TDMA Network Design using Decode-and-Forward Relays with Finite Set Modulation

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***Abstract*— This paper studies the uplink channel of a time division multiple access (TDMA) cellular network assisted by wireless relays. Aiming at increasing the network´s transmission rate, we optimally place multiple decode-and-forward (D&F) dedicated relays within every cell. In the first part of the contribution, we consider the relays transmitting Gaussian codewords. Results show that there is almost 39% gain in the transmission rate of each cell with respect to no relaying. In the second part, we analyze the performance impact of finite set modulation at user and relays (*i.e.* M-PAM) . With such a modulation, the gain is slightly decreased to 27%. Finally, we show that D&F relaying provides large gains in the outage capacity at the edge of the cell, with similar performance for Gaussian and M-PAM modulation.**

I. Introduction

Reliable communication and increased transmission rate are the demands of the current and future wireless networks. Aiming at overcoming channel impairments, *e.g.* path loss, shadowing and multi-path fading, multiple antenna techniques are widely proposed. However, deploying multiple antennas on wireless nodes is not always worth-while or possible due to cost and space constraints, respectively. One approach to overcome this problem is *relaying* [1].

The analysis of relay-based communications was pioneered by Cover and El Gamal in [2]. Achievable rates were presented therein for point-to-point channels assisted by a single relay, showing significant spectral efficiency gains. The extension of these results to multiple access channels with a single relay is presented in [3]. However, despite these results, few achievable rates are known for channels assisted by multiple relays, which motivates our work. In particular, we analyze the impact of multiple wireless relays on current cellular access networks.

The aim of the paper is three-fold: 1) To contribute the maximum transmission rate within the uplink channel of a TDMA network assisted by multiple D&F relays. 2) To optimally place the relays within such a network, so that the users' transmission rate is maximized. In particular, we design and compare three different relay placement topologies. 3) To analyze the impact of finite set modulation onto the rate performance of the network.

The rest of the paper is organized as follows: in Section II, we present the network and signal model. In Section III, we derive the maximum achievable rate of multiple relay channels. Section IV designs the topologies for placing relays. In Section V, we derive the maximum transmission rate assuming M-PAM signal set at sources and relays. Numerical results and conclusion are given in Section VI and VII, respectively.

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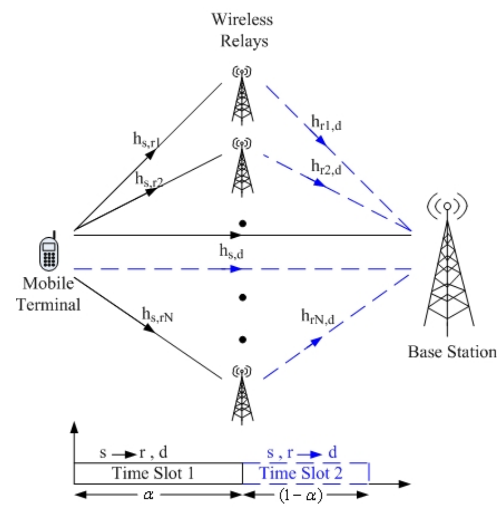

Fig. 1. Multiple Relay Channel

A. Notation

In this paper, $I(X;Y)$ denotes the mutual information between random variable X and Y, $H(.)$ denotes the entropy, $\mathcal{C}(.) = \frac{1}{2}\log_2(1+.)$.

II. Network Model

TDMA is a multiplexing scheme that involves separating the multiple users in time. That is, on every frequency band, only one user transmits at a time. In this paper, we consider the TDMA uplink channel of a single cell, of area 1.4km × 1.4km. Within the cell, N wireless D&F relays are present in order to aid the user-to-Base Station (BS) communication. Relays are fixed and dedicated i.e. they have no own data to transmit. The BS is located at the center of cell, at the height 30m and mobile terminals within the cell at the height 1.8m. The wireless relays are assumed to be at the height 10m. NLOS[1] propagation is considered between source-relay and source-destination, while

[1]NLOS - No line of Sight ; LOS - Line of Sight

LOS propagation is considered between relay-destination. All the nodes have only one antenna.

