Sum Power Minimization for TDD-Enabled Full-Duplex Bi-Directional MIMO Systems Under Channel Uncertainty

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***Abstract*—In this paper we address a sum power minimization problem for a bi-directional and full-duplex (FD) communication system, where the required rate constraints are imposed on the guaranteed communication rates in each direction. In this regard, the impact of channel-state information (CSI) error, as well as the signal distortion due to hardware impairments are jointly taken into account. In order to ensure backwards compatibility to an equivalent half-duplex setup, we assume a time-division-duplex capable system where the FD communication process takes place in multiple independent time segments. Due to the intractable structure of the resulting optimization problem, a weighted minimum mean squared-error based method is applied to cast the power minimization problem into a separately convex structure, which can be iteratively solved with a guaranteed convergence. The resulting computational complexity of the algorithm is then discussed analytically. Finally, the performance of the proposed algorithm is numerically evaluated over different levels of rate demand, CSI error and transmitter/receiver dynamic range.**

1. Introduction

A full-duplex transceiver has the capability to transmit and receive at the same time and frequency, and hence it has the potential to enhance the spectral efficiency [1]. Nevertheless, such systems suffer from the inherent self-interference from their own transmitter. Recently, specialized cancellation techniques, e.g., [2]–[4], have demonstrated adequate levels of isolation between transmit (Tx) and receive (Rx) directions to facilitate a full-duplex (FD) communication and motivated a wide range of related studies, see, e.g., [1], [5], [6]. The common idea of such techniques is to cancel/subtract the main part of self-interference signal in the radio-frequency (RF) analog domain, so that the remaining signal can be processed for further interference reduction in the baseband, i.e., digital domain. Nevertheless, such methods are far from perfect in a realistic environment due to i) aging and inherent inaccuracy of the hardware (analog) components, as well as ii) inaccurate estimation of the CSI in the interference paths due to the limited channel coherence time, noise, and limited processing power in digital domain. In this regard, a widely used model for the operation of a multiple-antenna FD transceiver is proposed in [7], where the aforementioned inaccuracies are taken into account. A convex optimization design framework is proposed in [8]–[10] by defining a price/threshold for the self-interference power, assuming the availability of perfect channel state information (CSI) and accurate transceiver operation. While this approach provides a design with relatively low computational complexity, it does not provide a reliable performance for a scenario with erroneous CSI, particularly regarding the self-interference path [11]. Consequently, the consideration of CSI error in a FD point-to-point (P2P) transmission is further studied in [12], [13] by maximizing the average system sum rate, and in [14] by minimizing the sum mean-squared-error (MSE). Furthermore, the work in [15] has presented a minimum power consumption design, where the explicit rate constrains are satisfied for a multiple-input-multiple-output (MIMO) FD system. Nevertheless, this approach is not yet extended with the consideration of CSI imperfection.

In this paper, we extend our previous work in [15] where a sum power minimization problem is studied under rate constraints for a TDD-enabled FD-P2P communication system. In particular, the imperfections of the hardware components, as well as the CSI estimation error in all links are jointly taken into account. Please note that as the intensity of the aforementioned imperfections increases, the performance of an FD system degrades rapidly and falls below an equivalent half-duplex (HD) setup, as observed in the related studies [8]–[13], [16]. This is particularly undesirable for our purpose, as it may lead to the infeasibility of a required rate constraint for a FD system, regardless of the available transmit power[1]. In contrast, an HD setup can always reach a required communication rate in both communication directions, by dividing the channel resources via the utilization of time-division duplexing (TDD). In order to close this performance gap, we present a TDD-capable FD system, where the FD communication is enabled together with dividing the channel resource into time-orthogonal sub-channels. Consequently, independent transmit/receive strategies are utilized in each communication time slot, which in turn leads to backwards compatibility to a traditional HD setup.

In Section 2 the system model is defined, taking into account the impact of CSI error, as well as the impact of the hardware inaccuracies. The optimization strategy is then defined as a sum power minimization problem, with the consideration of explicit rate requirements. Due to the intractable nature of the resulting problem, a variation of the weighted minimum mean-squared error (WMMSE) optimization method [17] is proposed to cast the problem into a separately convex structure.

[1] In an FD system with imperfect self-interference cancellation, a higher rate and consequently higher transmit power in one direction results in the degradation of the other communication direction due to the residual interference. This effect leads to the infeasibility of rate constraints, when required rates in both direction are high.

An alternating optimization procedure is then proposed with a guaranteed convergence, where semi-definite-programming (SDP) is applied in each step. The performance of the proposed method is then numerically evaluated in Section 4 under different rate requirements, and transceiver/CSI inaccuracy regimes.

A. Mathematical Notation:

Throughout this paper, column vectors and matrices are denoted as lower-case and upper-case bold letters, respectively. Mathematical expectation, trace, inverse, determinant, transpose, conjugate and Hermitian transpose are denoted by $\mathbb{E}(\cdot)$, $\text{tr}(\cdot)$, $(\cdot)^{-1}$ $|\cdot|$, $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^H$, respectively. The Kronecker product is denoted by $\otimes$. The identity matrix with dimension K is denoted as $\mathbf{I}_K$ and $\text{vec}(\cdot)$ operator stacks the elements of a matrix into a vector. $\|\cdot\|_2$ and $\|\cdot\|_F$ respectively represent the Euclidean and Frobenius norms. $\lfloor \mathbf{A}_i \rfloor_{i=1,\ldots,K}$ denotes a tall matrix, obtained by stacking the matrices $\mathbf{A}_i$, $i = 1,\ldots,K$. $\mathbf{\Gamma}_K^l$ represents a square matrix with dimension K, with all zero elements except of the l-th diagonal element equal to 1. The set of all positive definite matrices with dimension l is presented as $\mathcal{H}_l$.

