\ \hat{\mathcal{E}}_{\text{ext}}\ | = n_{\text{link}} = \frac{n_{\text{ext}}}{2}$. Note that $G_{\text{ext}}(\mathcal{V}_{\text{ext}}, \mathcal{E}_{\text{ext}})$ is the extended directed graph for the communication network, whereas $\hat{G}_{\text{ext}}(\mathcal{V}_{\text{ext}}, \hat{\mathcal{E}}_{\text{ext}})$ is the undirected graph for the same network.

Our planning approach is based on the following assumptions: First, we assume that the location v_{SPDC} of SPDC is predetermined and known beforehand. We denote the set of possible PMU locations by $\mathcal{P}_{\text{PMU}} \subseteq \mathcal{V}_{\text{pow}}$. Without loss of generality, we will assume, in the rest of this work, that a PMU can be installed at any $v_i \in \mathcal{V}_{\text{pow}}$, i.e. $\mathcal{P}_{\text{PMU}} = \mathcal{V}_{\text{pow}}$. Furthermore, there are $n_{\text{pow}} < n_{\text{bus}}$ power system nodes in addition to n_{com} communication nodes where a PDC can be located. These possible PDC locations are denoted by the set $\mathcal{P}_{\text{PDC}}$ with $|\mathcal{P}_{\text{PDC}}| = n_p = n_{\text{pow}} + n_{\text{com}}$. Furthermore, we assume that only one communication technology among all possible ones can be deployed between any two communication nodes.

We define the adjacency matrix of the power system $\boldsymbol{A} \in \mathcal{B}^{n_{\text{bus}} \times n_{\text{bus}}}$, $\mathcal{B} = \{0, 1\}$, as

$$A_{ij} = \begin{cases} 1, & \text{if } i = j \text{ or } (v_i, v_j) \in \mathcal{E}_{\text{pow}}, \quad v_i, v_j \in \mathcal{V}_{\text{pow}}, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

The communication links, which are available for the design of the WAMS, are denoted for the extended directed communication graph G_{ext} by the matrix $\boldsymbol{S} \in \mathcal{B}^{n_{\text{ext}} \times n_{\text{ext}}}$ as

$$S_{ij} = \begin{cases} 1, & \text{if } (v_i, v_j) \in \mathcal{E}_{\text{ext}}, \quad v_i, v_j \in \mathcal{V}_{\text{ext}}, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Furthermore, for each link which is available, n_t communication technologies can be considered for the planning, which are denoted by the set $\mathcal{T} = \{\tau_\phi \mid \phi = 1, \ldots, n_t\}$ in the technology availability array $\boldsymbol{T}$. For a possible communication link $(v_i, v_j) \in \mathcal{E}_{\text{ext}}$ denoted by a 1 in S_{ij}, the availability of communication technology τ_ϕ is denoted by $T_{ij\phi} = 1$. We denote the capacity and the delay of communication technologies by multidimensional arrays $\boldsymbol{M} \in \mathcal{R}_+^{n_{\text{ext}} \times n_{\text{ext}} \times n_t}$ and $\boldsymbol{D} \in \mathcal{R}_+^{n_{\text{ext}} \times n_{\text{ext}} \times n_t}$, respectively, while the costs of all communication links and technologies are denoted by $\boldsymbol{C} \in \mathcal{R}_+^{n_{\text{link}} \times n_t}$.

The aim of the design problem is to find the minimum cost network infrastructure by determining the optimal numbers and locations of PMU and PDC units, and the optimum heterogenous communication network topology under the constraints of power system observability and data communication requirements. For the decision variables, first we define the optimization variables for PMU locations and PDC locations as $\boldsymbol{x} \in \mathcal{B}^{n_{\text{bus}}}$ and $\boldsymbol{y} \in \mathcal{B}^{n_p}$, respectively.

For the minimum cost planning, we tackle the communication network design problem with a multi-commodity flow formulation. As both the data source locations and the data concentrator locations have to be optimized, we define the following flows:

i) the flows for all pairs of possible PMU locations and possible PDC locations,

ii) the flows from possible PDC locations to the SPDC location.