We define $\mathcal{N} = \{1, 2, \cdots, N\}$ the set of relays within the network, which works under a half-duplex constraint and helps the users as follows: the source transmits into two consecutive time slots of duration α and $(1-\alpha)$, respectively. During the time slot 1, the source broadcasts its message to relays and destination. During time slot 2, both source and relays mimic an antenna array and re-transmit the message to the destination (see Fig. 1).

Let h_{s,r_i}, $h_{s,d}$ and $h_{r_i,d}$ be real scalars that represent the *time-invariant* and *memoryless* channel from source-to-relay, source-to-destination and relay-to-destination, respectively. Also, let $x_{s,t}$ be the transmitted signal by the source during time slot $t = 1, 2$, $x_{r_i,2}$ be the transmitted signal by relay i during times slot 2, $z_{d,t} \sim \mathcal{N}(0, N_o)$ the additive white Gaussian noise (AWGN) at the destination during t and $z_{r_i,1} \sim \mathcal{N}(0, N_o)$ the AWGN at relay i during time slot 1. Then, the received signal at relays and destination is,

$$y_{r_i,1} = h_{s,r_i} \cdot x_{s,1} + z_{r_i,1} \quad ; \quad i \in \mathcal{N} \tag{1}$$

$$y_{d,1} = h_{s,d} \cdot x_{s,1} + z_{d,1} \tag{2}$$

$$y_{d,2} = h_{s,d} \cdot x_{s,2} + \sum_{i=1}^{N} h_{r_i,d} \cdot x_{r_i,2} + z_{d,2} \tag{3}$$

In the network, a power constraint is enforced for both time slots. That is, $E\{|x_{s,1}|^2\} \leq P$ and $E\{|x_{s,2}|^2\} + \sum_{i=1}^{N} E\{|x_{r_i,2}|^2\} \leq P$, respectively.

For design purposes, we assume realistic channel model, with path loss, shadowing, and multi-path fading. First, the path loss gain is calculated as [4]:

$$|h_{pathloss}|^2 = \left(\frac{\lambda}{4\pi d}\right)^{\beta} \cdot F_T \cdot F_R \tag{4}$$

where, transmitted power is 23 dbm, d is the distance between transmitter and receiver, with antenna gain factors F_T and F_R, respectively. Also, λ is the transmitted signal wavelength corresponding to a frequency carrier $f_c = 1$ GHz and $\beta = 2.6$ and $\beta = 4.05$ is the path loss exponent for LOS and NLOS propagation, respectively. Also, shadowing is considered to be Lognormal, with zero mean and standard deviation $\sigma = 4$dB, and $\sigma = 10$dB for LOS and NLOS respectively. Finally, multi-path fading is unitary-mean, Rayleigh-distributed.

III. Network Achievable Transmission Rate

Let the TDMA uplink, where a user transmits to a BS with the aid of N D&F relays. Its maximum transmission rate is presented in the next proposition.

Proposition 1: Let $A_n = \mathcal{C}\left(h_{s,r_n}^2 \frac{P}{N_o}\right)$, $B = \mathcal{C}\left(h_{s,d}^2 \frac{P}{N_o}\right)$ and $D_n = \mathcal{C}\left(\left(h_{s,d}^2 + \sum_{i=1}^{n} h_{r_i,d}^2\right)\frac{P}{N_o}\right)$, with the source-relay ordering:

$$h_{s,r_1} \geq h_{s,r_2} \geq \cdots \geq h_{s,r_N}. \tag{5}$$

Define $\alpha_{opt}^n = \min\left\{1, \frac{D_n}{A_n - B + D_n}\right\}$. Then, the maximum transmission rate R of the user assisted by multiple D&F relays is:

$$R = \max_{1 \leq n \leq N} R_n \tag{6}$$

where

$$R_n = \begin{cases} B & \text{if } \alpha_{opt}^n = 1 \\ \alpha_{opt}^n A_n & \text{if } \alpha_{opt}^n < 1 \end{cases} \tag{7}$$