2. System Model

A MIMO and bi-directional communication between two FD transceivers is considered. Each node is equipped with N_i (M_i) transmit (receive) antennas, where $i \in \mathcal{I}$ represents the index of the communication directions, and $\mathcal{I} := \{1,2\}$, see Fig. 1. Moreover, the communication is divided into multiple independent time slots, where $t \in \mathcal{T}$ represents the time slot index, and $\mathcal{T} := \{1,\ldots,T\}$. $\mathbf{H}_{ii} \in \mathbb{C}^{M_i \times N_i}$ and $\mathbf{H}_{ji} \in \mathbb{C}^{M_j \times N_i}$ respectively represent the desired channel in the communication direction i and the self-interference channel from i to j-th communication direction. All channels are assumed to follow a flat-fading model, and remain constant during the multiple time slots. Moreover, the CSI matrices are estimated with limited accuracy. In this respect we follow the so-called deterministic model [18], where the channel matrices are located within a known feasible region. This is written as

$$\mathbf{H}_{ij} = \tilde{\mathbf{H}}_{ij} + \mathbf{\Delta}_{ij}, \;\; \mathbf{\Delta}_{ij} \in \mathbb{D}_{ij}, \;\; i,j \in \mathcal{I}, \tag{1}$$

and

$$\mathbb{D}_{ij} := \left\{ \mathbf{\Delta}_{ij} \;\middle|\; \|\mathbf{\Delta}_{ij}\|_F \le \zeta_{ij} \right\}, \;\; \forall i,j \in \mathcal{I}, \tag{2}$$

where $\tilde{\mathbf{H}}_{ij}$ is the estimated channel matrix, $\mathbf{\Delta}_{ij}$ represents the channel estimation error, and $\mathbb{D}_{ij}$ is the norm-2 ball representing the CSI error feasible region with the radius $\zeta_{ij} \ge 0$, see [18]–[20] for more elaboration on the used error model. The transmitted signal in the direction i and the time instance t is written as

$$\mathbf{x}_i(t) = \mathbf{v}_i(t) + \mathbf{e}_{\text{t},i}(t), \;\; \mathbf{v}_i(t) := \mathbf{V}_i(t)\mathbf{s}_i(t), \tag{3}$$

where $\mathbf{s}_i(t) \in \mathbb{C}^{d_i}, \mathbf{V}_i(t) \in \mathbb{C}^{N_i \times d_i}$, and $\mathbf{v}_i(t) \in \mathbb{C}^{N_i}$ respectively represent the vector of the data symbols, the transmit precoding matrix and the intended (undistorted) transmit signal, at the time instance t. The number of the data streams in direction i is denoted as d_i, and $\mathbb{E}\{\mathbf{s}_i(t)\mathbf{s}_i(t)^H\} = \mathbf{I}_{d_i}$. Moreover, the inaccurate behavior of the transmit chain elements is modeled as an additional distortion term $\mathbf{e}_{\text{t},i}(t)$ such that

$$\begin{aligned} &\mathbf{e}_{\text{t},i}(t) \sim \mathcal{CN}\left(\mathbf{0}, \kappa_i \text{diag}\left(\mathbb{E}\left\{\mathbf{v}_i(t)\mathbf{v}_i(t)^H\right\}\right)\right), \\ &\mathbf{e}_{\text{t},i}(t) \perp \mathbf{v}_i(t), \end{aligned} \tag{4}$$

where $\perp$ denotes the statistical independence and κ_i is the transmit distortion coefficient, see [7, Section II.C]. The received signal at the destination can be consequently written as

$$\mathbf{y}_i(t) = \underbrace{\mathbf{H}_{ii}\mathbf{x}_i(t) + \mathbf{H}_{ij}\mathbf{x}_j(t) + \mathbf{n}_i(t)}_{=:\mathbf{u}_i(t)} + \mathbf{e}_{\text{r},i}(t), \tag{5}$$

where $\mathbf{n}_i(t)$ is the additive thermal noise with variance $\sigma_{\text{n},i}^2$, $\mathbf{u}_i(t) \in \mathbb{C}^{M_i}$ is the undistorted received signal, and the additive distortion term $\mathbf{e}_{\text{r},i}(t)$ models the inaccuracies in the receive chains such that

$$\begin{aligned} &\mathbf{e}_{\text{r},i}(t) \sim \mathcal{CN}\left(\mathbf{0}, \beta_i \text{diag}\left(\mathbb{E}\left\{\mathbf{u}_i(t)\mathbf{u}_i(t)^H\right\}\right)\right), \\ &\mathbf{e}_{\text{r},i}(t) \perp \mathbf{u}_i(t), \end{aligned}$$

where β_i is the receiver chain distortion coefficient, see [7, Section II.D]. Please note that the distortion terms $\mathbf{e}_{\text{r},i}(t)$ and $\mathbf{e}_{\text{t},i}(t)$ model the combined effects of the chain inaccuracies, e.g., digital-to-analog and analog-to-digital converter error, power amplifier noise, oscillator phase noise and the automatic gain control noise at the respective chains. Hence, unlike the thermal noise components, the variances of the distortion terms depend on the power of the intended transmit/receive signal at each antenna and play an important role in an FD setup due to the strong self-interference path, see [7], [21] and the references therein. The *known* part of the self-interference signal, i.e, $\tilde{\mathbf{H}}_{ij}\mathbf{v}_j$, can be canceled at the receiver side, resulting in