Thus, we define the optimization variable $f_{1,ij}^{(r,k)} \in \mathcal{B}$ for the flow from the PMU at $q_r \in \mathcal{P}_{\text{PMU}}$ to the possible PDC location $p_k \in \mathcal{P}_{\text{PDC}}$ on the communication link $(v_i, v_j) \in \mathcal{E}_{\text{ext}}$, where $v_i, v_j \in \mathcal{V}_{\text{ext}}$. Similarly, the optimization variable $f_{2,ij}^{k} \in \mathcal{B}$ is defined for the flow from PDC location $p_k \in \mathcal{P}_{\text{PDC}}$ to SPDC on the communication link $(v_i, v_j) \in \mathcal{E}_{\text{ext}}$, where $v_i, v_j \in \mathcal{V}_{\text{ext}}$. Furthermore, we define $t_{ij\phi} \in \mathcal{B}$ for the selection of the technology τ_ϕ over link $(v_i, v_j) \in \mathcal{E}_{\text{ext}}$, where $v_i, v_j \in \mathcal{V}_{\text{ext}}$. The optimization variable $l_{e\phi} \in \mathcal{B}$ is defined for the decision of the technology τ_ϕ over link $e \in \hat{\mathcal{E}}_{\text{ext}}$. Note that the variable $t_{ij\phi}$ is for the directed link $(v_i, v_j) \in G_{\text{ext}}$, whereas $l_{e\phi}$ is for the undirected links in $\hat{\mathcal{E}}_{\text{ext}}$. Table I shows the used notation for the set definitions and input parameters, whereas Table II provides an overview of the defined optimization variables. In the following, we introduce the constraints, and then the objective function of the proposed optimization model.

A. Power System Observability

A power system is observable, if the voltage values of all system nodes can be calculated or accurately estimated by using the available measurement set [16]. In the case where the measurement set consists of PMU measurements only, the vector $\boldsymbol{x}$, whose nonzero entries denote the locations of PMUs, should satisfy

$$\boldsymbol{A}\boldsymbol{x} \succeq \mathbf{1}, \tag{3}$$

where $\succeq$ denotes the element-wise greater-or-equal operator for matrices of the same size, and $\mathbf{1}$ is the vector of all-ones of size n_{bus}.

B. Communication Network Connectivity

The constraints for the communication network connectivity must ensure a connected network for the transmission of the measurement data from each installed PMU to the SPDC over intermediate PDCs. First, we note that the selection of communication links and technologies are constrained by their availability. Therefore, we may write

$$f_{1,ij}^{(r,k)} \leq S_{ij}, \quad \forall (q_r, p_k) \in \mathcal{P}_{\text{PMU}} \times \mathcal{P}_{\text{PDC}}, \forall (v_i, v_j) \in \mathcal{E}_{\text{ext}}, \tag{4}$$

$$f_{2,ij}^{k} \leq S_{ij}, \quad \forall p_k \in \mathcal{P}_{\text{PDC}}, \quad \forall (v_i, v_j) \in \mathcal{E}_{\text{ext}}, \tag{5}$$

$$t_{ij\phi} \leq T_{ij\phi}, \quad \forall (v_i, v_j) \in \mathcal{E}_{\text{ext}}, \quad \forall \phi = 1, \ldots, n_t, \tag{6}$$

where $v_i, v_j \in \mathcal{V}_{\text{ext}}$. In order to ensure a communication connection from selected PMU locations to a selected PDC, we write the flow conservation constraints as

$$\sum_{(v_i,v_j)\in\mathcal{E}_{\text{ext}}} \left(f_{1,ij}^{(r,k)} - f_{1,ji}^{(r,k)}\right) = \begin{cases} x_r y_k, & v_i = v_r, \\ 0, & v_i \neq p_k, v_r, \\ -x_r y_k, & v_i = p_k, \end{cases} \quad (7)$$

for each $v_i \in \mathcal{V}_{\text{ext}}$ and for all $(q_r, p_k) \in \mathcal{P}_{\text{PMU}} \times \mathcal{P}_{\text{PDC}}$, where $v_r \in \mathcal{V}_{\text{pow}}$ and $v_i, v_j \in \mathcal{V}_{\text{ext}}$. Similarly, the flow conservation constraints for connections from PDCs to the SPDC are formulated as