Proof: Let (w.l.o.g) the source-relay channels satisfy (5), and consider that only the subset $\mathcal{R}_n = \{1, 2, \cdots, n\} \subseteq \mathcal{N}$ is active, with $n \leq N$ [5]. The source encodes at a rate R, that can be decoded by all relays in $\mathcal{R}_n$ *iff*

$$\begin{aligned} R &\leq \min_{i \in \mathcal{R}_n} \alpha \cdot \mathcal{C}\left(h_{s,r_i}^2 \frac{P}{N_o}\right) \\ &= \alpha \cdot \mathcal{C}\left(h_{s,r_n}^2 \frac{P}{N_o}\right) \end{aligned} \tag{8}$$

where equality is due to ordering. During time-slot 2, relays in $\mathcal{R}_n$ cooperatively transmit with the source. Hence, the destination is able to decode the source message *iff*:

$$\begin{aligned} R &\leq \alpha \cdot \mathcal{C}\left(h_{s,d}^2 \frac{P}{N_o}\right) \\ &+ (1-\alpha) \cdot \mathcal{C}\left(\left(h_{s,d}^2 + \sum_{i=1}^{n} h_{r_i,d}^2\right)\frac{P}{N_o}\right) \end{aligned} \tag{9}$$

Therefore, the maximum transmission rate (given the subset of active relays $\mathcal{R}_n$) is the minimum of both (8) and (9). Now, notice that (8) is an increasing constraint on $\alpha \in (0, 1]$, while (9) is decreasing. Hence, the transmission rate R will be maximum at the point where both constraints intersect each other. We call this point as α_{opt}^n and is calculated by equating (8) and (9). Let define A_n, B and D_n as in proposition 1, then

$$\begin{aligned} \alpha_{opt}^n &= \{\alpha : \alpha \cdot A_n = \alpha \cdot B + (1-\alpha) \cdot D_n\} \\ &= \frac{D_n}{A_n - B + D_n} \end{aligned} \tag{10}$$

Therefore, the transmission rate must satisfy

$$R \leq \alpha_{opt}^n A_n. \tag{11}$$

However, α has to be lower than or equal to 1; thus, α_{opt}^n follows proposition 1. Notice that for $\alpha_{opt}^n = 1$ there is no time allocated on the relaying part of the communication, and so the maximum rate is that of the source-destination communication, *i.e.*, $R \leq B$. Finally, as we may arbitrary choose the subset of active relays from $\mathcal{R}_1, \cdots, \mathcal{R}_N$, then it concludes the proof.

IV. Topology Design for Relay Placement

The first task on the network design is to decide the number of relays to be deployed. To that end, consider Fig. 2, where the aggregate transmission rate of the network with different number of relays is plotted. The aggregate transmission rate $\bar{R}$ is calculated as,

$$\bar{R} = \frac{1}{Area} \cdot \int_x \int_y R(x, y) \cdot dy \cdot dx \tag{12}$$

where, $R(x, y)$ is the transmission rate at the cell point (x, y) following proposition 1, and assuming path-loss only. In the

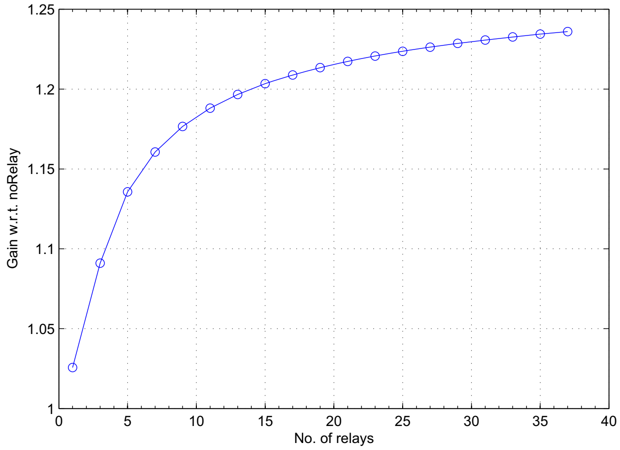

Fig. 2. Plot of the Gain in aggregate rate against the No. of Relays

plot the gain is normalized with respect to no relay case. We observe that, as the number of relays increases, the relative gain of introducing a new relay diminishes (as shown in e.g. [6]). Hence, we select to place $N = 16$ relays as a good trade-off between cost and gain. To place the relays, we analyze three different topologies, working on one-hop relaying.