$$\tilde{\mathbf{y}}_i(t) := \mathbf{y}_i(t) - \tilde{\mathbf{H}}_{ij}\mathbf{v}_j = \mathbf{H}_{ii}\mathbf{V}_i(t)\mathbf{s}_i(t) + \mathbf{n}_i(t) + \mathbf{m}_i(t). \tag{6}$$

where the combined effect of the interference signal in the direction i is given by

$$\mathbf{m}_i(t) = \mathbf{H}_{ij}\mathbf{e}_{\text{t},j}(t) + \mathbf{H}_{ii}\mathbf{e}_{\text{t},i}(t) + \mathbf{e}_{\text{r},i}(t) + \mathbf{\Delta}_{ij}\mathbf{V}_j(t)\mathbf{s}_j(t). \tag{7}$$

Finally, the estimated data vector is obtained at the receiver as

$$\tilde{\mathbf{s}}_i(t) = \mathbf{U}_i(t)^H \tilde{\mathbf{y}}_i(t), \tag{8}$$

where $\mathbf{U}_i \in \mathbb{C}^{M_i \times d_i}$ is the linear receive filter. Please note that the combined effect of the residual self-interference, does not necessarily follow a Gaussian distribution. Following [7], an approximation on the achievable spectral efficiency in the direction i is written as

$$I_i = \frac{1}{T}\sum_{t\in\mathcal{T}} \log_2 \left| \mathbf{I}_{M_i} + \mathbf{\Sigma}_i^{-1}(t)\mathbf{H}_{ii}\mathbf{V}_i(t)\mathbf{V}_i(t)^H\mathbf{H}_{ii}^H \right|, \tag{9}$$

where I_i is the achievable rate, and $\mathbf{\Sigma}_i(t)$ represents the combined interference-plus-noise covariance which will be later defined in (12).

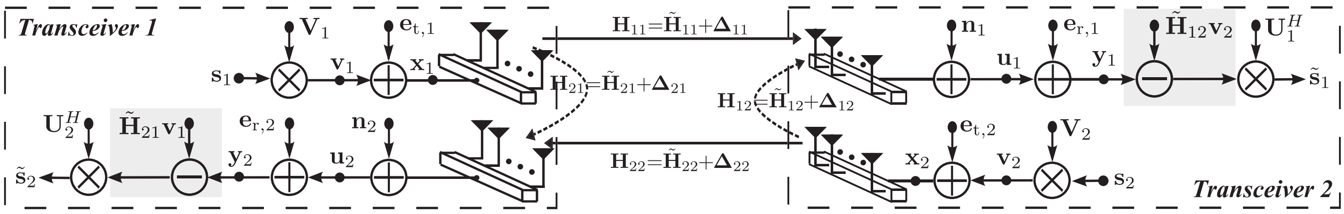

Figure 1. A full-duplex bi-directional system with multiple antennas. The communication quality suffers due to the additive white noise, i.e., $\mathbf{n}_i$, inaccuracies of transmit and receive chains, i.e., $\mathbf{e}_{\mathrm{t},i}$ and $\mathbf{e}_{\mathrm{r},i}$, as well as the CSI estimation error $\mathbf{\Delta}_{ij}$, $i,j \in \{1,2\}$.

A. Optimization Problem

Our goal is to obtain a design that guarantees the fulfillment of our pre-defined rate requirements, while consuming the minimum total power. Please note that the fulfillment of the rate constraints can only be guaranteed when the resulting communication rate remains above the rate demand for any feasible channel error, defined by (1). This is formulated as the following semi-infinite minimization

$$\min_{\mathcal{V}} \quad \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} \mathrm{tr}\left(\mathbf{V}_i(t)\mathbf{V}_i(t)^H\right) \tag{10a}$$

$$\text{s.t.} \quad I_i \geq R_{\mathrm{min},i}, \ \ \forall \mathbf{\Delta}_{ij} \in \mathbb{D}_{ij}, \ \ \forall i,j \in \mathcal{I}, \tag{10b}$$

Please note that the semi-infiniteness results from the rate constraint (10b) that needs to be satisfied for all feasible CSI error matrices. For notational simplicity we define $\mathcal{V} := \{\mathbf{V}_i(t), \ \forall i \in \mathcal{I}, \ t \in \mathcal{T}\}$. Moreover, $R_{\mathrm{min},i}$ is the required spectral efficiency in the direction i. In the following part we aim at providing a convex optimization framework for the above-defined problem, following the WMMSE method [17].