$$\sum_{(v_i,v_j)\in\mathcal{E}_{\text{ext}}} \left(f_{2,ij}^{k} - f_{2,ji}^{k}\right) = \begin{cases} y_k, & v_i = p_k, \\ 0, & v_i \neq p_k, v_{\text{SPDC}}, \\ -y_k, & v_i = v_{\text{SPDC}}, \end{cases} \quad (8)$$

for each $v_i \in \mathcal{V}_{\text{ext}}$ and for all $p_k \in \mathcal{P}_{\text{PDC}}$, where $v_i, v_j \in \mathcal{V}_{\text{ext}}$. Since the existence of a demand for a flow from q_r to p_k depends on both the selection of q_r as a PMU location and p_k as a PDC location in (7), we include the constraint

$$\sum_{p_k\in\mathcal{P}_{\text{PDC}}} \sum_{(v_r,v_j)\in\mathcal{E}_{\text{ext}}} f_{1,rj}^{(r,k)} = x_r, \quad \forall v_r \in \mathcal{P}_{\text{PMU}}, \quad (9)$$

to set q_r as a flow source independent from the selection of a PDC, in the case where a PMU is located at $v_r \in \mathcal{P}_{\text{PMU}}$, where $v_j \in \mathcal{V}_{\text{ext}}$. Similarly, each flow which is originated at a PMU location by (9) must end at a PDC location. We formulate this constraint as

$$\sum_{p_k\in\mathcal{P}_{\text{PDC}}} \sum_{(v_k,v_j)\in\mathcal{E}_{\text{ext}}} f_{1,jk}^{(r,k)} = x_r, \quad \forall v_r \in \mathcal{P}_{\text{PMU}}. \quad (10)$$

The constraint for the selection of a PDC location p_k is written as

$$y_k = \max\{f_{1,kj}^{(r,k)} \mid \forall (v_k, v_j) \in \mathcal{E}_{\text{ext}}, \forall v_r \in \mathcal{P}_{\text{PMU}}\}, \quad (11)$$

$\forall p_k \in \mathcal{P}_{\text{PDC}}$ which means that if a flow from a selected PMU ends in p_k, it must be selected as a PDC location. In addition, the decision for a technology on a link is constrained by

$$\begin{aligned} \textstyle\sum_{\phi=1}^{n_t} t_{ij\phi} \quad &= \max\big\{ \max\{f_{2,ij}^{l}, f_{1,ij}^{(k,r)}\} \mid \forall p_l \in \mathcal{P}_{\text{PDC}}, \\ &\forall (p_k, q_r) \in \mathcal{P}_{\text{PMU}} \times \mathcal{P}_{\text{PDC}}\big\}, \end{aligned} \quad (12)$$

for all $(v_i, v_j) \in \mathcal{E}_{\text{ext}}$. If a directed link between two nodes of G_{ext} is used in the flow transportation, the related technology will be selected for the deployment between these nodes. This constraint is formulated as

$$l_{e\phi} = \max\{t_{ij\phi}, t_{ji\phi}\}, \quad \forall e \in \hat{\mathcal{E}}_{\text{ext}}, \quad \forall \tau_\phi \in \mathcal{T}, \quad (13)$$

where $e = (v_i, v_j)$, $v_i, v_j \in \mathcal{V}_{\text{ext}}$, and $\hat{\mathcal{E}}_{\text{ext}}$ is the set of undirected communication links with $\hat{\mathcal{E}}_{\text{ext}} \subset \mathcal{E}_{\text{ext}}$.

C. Data Communication Requirements

The constraints discussed so far ensure the selection of PMU and PDC locations as well as the selection of communication links and technologies for the transmission of measurement data. In addition, we introduce the constraints for the data communication in order to take into account the capacity and delay capabilites of communication technologies, as well as the system requirements. For example, various protection and control applications have strict latency requirements which must be considered in the network planning [17].