I) *Circular Ring:* Relays are placed into one circular ring of radius R_c.

II) *Two Circular Rings:* Relays are placed into two circular rings of radius R_{c1} and R_{c2}, each ring with 8 relays.

III) *Two Squares:* Relays are placed into two concentric squares. The inner square with 4 relays and outer square with 12 relays. The width of the outer square W_o is three times the width of inner square W_i.

Let now obtain the the optimum distance at which relays must be placed on the three topologies. To do so, we consider the free space path loss channel model only. For topology I, we plot the gain in $\bar{R}$, by varying radius R_c (see Fig. 3, top). For topology II, $\bar{R}$ is plotted by keeping $R_{c1} = 300$m and varying R_{c2}. For topology III, $\bar{R}$ is plotted by varying the half of the width of the outer square from the base station. Now, the optimum distance is selected against the maximum $\bar{R}$ gain. We can clearly see in Fig. 3 (up) that the maximum gain is obtained for topology I when $R_c = 555$m, for topology II when $R_{c1} = 300$m and $R_{c2} = 625$m, and for topology III when half of W_o is at 500m from the base station. We observe that the best topology to place relays is topology III and the worst is the topology I. Also, the optimal distances shows that D&F relays are more helpful when they are deployed away from the base station and closer to the edge of the cell.

Now placing the relays at the optimal positions, we analyze the gain in $\bar{R}$ for realistic channel models: ***i)*** Path loss only, ***ii)*** Path loss and Shadowing only, ***iii)*** Path loss and multi-path Fading only, and ***iv)*** the model with the three effects (see Table. I). We observe that, by increasing the channel impairments in the channel the aggregate rate gain is also increased *i.e.* the gain in $\bar{R}$ is increased by 15% from path loss only channel model to the realistic channel model with all three effects.

TABLE I

GAIN IN THE AGGREGATE RATE AT THE OPTIMUM DISTANCE(GAUSSIAN)

Topology	Optimum Distance	Gain in the Tx. rate *i*	***ii***	***iii***	***iv***
I	$R_c = 555$m	21 %	25 %	31 %	36 %
II	$R_{c1} = 300$m, $R_{c2} = 625$m	23 %	27 %	33 %	38 %
III	$\frac{1}{2}W_i = 167$m, $\frac{1}{2}W_o = 500$m	24 %	28 %	34 %	39 %

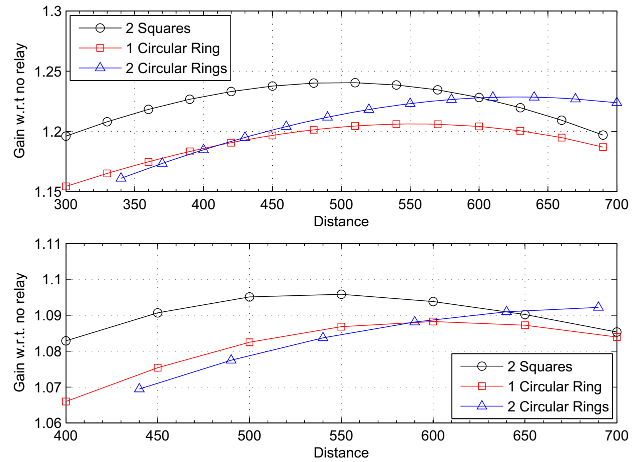

Fig. 3. Gain in the aggregate rate for Relay Channel with 16 decode and forward relays for different topologies, Gaussian (up) and 512-PAM (down)

V. M-PAM IMPACT ON THE COVERAGE

In this section we analyze the impact of finite set modulation onto the achievable rate of the relay channel. We assume M-PAM modulation at user and relays, and again find the optimum distance for placing relays.