3. WMMSE Method for Worst-Case Rate Maximization

The MSE matrix for the communication direction i and time segment t is formulated as

$$\begin{aligned}\mathbf{E}_i(t) := \mathbb{E}&\left\{\left(\tilde{\mathbf{s}}_i(t) - \mathbf{s}_i(t)\right)\left(\tilde{\mathbf{s}}_i(t) - \mathbf{s}_i(t)\right)^H\right\} \\ =& \left(\mathbf{U}_i(t)^H\mathbf{H}_{ii}\mathbf{V}_i(t) - \mathbf{I}_{d_i}\right)\left(\mathbf{U}_i(t)^H\mathbf{H}_{ii}\mathbf{V}_i(t) - \mathbf{I}_{d_i}\right)^H \\ &+ \mathbf{U}_i(t)^H\mathbf{\Sigma}_i(t)\mathbf{U}_i(t),\end{aligned} \tag{11}$$

where $\mathbf{\Sigma}_i(t)$ is calculated as

$$\begin{aligned}\mathbf{\Sigma}_i(t) =& \mathbb{E}\left\{\left(\mathbf{n}_i(t) + \mathbf{m}_i(t)\right)\left(\mathbf{n}_i(t) + \mathbf{m}_i(t)\right)^H\right\} \\ =& \kappa_j\mathbf{H}_{ij}\mathrm{diag}\left(\mathbf{V}_j(t)\mathbf{V}_j(t)^H\right)\mathbf{H}_{ij}^H \\ &+ \beta_i\mathrm{diag}\left(\mathbf{H}_{ij}\mathbf{V}_j(t)\mathbf{V}_j(t)^H\mathbf{H}_{ij}^H\right) \\ &+ \kappa_i\mathbf{H}_{ii}\mathrm{diag}\left(\mathbf{V}_i(t)\mathbf{V}_i(t)^H\right)\mathbf{H}_{ii}^H \\ &+ \beta_i\mathrm{diag}\left(\mathbf{H}_{ii}\mathbf{V}_i(t)\mathbf{V}_i(t)^H\mathbf{H}_{ii}^H\right) \\ &+ \mathbf{\Delta}_{ij}\mathbf{V}_j(t)\mathbf{V}_j(t)^H\mathbf{\Delta}_{ij}^H + \sigma_{\mathrm{n},i}^2\mathbf{I}_{M_i}.\end{aligned} \tag{12}$$

The MMSE receive filter can be then calculated as

$$\mathbf{U}_i^{\mathrm{mmse}}(t) = \left(\mathbf{\Sigma}_i(t) + \mathbf{H}_{ii}\mathbf{V}_i(t)\mathbf{V}_i(t)^H\mathbf{H}_{ii}^H\right)^{-1}\mathbf{H}_{ii}\mathbf{V}_i(t), \tag{13}$$

and the resulting MSE matrix is obtained as

$$\mathbf{E}_i^{\mathrm{mmse}}(t) = \left(\mathbf{I}_{d_i} + \mathbf{V}_i(t)^H\mathbf{H}_{ii}^H\mathbf{\Sigma}_i(t)^{-1}\mathbf{H}_{ii}\mathbf{V}_i(t)\right)^{-1}, \tag{14}$$

which presents the useful relation to the rate function (9) such that

$$I_i = \frac{-1}{T}\sum_{t\in\mathcal{T}}\log_2\left|\mathbf{E}_i^{\mathrm{mmse}}(t)\right|, \tag{15}$$

see also [22, Eq. (9)] for similar identities.

Lemma 1. *Let $\mathbf{E} \in \mathbb{C}^{d\times d}$ be a positive semi-definite matrix. The maximization of the term $-log\,|\mathbf{E}|$ is equivalent to the maximization*

$$\max_{\mathbf{E},\mathbf{L}} - tr\left(\mathbf{LE}\right) + log\left|\mathbf{L}\right| + d, \tag{16}$$

where $\mathbf{L} \in \mathcal{H}_d$.

Proof: The proof is given in [23, Lemma 2]. ■

By recalling (11) and (9), utilizing Lemma 1, and decomposing $\mathbf{L} = \mathbf{W}_i\mathbf{W}_i^H, \mathbf{W}_i \succ 0$, the original optimization problem (10) can be equivalently formulated as

$$\min_{\mathcal{V}} \quad \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} \mathrm{tr}\left(\mathbf{V}_i(t)\mathbf{V}_i(t)^H\right) \tag{17a}$$

$$\begin{aligned}\text{s.t.} \ \min_{\mathbf{\Delta}} \max_{\mathcal{U},\mathcal{W}} &\left(d_i - \sum_{i\in\mathcal{T}}\mathrm{tr}\left(\mathbf{W}_i(t)^H\mathbf{E}_i(t)\mathbf{W}_i(t)\right)\right. \\ &\left. + 2\log\left|\mathbf{W}_i(t)\right|\right) \geq \log(2)TR_{\mathrm{min},i},\end{aligned} \tag{17b}$$

$$\mathbf{W}_i(t) \in \mathcal{H}_{d_i}, \ \ \mathbf{\Delta}_{ij} \in \mathbb{D}_{ij}, \ \ \forall i,j \in \mathcal{I}, \tag{17c}$$

where $\mathcal{U}$ is a set defined similar to that of $\mathcal{V}$, $\mathcal{W} := \{\mathbf{W}_i \in \mathcal{H}_{d_i} \ \forall i \in \mathcal{I}\}$ and $\mathbf{\Delta} := \{\mathbf{\Delta}_{ij}, \ \forall i,j \in \mathcal{I}\}$. Please note that the defined equivalent problem is still intractable due to the inner maximization in (17b). In order to cast our the problem into a tractable form, we follow the max-min inequality for (17b), see [23, Eq. (12)], which ensures the satisfaction of the rate constraints by replacing a lower bound. This is stated as[2]

$$\min_{\mathbf{\Delta}} \max_{\mathcal{U},\mathcal{W}}\left(\,\cdot\,\right) \ \geq \ \max_{\mathcal{U},\mathcal{W}} \min_{\mathbf{\Delta}}\left(\,\cdot\,\right) \ \geq \ R_{\mathrm{min},i}. \tag{18}$$

[2] It follows the observation that if a lower bound of the rate function satisfies the rate constraint for all CSI error matrices, the actual rate constraints are also satisfied.