1) End-to-end Delay: We consider a maximum allowable communication delay from an installed PMU to the SPDC, denoted by δ_{th}. The total delay constraints are formulated as

$$\sum_{(v_i,v_j)\in\mathcal{E}_{\text{ext}}} f_{1,ij}^{(r,k)} \sum_{\phi=1}^{n_t} t_{ij\phi} D_{ij\phi} + f_{2,ij}^{k} \sum_{\phi=1}^{n_t} t_{ij\phi} D_{ij\phi} \leq \delta_{th}, \quad (14)$$

for all $(q_r, p_k) \in \mathcal{P}_{\text{PMU}} \times \mathcal{P}_{\text{PDC}}$, where $v_i, v_j \in \mathcal{V}_{\text{ext}}$.

2) Capacity: In addition to the maximum delay constraints, the total data flow on each communication link is constrained by the capacity of the selected technology. We formulate this constraint as

$$\begin{aligned} \sum_{(q_r,p_k)\in\mathcal{P}_{\text{PMU}}\times\mathcal{P}_{\text{PDC}}} g_r f_{1,ij}^{(r,k)} + \sum_{p_k\in\mathcal{P}_{\text{PDC}}} f_{2,ij}^{k} \sum_{r=1}^{n_{\text{bus}}} g_r x_r y_k \\ \leq \sum_{\phi=1}^{n_t} t_{ij\phi} M_{ij\phi}, \quad \forall (v_i, v_j) \in \mathcal{E}_{\text{ext}}, \end{aligned} \quad (15)$$

where $v_i, v_j \in \mathcal{V}_{\text{ext}}$, and $g_r \in \mathcal{R}_+$ is the bandwidth requirement of the installed PMU at $q_r \in \mathcal{P}_{\text{PMU}}$. In addition, in case where a mobile radio network is considered with a cell capacity, this constraint can be written as

$$\begin{aligned} \sum_{\substack{(v_i,v_j)\\ \in\mathcal{E}_{\text{ext}}}} \left(\sum_{\substack{(q_r,p_k)\in\\ \mathcal{P}_{\text{PMU}}\times\mathcal{P}_{\text{PDC}}}} g_r f_{1,ij}^{(r,k)} + \sum_{p_k\in\mathcal{P}_{\text{PDC}}} f_{2,ij}^{k} \sum_{r=1}^{n_{\text{bus}}} g_r f_{1,ij}^{(r,k)} \right) \\ \leq M_{\text{cell,i}}, \quad \forall v_i \in \mathcal{V}_{\text{com}}, \end{aligned} \quad (16)$$

where $M_{\text{cell,i}}$ is the total capacity of the cell at $v_i \in \mathcal{V}_{\text{com}}$.

D. Objective Function

The objective of the network design is the minimization of the total deployment costs including PMU, PDC and communication link costs. The objective function is formulated as

$$F(\boldsymbol{x}, \boldsymbol{y}, \mathbf{l}, t_w) = \underbrace{\boldsymbol{c}_x^T \boldsymbol{x}}_{\text{PMU costs}} + \underbrace{\boldsymbol{c}_y^T \boldsymbol{y}}_{\text{PDC costs}} + F_{\text{comm}}(\mathbf{l}, t_w), \quad (17)$$

where $\boldsymbol{c}_x \in \mathcal{R}_+^{n_{\text{bus}}}$ and $\boldsymbol{c}_y \in \mathcal{R}_+^{n_p}$ are the given cost vectors for the PMU and PDCs, respectively, and $F_{\text{comm}}(\mathbf{l}, t_w)$ is the total cost for the communication network, which include the costs for each link as well as the fixed cost for WiMAX. The expression of F_{comm} can be written as

$$F_{\text{comm}}(t_w) = \sum_{e\in\hat{\mathcal{E}}_{\text{ext}}} \sum_{\phi=1}^{n_t} C_{e\phi} l_{e\phi} + t_w c_{\text{WiMAX}}, \quad (18)$$

where $C_{e\phi}$ is the cost of technology τ_ϕ over link $e \in \hat{\mathcal{E}}_{\text{ext}}$, c_{WiMAX} is the fixed licence fee for WiMAX and t_w is an additional binary optimization variable with the constraints

$$t_w = \max\{t_{ij\phi} \mid \forall (v_i, v_j) \in \mathcal{E}_{\text{ext}}, \tau_\phi = \text{WiMAX}\}. \quad (19)$$

