Optimal Power Allocation for Amplify and Forward Relaying with Finite Blocklength Codes and QoS Constraints

(Invited Paper)

Yulin Hu[1], M. Cenk Gursoy[2] and Anke Schmeink[1]
[1]Information Theory and Systematic Design of Communication Systems, RWTH Aachen University
52062 Aachen, Germany. Email: $hu|schmeink$@umic.rwth-aachen.de
[2]Department of Electrical Engineering and Computer Science, Syracuse University,
Syracuse, NY 13244, USA. E-mail: $mcgursoy$@syr.edu

Abstract

In this work, motivated by emerging low-latency applications, we consider an amplify-and-forward relaying network operating with finite blocklength (FBL) codes subject to delay quality of service (QoS) constraints. Hence, we address both transmission delay (via FBL codes) and queueing delay (via delay QoS requirements). We first derive the QoS-constrained throughput of the network. Subsequently, we state a resource allocation problem aiming at allocating the power between the source and the relay to maximize the throughput. The convexity of the problem is proved and the optimal power allocation policy is provided. Via simulations, we confirm the accurateness of our analytical model. In addition, we provide interesting insights on the system behavior by characterizing the impact of the error probability, the QoS-exponent and coding blocklength on the throughput performance.

Index Terms

Amplify-and-forward relaying, finite blocklength, power allocation, QoS, throughput.

I. Introduction

In wireless communications, cooperative relaying [1]–[4] is a promising technique to improve the wireless transmission performance by exploiting spatial diversity and providing better channel quality. Specifically, two-hop amplify-and-forward (AF) relaying protocols significantly improve the throughput and quality of service [5]–[7]. However, typically these studies on the advantage of relaying are performed under the ideal assumption of communicating arbitrarily reliably at Shannon's channel capacity, i.e., codewords are assumed to be infinitely long, which can be prohibitive in low-latency applications with deadline requirements.

In the finite blocklength regime, the data transmission is no longer arbitrarily reliable. Especially when the blocklength is short, the error probability (due to noise) becomes significant even if the rate is selected below the Shannon limit. Taking this into account, an accurate approximation of the achievable coding rate under the finite blocklength assumption for an additive white Gaussian noise (AWGN) channel was derived in [8] for a single-hop transmission system. Later on, a tighter approximation is provided in [9] with a third-order term. In addition, the initial work for AWGN channels was extended to Gilbert-Elliott channels [10] as well as quasi-static fading channels [11]–[13], quasi-static fading channels with retransmissions [14], [15], spectrum sharing networks [16] as well as transmissions with packet scheduling [17], [18]. It is shown in these works that the finite blocklength performance of a single-hop transmission is determined by the coding rate, error probability and code blocklength. In particular, the performance loss due to the additional decoding errors at finite blocklength is considerable and increases as the blocklength decreases. Also, if the channel and the blocklength are fixed, the error probability of the single-hop transmission is strictly increasing in the coding rate. In our previous work [19]–[23], we extended Polyanskiy's model of single-hop transmission to the relaying network, in which the relay halves the distance to provide a power gain but at the same time also halves the blocklength of the transmission. In particular, general analytical models of the physical-layer finite blocklength (FBL) performance [20], [21] and QoS-constrained performance [22], [23] of DF relaying networks are derived. However, all the above studies regarding the FBL performance of relaying are conducted under the assumption of decode and forward relaying. To the best of our knowledge, the FBL performance and optimal operation in AF relaying networks have not been addressed thus far.

In this work, we focus on an AF relaying network subject to delay QoS constraints and derive the FBL performance. In addition, we identify the optimal power allocation between the source and the relay to maximize the QoS-constrained FBL throughput. In particular, we first formulate a power allocation problem, then show its convexity and finally solve the problem. Moreover, we validate our analytical model and further investigate the throughput performance of the network via Monte Carlo simulations.

The rest of the paper is organized as follows. In Section II, we provide our system model and introduce the metric adopted to measure the QoS-constrained performance of the AF network. In Section III, we derive the QoS-constrained FBL throughput of AF relaying and state the optimization problem. We show the convexity of the problem and provide a solution in Section IV. Finally, we present our numerical results in Section V and conclude the work in Section VI.