Moreover we calculate

$$\sum_{t\in\mathcal{T}} \mathrm{tr}\left(\mathbf{W}_i(t)^H\mathbf{E}_i(t)\mathbf{W}_i(t)\right)$$
$$=\sum_{t\in\mathcal{T}}\Bigg(\left\|\mathbf{W}_i(t)^H\left(\mathbf{U}_i(t)^H\mathbf{H}_{ii}\mathbf{V}_i(t)-\mathbf{I}_{d_i}\right)\right\|_F^2 + \sum_{l\in\mathbb{F}_{N_j}}\kappa_j\left\|\mathbf{W}_i(t)^H\mathbf{U}_i(t)^H\mathbf{H}_{ij}\mathbf{\Gamma}_l\mathbf{V}_j(t)\right\|_F^2 + \sum_{l\in\mathbb{F}_{M_i}}\beta_i\left\|\mathbf{W}_i(t)^H\mathbf{U}_i(t)^H\mathbf{\Gamma}_l\mathbf{H}_{ij}\mathbf{V}_j(t)\right\|_F^2 + \sum_{l\in\mathbb{F}_{N_i}}\kappa_i\left\|\mathbf{W}_i(t)^H\mathbf{U}_i(t)^H\mathbf{H}_{ii}\mathbf{\Gamma}_l\mathbf{V}_i(t)\right\|_F^2 + \sum_{l\in\mathbb{F}_{M_i}}\beta_i\left\|\mathbf{W}_i(t)^H\mathbf{U}_i(t)^H\mathbf{\Gamma}_l\mathbf{H}_{ii}\mathbf{V}_i(t)\right\|_F^2 + \left\|\mathbf{W}_i(t)^H\mathbf{U}_i(t)^H\mathbf{\Delta}_{ij}\mathbf{V}_j(t)\right\|_F^2 + \sigma_{\mathrm{n},i}^2\left\|\mathbf{W}_i(t)^H\mathbf{U}_i(t)^H\right\|_F^2\Bigg) \tag{19}$$

$$=\boxed{\sum_{j\in\mathcal{I}}\|\mathbf{c}_{ij}+\mathbf{C}_{ij}\mathrm{vec}\left(\mathbf{\Delta}_{ij}\right)\|_2^2}, \tag{20}$$

where

$$\mathbf{c}_{ii} := \begin{bmatrix} \mathrm{vec}\left(\mathbf{W}_i(t)^H\left(\mathbf{U}_i(t)^H\tilde{\mathbf{H}}_{ii}\mathbf{V}_i(t)-\mathbf{I}_{d_i}\right)\right) \\ \left\lfloor\sqrt{\kappa_i}\mathrm{vec}\left(\mathbf{W}_i(t)^H\mathbf{U}_i(t)^H\tilde{\mathbf{H}}_{ii}\mathbf{\Gamma}_{N_i}^l\mathbf{V}_i(t)\right)\right\rfloor_{l\in\mathbb{F}_{N_i}} \\ \left\lfloor\sqrt{\beta_i}\mathrm{vec}\left(\mathbf{W}_i(t)^H\mathbf{U}_i(t)^H\mathbf{\Gamma}_{M_i}^l\tilde{\mathbf{H}}_{ii}\mathbf{V}_i(t)\right)\right\rfloor_{l\in\mathbb{F}_{M_i}} \\ \sigma_{\mathrm{n},i}\mathrm{vec}\left(\mathbf{W}_i(t)^H\mathbf{U}_i(t)^H\right)\end{bmatrix}_{t\in\mathcal{T}}, \tag{21}$$

$$\mathbf{C}_{ii} := \begin{bmatrix} \mathbf{V}_i(t)^T\otimes\left(\mathbf{W}_i(t)^H\mathbf{U}_i(t)^H\right) \\ \left\lfloor\sqrt{\kappa_i}\left(\mathbf{\Gamma}_{N_i}^l\mathbf{V}_i(t)\right)^T\otimes\left(\mathbf{W}_i(t)^H\mathbf{U}_i(t)^H\right)\right\rfloor_{l\in\mathbb{F}_{N_i}} \\ \left\lfloor\sqrt{\beta_i}\mathbf{V}_i^T\otimes\left(\mathbf{W}_i(t)^H\mathbf{U}_i(t)^H\mathbf{\Gamma}_{M_i}^l\right)\right\rfloor_{l\in\mathbb{F}_{M_i}} \\ \mathbf{0}_{M_id_i\times N_iM_i}\end{bmatrix}_{t\in\mathcal{T}}, \tag{22}$$

$$\mathbf{c}_{ij} \overset{i\neq j}{:=} \begin{bmatrix} \left\lfloor\sqrt{\kappa_j}\mathrm{vec}\left(\mathbf{W}_i(t)^H\mathbf{U}_i(t)^H\tilde{\mathbf{H}}_{ij}\mathbf{\Gamma}_{N_j}^l\mathbf{V}_j(t)\right)\right\rfloor_{l\in\mathbb{F}_{N_j}} \\ \left\lfloor\sqrt{\beta_i}\mathrm{vec}\left(\mathbf{W}_i(t)^H\mathbf{U}_i(t)^H\mathbf{\Gamma}_{M_i}^l\tilde{\mathbf{H}}_{ij}\mathbf{V}_j(t)\right)\right\rfloor_{l\in\mathbb{F}_{M_i}} \\ \mathbf{0}_{d_id_j\times 1}\end{bmatrix}_{t\in\mathcal{T}}, \tag{23}$$