E. Optimization Problem

The optimization problem for the integrated planning of the WAMS can be written as

$$\begin{aligned} \underset{\boldsymbol{x},\boldsymbol{y},\mathbf{f}_1,\mathbf{f}_2,\boldsymbol{t},t_w,\mathbf{l}}{\text{minimize}} \quad & F(\boldsymbol{x},\boldsymbol{y},\mathbf{l},t_w), \\ \text{subject to} \quad & (3)-(15), \end{aligned} \tag{20}$$

with additional binary constraints for the optimization variables. Note that the optimization problem in (20) is a binary non-linear problem due to the multiplication of optimization variables in (7), (14), (15), (16), and the maximum over the binary set of optimization variables in (11), (12), and (19). Since this non-linear and NP-hard problem is analytically intractable, meta-heuristic approaches seem to be the only way which can be used to obtain accurate solutions. On the other hand, since the optimization variables are binary, the non-linear constraints can be linearized [18], and the linearized problem can be solved for reasonable problem sizes optimally by means of available solvers which apply techniques such as branch-and-bound and cutting plane methods. In the next subsection, we describe the linearization steps.

F. Linearization of the Optimization Problem

The linearity of the optimization problem in (20) is violated by the multiplication of binary variables in (7), (14), (15), (16), and the maximum over the binary set of optimization variables in (11), (12), and (19). A multiplication of two binary variables a_1 and a_2 corresponds to an *AND* operation. A constraint involving a term a_1a_2 can hence be linearized by defining a new binary optimization variable $a' = a_1a_2$ with additional constraints

$$a' \quad \leq a_i, \quad \forall i = 1, 2, \tag{21a}$$

$$a' \quad \geq a_1 + a_2 - 1. \tag{21b}$$

Similarly, the maximum of a binary set corresponds to an *OR* operation of the binary elements in the set. To illustrate, a constraint involving the term $\max\{b_i \mid i = 1, \ldots, n\}$ can be linearized by defining a new binary optimization variable $b' = \max\{b_i \mid i = 1 \ldots n\}$ with additional constraints

$$b' \quad \geq b_i, \quad \forall i = 1, \ldots, n, \tag{22a}$$

$$b' \quad \leq \sum_{i=1}^{n} b_i. \tag{22b}$$

As a result of these steps, the optimization problem in (20) is linearized [18].

III. Test Cases

In order to assess the advantage of the proposed integrated optimization model, we apply it with representative technology and cost assumptions to MV test networks, which are stochastically generated by the tool presented in [19]. We compare the optimal cost value of the proposed model with a multi-stage approach, where the communication network design and PDC placement are carried out after the placement of the minimum required number of PMUs for observability. In addition, we include the cost reduction against the integrated approach with fiber links only. In the following, we briefly mention the details of the parameter assumptions and the optimization process for a given network. The bandwidth requirement at each PMU location is calculated according to the frame sizes of the IEEE C37.118 standard as described in [4] with the communication overhead of UDP/IP protocol layers. At each possible PMU location, we take into account the 3-phase voltage phasor measurements and 3-phase current phasor measurements on all incident power system branches. We consider broadband PLC (BPLC), fiber, and WiMAX as available technologies, which are used in real world applications [12]. The assumptions for the cost and parameters of the communication technologies are based on available standards and studies in the literature [12], [20]–[22]. The summary of link parameter and cost assumptions is given in Table III. We assume the costs for a PMU and PDC as €7500 and €12500, respectively, based on the prices of commercially available metering products [23].

TABLE III
LINK PARAMETERS AND COST ASSUMPTIONS [12], [20]–[22]

Technology	Range	Capacity	Cost(€)
BPLC	2 km	1 Mbps	500 / link
WiMAX	3 km	30 Mbps	1000/link 20000 license fee
Fiber	100 km	10 Gbps	1000 / km

Note that in these test cases, we do not consider the delay constraints introduced in Section II.C. A thorough modeling of link delays in WAMS and the analysis of their effects on the planning for specific applications are planned in the future work.