Definition: *Let an AWGN channel $Y = hX + Z$, with $Z \sim \mathcal{N}(0, N_o)$. Consider the input of the channel to be a M-PAM constellation $\mathcal{X} = \{X_1, X_2, \cdots, X_M\}$, satisfying $\frac{1}{M}\sum_{i=1}^{M} |X_i|^2 = P$. Then, the maximum transmission rate is:*

$$R_{\text{MPAM}}\left(h^2\frac{P}{N_o}\right) = -\int\left[\sum_{X_0 \epsilon \mathcal{X}} \frac{1}{M} \cdot \frac{e^{\frac{-(Y-h.X_0)^2}{2N_o}}}{\sqrt{2\pi N_o}} \cdot \log_2\left(\sum_{X_0 \epsilon \mathcal{X}} \frac{1}{M} \cdot \frac{e^{\frac{-(Y-h.X_0)^2}{2N_o}}}{\sqrt{2\pi N_o}}\right) \cdot dY\right] - \frac{1}{2}\log_2(2\pi e N_o) \tag{13}$$

Proof [7]: Let $\mathcal{X} = \{X_1, X_2, \cdots, X_M\}$ is a set of possible M signal sets, then M-PAM constellation satisfy,

$$\frac{1}{M}\sum_{i=1}^{M}|X_i|^2 = P \tag{14}$$

Consider an M-PAM channel input $X \in \mathcal{X}$ such that $p(X_i) = \frac{1}{M} \quad \forall i$. Then the maximum transmission rate for M-PAM $R_{\text{MPAM}}\left(h^2 \frac{P}{N_o}\right)$ is given using [8] as,

$$R_{\text{MPAM}}\left(h^2 \frac{P}{N_o}\right) = I(X;Y) = H(Y) - H(Y|X) \qquad (15)$$

where, $Y = hX + Z$ is channel output and is equal to the channel input X added with AWGN Z. h represents the channel gain. Then,

$$H(Y|X) = \frac{1}{2} \log_2 2\pi e N_o \qquad (16)$$

Also, the entropy $H(Y)$ is given as,

$$H(Y) = -\int p(Y) \cdot \log_2(p(Y)) \cdot dY \qquad (17)$$

Now $p(Y)$ can be given as,

$$p(Y) = \sum_{X_0 \epsilon \mathcal{X}} p(Y|X = X_0) \cdot p(X = X_0)$$

Where, $p(X = X_0) = \frac{1}{M}$ and conditional probability density [7] is $p(Y|X = X_0) = \frac{1}{\sqrt{2\pi N_o}} \cdot e^{\frac{-(Y - h.X_o)^2}{2N_o}}$. Substituting values in equation (15) yields equation (13). [7] showed that the maximum transmission rate that M-PAM can achieve is $\log_2(M)$ *i.e.* with 512-PAM, the maximum transmission rate for the communication will be 9 bits/symbol.

A. D&F relays and M-PAM Signal Set

Easily, it can be proven that the multiple-relay achievable rate with M-PAM modulation (at source and relays) follows proposition 1 where $R_{\text{MPAM}}(\cdot)$ is used instead of $\mathcal{C}(\cdot)$. Considering such a rate, we now find the optimal distance for placing the relays. We study the same three topologies discussed in Section. IV, and use 512-PAM modulation. A free space path loss channel model is considered and the aggregate rate $\bar{R}$ gain is plotted in the same way as it is done for Gaussian case (see Fig. 3 bottom). The optimum distance is selected at the point where there is maximum $\bar{R}$ gain. We observe that, the maximum gain is obtained for topology I when $R_c = 600$m, for topology II when $R_{c1} = 400$m and $R_{c2} = 690$m, and for topology III when the half of the outer square is at 550m from base station. Again the best topology to place relays is topology III and the worst is the topology I.