$$\mathbf{C}_{ij} \overset{i\neq j}{:=} \begin{bmatrix} \left\lfloor\sqrt{\kappa_j}\left(\mathbf{\Gamma}_{N_j}^l\mathbf{V}_j(t)\right)^T\otimes\left(\mathbf{W}_i(t)^H\mathbf{U}_i(t)^H\right)\right\rfloor_{l\in\mathbb{F}_{N_j}} \\ \left\lfloor\sqrt{\beta_i}\mathbf{V}_j(t)^T\otimes\left(\mathbf{W}_i(t)^H\mathbf{U}_i(t)^H\mathbf{\Gamma}_{M_i}^l\right)\right\rfloor_{l\in\mathbb{F}_{M_i}} \\ \mathbf{V}_j(t)^T\otimes\left(\mathbf{W}_i(t)^H\mathbf{U}_i(t)^H\right)\end{bmatrix}_{t\in\mathcal{T}}, \tag{24}$$

where $\mathbf{c}_{ij}\in\mathbb{C}^{\tilde{d}_{ij}}, \mathbf{C}_{ij}\in\mathbb{C}^{\tilde{d}_{ij}\times M_iN_j}$ such that

$$\tilde{d}_{ii} = T\left(\left(1+N_i+M_i\right)d_i^2+M_id_i\right),\quad \forall i\in\mathcal{I},$$
$$\tilde{d}_{ij} = T\left(1+N_j+M_i\right)d_id_j,\quad \forall i,j\in\mathcal{I},\quad i\neq j,$$

and (21)-(24) are obtained by recalling (11) and (12), and following the known matrix equalities [24, Eq. (496), (516)]. The resulting problem can be hence written as

$$\min_{\mathcal{V},\mathcal{U},\mathcal{W},\tau}\ \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\mathrm{tr}\Big(\mathbf{V}_i(t)\mathbf{V}_i(t)^H\Big) \tag{25a}$$
$$\text{s.t.}\ \sum_{t\in\mathcal{T}}\left(d_i+2\log\left|\mathbf{W}_i(t)\right|\right)-\sum_{j\in\mathcal{I}}\tau_{ij}\geq\log(2)TR_{\min,i}, \tag{25b}$$
$$\Big(\max_{\mathbf{\Delta}_{ij}\in\mathbb{D}_{ij}}\ \|\mathbf{c}_{ij}+\mathbf{C}_{ij}\mathrm{vec}\left(\mathbf{\Delta}_{ij}\right)\|_2^2\Big)\leq\tau_{ij},\ \forall i,j\in\mathcal{I}, \tag{25c}$$

where $\tau := \{\tau_{ij},\ \forall i,j\in\mathcal{I}\}$. By applying the Schur's complement lemma, the norm constraint in (25c) is equivalently written as

$$\begin{bmatrix} 0 & \mathrm{vec}\left(\mathbf{\Delta}_{ij}\right)^H\mathbf{C}_{ij}^H \\ \mathbf{C}_{ij}\mathrm{vec}\left(\mathbf{\Delta}_{ij}\right) & \mathbf{0}_{\tilde{d}_{ij}\times\tilde{d}_{ij}}\end{bmatrix}+\begin{bmatrix}\tau_{ij} & \mathbf{c}_{ij}^H \\ \mathbf{c}_{ij} & \mathbf{I}_{\tilde{d}_{ij}}\end{bmatrix}\succeq 0,$$
$$\forall\mathbf{\Delta}_{ij}:\ \ \|\mathrm{vec}\left(\mathbf{\Delta}_{ij}\right)\|_2\leq\zeta_{ij},\ \ \forall i,j\in\mathcal{I}, \tag{26}$$

where $\tilde{d}_{ij}$ is equal to the size of $\mathbf{c}_{ij}$.

Lemma 2. *Generalized Petersen's sign-definiteness lemma: Let* $\mathbf{Y}=\mathbf{Y}^H$*, and* $\mathbf{X},\mathbf{P},\mathbf{Q}$ *are arbitrary matrices with complex valued elements. Then we have*

$$\mathbf{Y}\succeq\mathbf{P}^H\mathbf{X}\mathbf{Q}+\mathbf{Q}^H\mathbf{X}^H\mathbf{P},\ \ \forall\mathbf{X}\ :\ \|\mathbf{X}\|_F\leq\zeta, \tag{27}$$

if and only if

$$\exists\lambda\geq 0,\ \begin{bmatrix}\mathbf{Y}-\lambda\mathbf{Q}^H\mathbf{Q} & -\zeta\mathbf{P}^H \\ -\zeta\mathbf{P} & \lambda\mathbf{I}\end{bmatrix}\succeq 0. \tag{28}$$