In the optimization procedure for a given network, we select the SPDC location as the node with the largest nodal degree in the power system graph $\mathcal{V}_{\text{pow}}$, whereas n_{pow} PDC locations are randomly selected over 4 equally divided subregions of the total area to ensure a uniform distribution of the PDC locations over the whole area. Next, we identify all possible communication links and technologies along with their parameters and costs, by using the power system branch distances and communication technology limitations, namely branch distances and maximum communication ranges. As a result of this discovery process, the matrices $\boldsymbol{S}$, and $\boldsymbol{T}$ are generated, as well as the problem data $\boldsymbol{A}$, $\boldsymbol{M}, \boldsymbol{D}, \boldsymbol{C}$, and $\boldsymbol{g}$. Then, the problem is solved by the ILP solver Gurobi [24].

IV. Results & Discussion

Figure 2 illustrates an example result of the optimal network design for a 10-node power network over a region of size $5\,\text{km} \times 5\,\text{km}$. In addition to the nodes in power system, illustrated by squares, there are 2 WiMAX base stations shown by circles, which enable the use of wireless links shown by the dashed lines. As shown in Figure 2, the optimization model introduced in Section II delivers the optimum PMU locations P1, P8, and P9, shown by the red-filled boxes, the optimum PDC location P4, shown by the square with outer green line, and the optimum communication network with the links and their technologies, shown by the green graph edges.

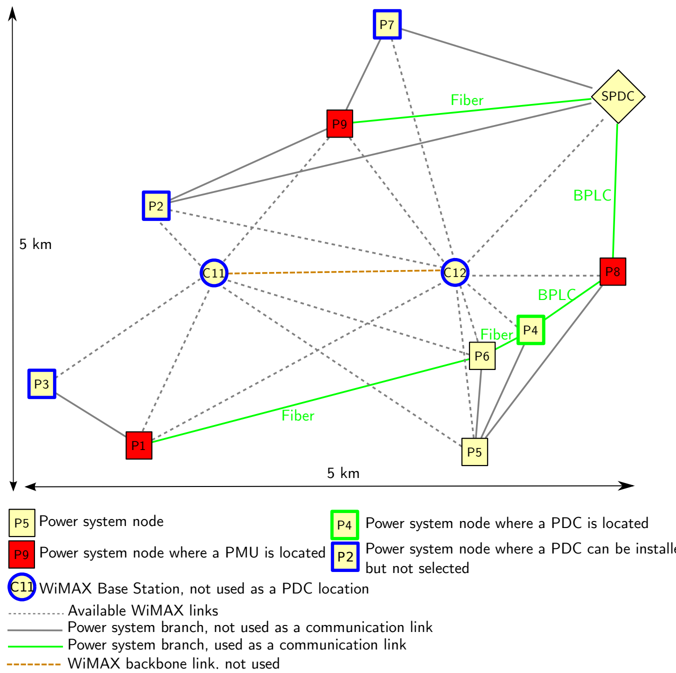

Fig. 2. An example optimal WAMS design of a power network with 10 nodes. Two WiMAX base stations (nodes C1, C2) are available for the communication network design in addition to the technologies BPLC and fiber. The graph edges represent the available communication links. The result of the optimization model includes the optimum locations of PMUs (boxes colored with red, P1, P28, P9), and the optimum location of PDC (the box marked with outer green line, P4) rather than the other possible ones (nodes marked with outer blue lines, P2, P3, P7, C11, and C12) and the optimum communication network represented by green links. WiMAX technology is not used due to the assumed high fixed costs