Now, at the optimum distance, we calculate the $\bar{R}$ gain for 512-PAM considering the same realistic channel models discussed in previous section (see Table. II). We observe that, by increasing the channel impairments in a channel, $\bar{R}$ gain also increases. Comparing Table. I and II, we observe a decrease in the $\bar{R}$ gain for 512-PAM modulation *i.e.* for path loss channel only, there is a large loss in the $\bar{R}$ gain. However, with the realistic channel model with all three effects the decrease is not that significant.

TABLE II

GAIN IN AGGREGATE RATE AT THE OPTIMUM DISTANCE(512-PAM)

Topology	Optimum Distance	Gain in the Tx. rate			
		i	***ii***	***iii***	***iv***
I	$R_c = 600$m	08 %	16 %	24 %	26 %
II	$R_{c1} = 400$m, $R_{c2} = 690$m	09 %	16 %	24 %	26 %
III	$\frac{1}{2}W_i = 184$m, $\frac{1}{2}W_o = 550$m	10 %	17 %	25 %	27 %

VI. NUMERICAL RESULTS

Fig. 4 and 5 depict the coverage of the TDMA uplink with 16 relays, considering Gaussian and 512-PAM channel input respectively. A free space path loss model is considered and relays are placed following topology III. We observe that for the Gaussian input, the transmission rate at points close to base station is extended. Also the transmission rate at the edge of the cell is improved. On the other hand, with 512-PAM modulation, the inner square of the topology III is not worthwhile due to the fact that the transmission rate is saturated for the PAM modulation as shown by P.McIllree in [7]. Only the outer square comes into play and improves the coverage away from base station.

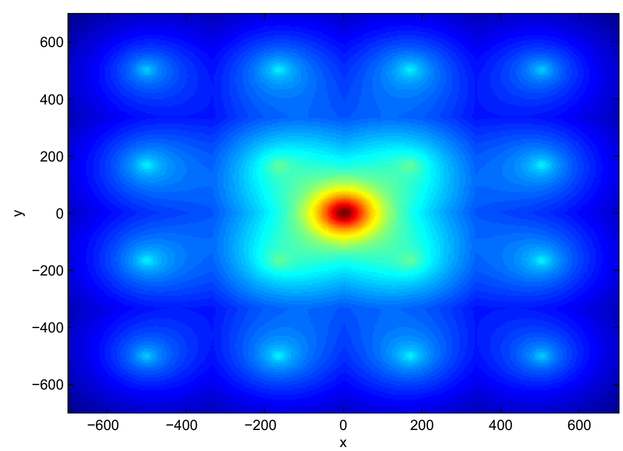

Fig. 4. Coverage within a square Cell (Gaussian)

Fig. 6 and 7 show the transmission rate behaviour, for all the relay placement topologies considered. We have cut the cell in the diagonal ($x = y$) and observe that with the use of relays, we have improved the transmission rate at the edge of the cell. For the Gaussian input case, transmission rates within the radius of 100m around base station is the same for all the relaying and no relaying schemes. But as the distance is further increased, the relaying scheme exhibits a better transmission rates than no relay scheme. The peak shows the presence of the relays. For 512-PAM, the transmission rate is saturated close to the base station and the maximum transmission rate achieved is 9 bits/symbols for all the relaying and no relay schemes. We observe that, with the use of relaying schemes, the transmission rate is improved at areas away from the base station and specially at the edge of the cell.

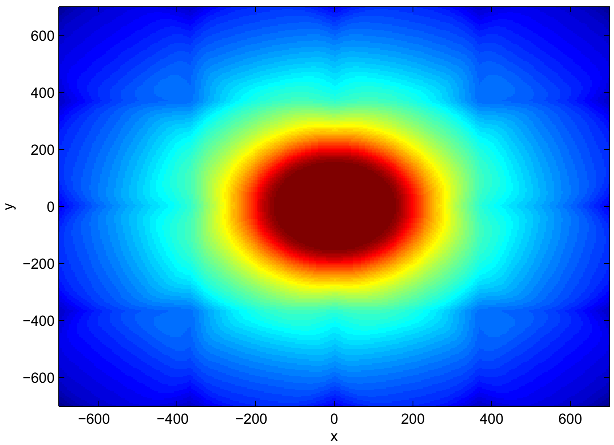

Fig. 5. Coverage within a square cell (512-PAM)