Proof: See [25, Proposition 2], [26]. ■

By choosing the matrices in Lemma 2 such that $\mathbf{X}=\mathrm{vec}\left(\mathbf{\Delta}_{ij}\right)$, $\mathbf{Q}=\Big[-1,\ \mathbf{0}_{1\times\tilde{d}_{ij}},\Big]$ and

$$\mathbf{Y}=\begin{bmatrix}\tau_{ij} & \mathbf{c}_{ij}^H \\ \mathbf{c}_{ij} & \mathbf{I}_{\tilde{d}_{ij}}\end{bmatrix},\mathbf{P}=\begin{bmatrix}\mathbf{0}_{M_iN_j\times 1}, & \mathbf{C}_{ij}^H\end{bmatrix}, \tag{29}$$

the optimization problem in (25) is equivalently written as

$$\min_{\mathcal{V},\mathcal{U},\mathcal{W},\tau,\lambda}\ \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\mathrm{tr}\Big(\mathbf{V}_i(t)\mathbf{V}_i(t)^H\Big) \tag{30a}$$
$$\text{s.t.}\ \underbrace{\begin{bmatrix}\tau_{ij}-\lambda_{ij} & \mathbf{c}_{ij}^H & \mathbf{0}_{1\times M_iN_j} \\ \mathbf{c}_{ij} & \mathbf{I}_{\tilde{d}_{ij}} & -\zeta_{ij}\mathbf{C}_{ij} \\ \mathbf{0}_{M_iN_j\times 1} & -\zeta_{ij}\mathbf{C}_{ij}^H & \lambda_{ij}\mathbf{I}_{M_iN_j}\end{bmatrix}}_{\mathbf{\Theta}_{ij}}\succeq 0,$$
$$\forall i,j\in\mathcal{I}, \tag{30b}$$

where $\lambda := \{\lambda_{ij},\ \forall i,j\in\mathcal{I}\}$. This is equivalently written as

$$\min_{\mathcal{V},\mathcal{U},\mathcal{W},\tau,\lambda,p_{\text{tot}}}\ p_{\text{tot}} \tag{31a}$$
$$\text{s.t.}\ \mathbf{\Theta}_{ij}\succeq 0,\ \ \forall i,j\in\mathcal{I}, \tag{31b}$$
$$\begin{bmatrix}p_{\text{tot}} & \mathbf{b}^H \\ \mathbf{b} & \mathbf{I}\end{bmatrix}\succeq 0, \tag{31c}$$

where

$$\mathbf{b} = \left\lfloor \lfloor \text{vec}\left(\mathbf{V}_i\left(t\right)\right) \rfloor_{i \in \mathcal{I}} \right\rfloor_{t \in \mathcal{T}}. \tag{32}$$

It can be observed that (31) holds a separately convex structure. This facilitates an iterative optimization over separated variable sets, where in each iteration a convex sub-problem is solved. In this regard, the minimization over $\mathcal{V}, \mathcal{U}$ can be separately cast as a general SDP, where the optimization over $\mathcal{W}$ can be efficiently implemented using the MAX-DET algorithm [27]. Moreover, due to the monotonic increase of the objective in each optimization iteration, and the fact that the consumed sum-power is necessarily non-negative, i.e., bounded from below, the algorithm convergences to a stationary point. The convergence behavior of the proposed algorithm is numerically evaluated in Fig. 2.

A. Computational Complexity

As mentioned, the power minimization over $\mathcal{V}, \mathcal{U}$ are cast as variations of SDP problems. Regarding the calculation of $\mathcal{W}$, the MAX-DET algorithm is used, which leads to an SDP as a special case. A general SDP problem is defined as

$$\min_{\mathbf{x}}, \;\; \mathbf{c}^T\mathbf{x} \;\; \text{s.t.} \;\; \|\mathbf{x}\|_2 \leq R, \mathbf{x} \in \mathcal{R}^n, \mathbf{A}_0 + \sum_{j=1}^{n} x_j \mathbf{A}_j \succeq 0, \tag{33}$$

where the fixed matrices $\mathbf{A}_j$ are symmetric block-diagonal, with K diagonal blocks of the sizes $l_k \times l_k$, $k \in 1, \ldots, K$ and define the specific problem structure, see [28, Subsection 4.6.3]. The arithmetic complexity of ϵ-solution, i.e., the convergence to the ϵ-distance vicinity of the optimum solution for the defined SDP problem is obtained as

$$\mathcal{O}(1)\left(1 + \sum_{k=1}^{K} a_K\right)^{\frac{1}{2}} \left(n^3 + n^2 \sum_{k=1}^{K} a_k^2 + n \sum_{k=1}^{K} a_k^3\right) \text{digit}\left(\epsilon\right) \tag{34}$$

where digit(ϵ) is obtained from [28, Subsection 4.6.3], and affected by the required solution precision. The required computation of each step is hence determined by size of the variable space, and the corresponding block diagonal matrix structure.

1) Calculation of $\mathcal{V}, \tau, \lambda$: The size of the variable space is given as $n = 1 + 4|\mathcal{I}| + 2T \sum_{i \in \mathcal{I}} N_i d_i$, $K = 2|\mathcal{I}| + 1$, $a_k = 2\left(1 + \tilde{d}_{ij} + M_i N_j\right)$, $\forall i, j$, corresponding to (31b), and $a_k = 2 + 2T \sum_i N_i d_i$ corresponding to (31c).

2) Calculation of $\mathcal{U}, \tau, \lambda$: The size of the variable space is given as $n = 1 + 4|\mathcal{I}| + 2T \sum_{i \in \mathcal{I}} M_i d_i$, $K = 2|\mathcal{I}|$, $a_k = 2\left(1 + \tilde{d}_{ij} + M_i N_j\right)$, $\forall i, j$.

3) Calculation of $\mathcal{W}, \tau, \lambda$: Similar to the last parts we calculate $n = 1 + 4|\mathcal{I}| + 2T \sum_{i \in \mathcal{I}} d_i^2$, $K = 2|\mathcal{I}|$, $a_k = 2\left(1 + \tilde{d}_{ij} + M_i N_j\right)$, $\forall i, j$.