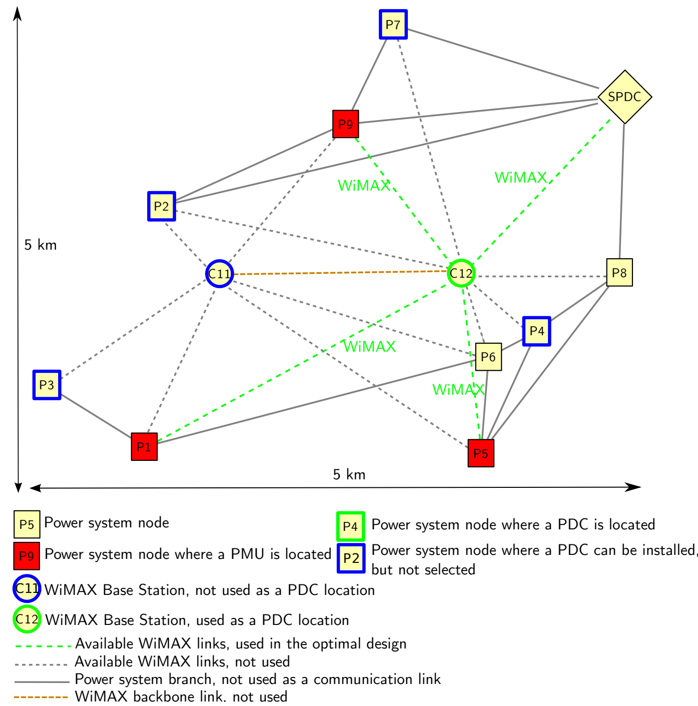

Fig. 3. Another example optimal design of the same network in Figure 2 with the difference that fiber is not available for the deployment, and BPLC and WiMAX are considered for the communication network design. Note that the minimum cost WAMS relies on WiMAX as the communication technology.

TABLE IV
COST COMPARISON RESULTS

n_{bus}	Region Size	n_{pow}	n_{com}	Reduction(%) vs. Multi Stage		Reduction(%) vs. Fiber Only	
				Com.	Tot.	Com.	Tot.
25	10 × 10 km^2	4	2	10.9	0.6	49.8	4.7
50	20 × 20 km^2	5	4	9.2	0.6	28.9	2.2

Table IV shows the percentage reduction in the communication network and total deployment costs achieved by our model against a multi stage planning and a planning with single communication technology as fiber, for two networks with 25 and 50 nodes with different region sizes and input parameters. Note that the percentage reduction in the total deployment cost is less than the percentage reduction in the cost of the communication network in both cases since the costs of each PMU and each PDC are assumed to be equal at all locations, and their number remain the same in the compared optimization models although their locations can be different.

The results show that the proposed integrated optimization model provides a more cost-efficient WAMS due to the consideration of multiple communication technologies and the simultaneous optimization of PMU and PDC locations. The deployment cost for the communication network is decreased by 49.8% in 25-node network and 28.9% in 50-node network compared with a planning with only one technology as optical fiber. Against a multi-stage planning, a reduction of about 10% is similarly achieved in communication costs in both cases.

Please note that the minimum cost network design, which is delivered as a result of our optimization model, reveals i) the required number and exact locations of PMU and PDCs for full observability of the power system, ii) the required communication network, which includes the locations and capabilities of necessary telecommunication equipments to install along with required links and their technologies, iii) a guarantee for the fulfillment of the capacity and delay specifications, and iv) insights about the operation of the network, such as the utilization of the communication links and the overall robustness and the reliability of the network.

Note also that the specific requirements and conditions in the planning process can lead to different WAMS topologies for the same power network. Figure 3 illustrates a WAMS design for the same power network as in Figure 2 with the only difference that optical fiber is not available for the deployment. As shown in Figure 3, also BPLC is not used any more in the optimal design, since the links are longer than the allowed range. In this setting, WiMAX becomes the technology which leads to the minimum cost design while fulfilling the data communication requirements. This example tangibly illustrates the importance of considering the data communication requirements for the placement of PMUs and PDCs in certain applications. Furthermore, from the perspective of communication network design, this model is flexible enough to be improved for further requirements and design objectives such as security and reliability, and can enable multi-objective planning approaches for network planners.

V. Conclusion

In this paper, we have introduced an optimization technique for a minimum cost planning of a WAMS in smart grids. The proposed model enables a simultaneous optimization of PMU and PDC locations in addition to the topology of a heterogeneous communication network for the transmission of measurement data. The proposed model have been applied to test networks and its advantages have been validated. Our model can enable significant cost savings in a real deployment while also satisfying the technical system requirements. Since the complexity of the NP-hard problem is associated with high computational time in large networks, the development of an efficient algorithm with an acceptable trade-off between the computational complexity and the accuracy is an important step to enable the planning of large networks. In this sense, our novel method yields a bunch of benefits in comparison with common techniques.

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