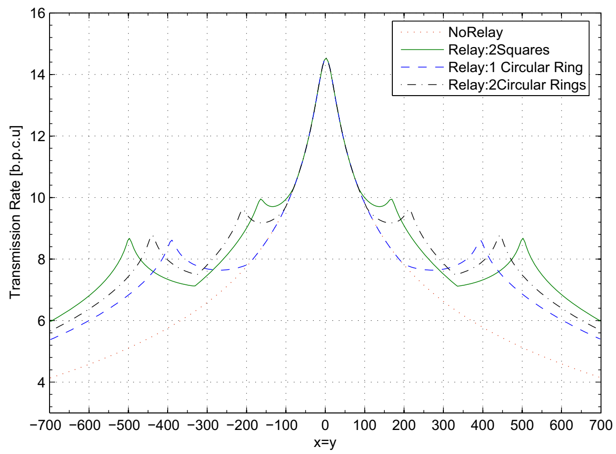

Fig. 6. Diagonal cut of the cell coverage (Gaussian)

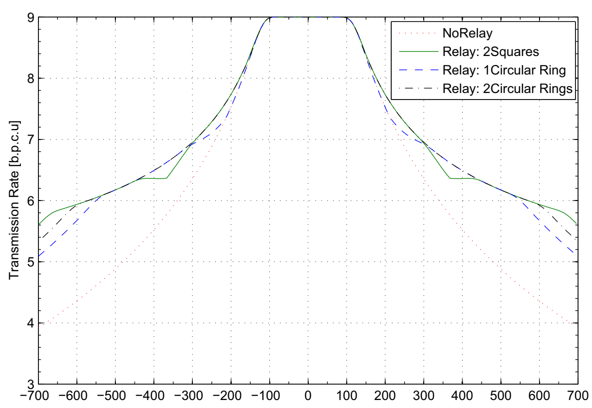

Fig. 7. Diagonal cut of the cell coverage (512-PAM)

Fig. 8 depicts the outage capacity at the edge of the cell for the no relay scheme and a relay scheme based on topology III, considering Gaussian and 512-PAM modulation, respectively. We show that the relaying scheme has improved the transmission rate at the edge of the cell. For a certain communication to be at least 95% successful at the edge of the cell with no relays, transmission can be done at a rate 0.6 bits/symbols. But with the introduction of the relays in a network, transmission rate is increased to 4.5 bits/symbols, which is a good improvement at the edge. Using the 512-PAM modulation, the transmission rate improvement at the edge of the cell is significant.

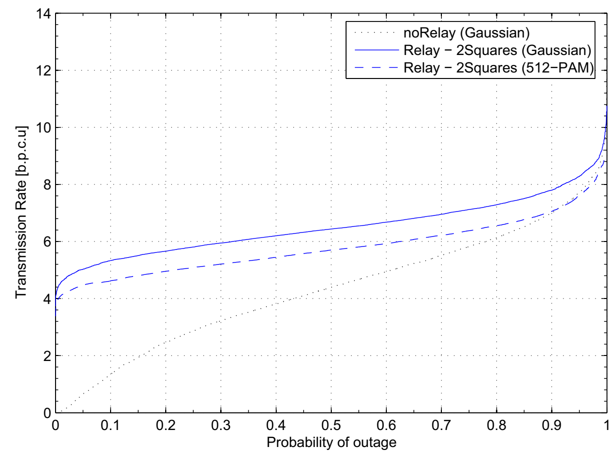

Fig. 8. Outage Capacity at the edge of the cell

VII. Conclusion

This paper considered the design of a TDMA network using decode-and-forward wireless relays. Different relay placement topologies were considered and the transmission rate coverage and gain was studied for Gaussian and M-PAM channel input. It is concluded that D&F relays provide significant gains in the rate coverage comparing networks without relays. For the M-PAM modulation, the transmission rate gain decreases compared to that of Gaussian input. However, both scenarios provide immense increase in transmission rate gain at the edge of the cell.

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