Please note that the discussed complexity refers to the required arithmetic complexity of each optimization step. The overall arithmetic complexity is also impacted by the number of necessary optimization iterations, which is numerically studied in Section 4.

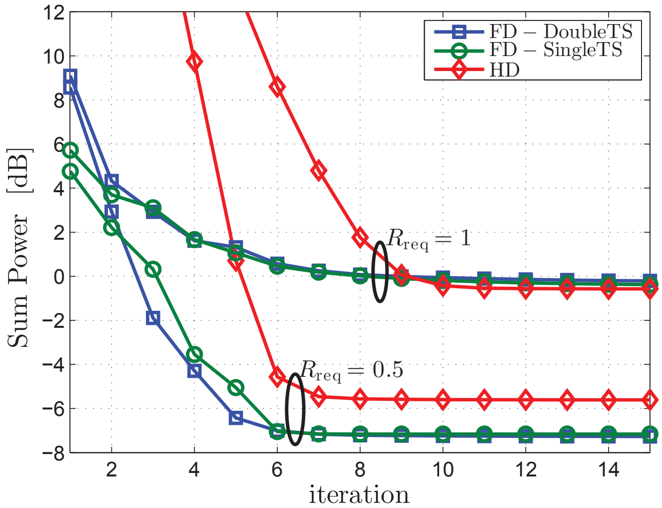

Figure 2. Average convergence behavior of the proposed iterative method.

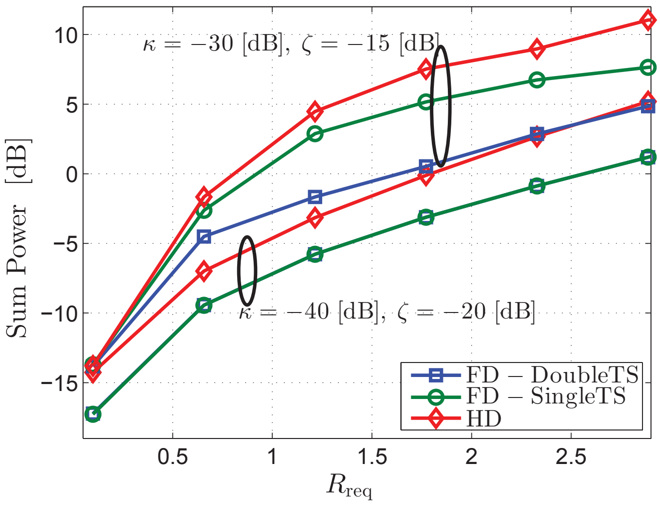

Figure 3. Sum power consumption vs. required communication rate R_{req}. Higher rate demand results in a higher system power consumption.

4. Simulation Results

In this section we numerically evaluate the resulting sum power consumption of the defined FD bi-directional system, using the proposed design, where a required communication rate in each direction needs to be guaranteed. We consider the case that $\mathbf{H}_{ii}$ follows an uncorrelated Rayleigh flat-fading distribution, with variance 0.01 for each element and $\mathbf{H}_{ij} \sim \mathcal{CN}\left(\sqrt{\frac{K_R}{1+K_R}}\mathbf{H}_0, \frac{1}{1+K_R}\mathbf{I}_{N_j} \otimes \mathbf{I}_{M_i}\right)$, $i \neq j$. $\mathbf{H}_0$ is a matrix with all elements equal to 1 and K_R is the Rician coefficient. Unless otherwise stated, we use the following values to the define our default setup: $N_j = M_i = 2$, $K_R = 1$, $\kappa_i = \beta_j = \kappa = 0.001$, $\sigma_{\text{n},i}^2 = 0.1$, $\mathbf{D}_{ij} = \mathbf{I}$, $R_{\text{req},i} = R_{\text{req}} = 1$, $\zeta_{ij} = \zeta = -15$ dB, $\forall i, j$.

In Fig. 2-3 the consumed sum power of the system is evaluated. In this respect, 'FD-DoubleTS' and 'FD-SingleTS' respectively represent the proposed design in Section 3, with $T = 2$, i.e., representing a TDD-capable FD system, and with $T = 1$, i.e, an FD setup with no temporal diversity. Moreover, 'HD' represents the performance of an equivalent HD setup.

In Fig. 2 the average convergence behavior of the proposed iterative method is depicted. As it is observed, the convergence is obtained within 10-20 optimization iterations, which verifies the efficiency of the proposed iterative algorithm in terms of the required computational effort.

In Fig. 3 the sum power consumption of the system is evaluated for different values of rate requirements. It is observed that higher required communication rate results in a higher system power consumption. On the other hand, while the application of a TDD-capable setup appears to be gainful for all cases, a significant gain is observed where the system suffers from a lower dynamic range.

5. Conclusion

While the application of bi-directional FD communication paradigm presents a potential for improving the spectral efficiency, such systems are limited due to the imperfect self-interference cancellation. In this work we have presented a design with sum power minimization, where a given set of rate requirements are satisfied. It is observed that the consideration of both CSI error, as well as the impact of hardware impairments are necessary in obtaining a reliable design. Moreover, our design is generalized to a TDD-enabled FD system with the backwards compatibility to the traditional HD setup. This capability is observed to be essential, especially as the transceiver accuracy decreases or as the required communication rate increases.

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