II. Preliminaries

In this section, we first describe our system model. Subsequently, the statistical queuing constraints are briefly discussed and reviewed.

A. System model

We consider a simple scenario with a source S, a destination D and a relay R as schematically shown in Fig. 1 below. Time is divided into frames with length $2m$ (symbols), each frame is further divided into two phases which are referred to as backhaul phase and relaying phase. The relay is assumed to operate in an AF mode. In particular, the source sends a signal with data block to the relay. Afterwards, the relay amplifies and forwards the received signal (with noise) to the destination. Hence, the lengths of the backhaul phase and the relaying phase (therefore their coding blocklengths) are required to be the same, i.e., the blocklength of both the backhaul phase and the relaying phase is $m_1 = m_2 = m$.

Fig. 1. Example of the considered relaying scenario.

We consider complex channels and denote the channel fading coefficients of the S-R backhaul link and the R-D relaying link by h_1 and h_2, respectively. We assume perfect channel state information (CSI) at the receivers and in particular at the source. In addition, the transmit power levels at the source and the relay are denoted by p_1 and p_2, respectively. Hence, for AF relaying the signals at the relay and the destination are given, respectively, by

$$\mathbf{y}_1 = \sqrt{p_1} h_1 \mathbf{x} + \mathbf{n}_1, \tag{1}$$

$$\mathbf{y}_2 = \sqrt{p_2} h_2 (\sqrt{p_1} h_1 \mathbf{x} + \mathbf{n_1}) + \mathbf{n_2}. \tag{2}$$

The transmitted signal $\mathbf{x}$ and received signals $\mathbf{y}_1$ and $\mathbf{y}_2$ are m-dimensional vectors. Furthermore, $\mathbf{n}_k, k = 1, 2$ represents the Gaussian noise vector $\mathbf{n}_k \sim \mathcal{N}\left(\mathbf{0}, \sigma^\mathbf{2} \mathbf{I_m}\right)$, where $\mathbf{I}_m$ denotes an $m \times m$ identity matrix. Denote by z_1 and z_2 the instantaneous channel gains of the backhaul and relaying links, i.e., $z_1 = |h_1|^2$ and $z_2 = |h_2|^2$. Then, the SNR of received signal at the AF relay is $\gamma_1 = \frac{p_1 z_1}{\sigma^2}$ while the SNR at the destination is given by

$$\gamma_2 = \frac{p_1 p_2 z_1 z_2}{\sigma^2 \left(1 + p_2 z_2\right)}. \tag{3}$$

Moreover, a total power constraint is considered[1], given by $p_1 + p_2 = p_{\mathrm{tot}}$, while the average transmit power of the two-hop relaying is $p_{\mathrm{tot}}/2$. Finally, the AF relay network is expected to support the transmission of the source node under certain QoS requirements related to reliability guarantees and queuing constraints.

B. Statistical queuing constraints

Throughout this paper, we assume that the transmissions to the destination are performed under queuing constraints, which require the buffer overflow probabilities to decay exponentially fast [24]. Let us denote Q as the stationary queue length and θ as the decay rate of the tail of the distribution of the queue length Q. Then, the probability that the queue length Q exceeds a threshold q satisfies

$$P\left(Q \geq q\right) \approx \varsigma e^{-\theta q}, \tag{4}$$

where ς is probability of non-empty buffer. In addition, θ is called the QoS exponent, and is defined in [25] as

$$\lim_{q \to \infty} \frac{\log P\left(Q \geq q\right)}{q} = -\theta. \tag{5}$$

Note that small and large θ correspond to relatively loose and strict QoS constraints, respectively. More specifically, QoS exponent θ controls the exponential decay rate of the buffer overflow probability. Thus larger θ indicates stricter limitation on the buffer overflow probability, leading to more stringent QoS constraints, and vice versa for small θ.

According to [24], the effective capacity in bits/frame is given by

$$R_{\mathrm{E}}(\theta) = -\frac{\Lambda_c\left(-\theta\right)}{\theta}, \tag{6}$$

where $\Lambda_c\left(\theta\right) = \lim\limits_{t \to \infty} \log \mathbb{E}\{e^{\theta \sum_{k=1}^{t} c_k}\}$ denotes the asymptotic logarithmic moment generating function (LMGF) of the random process c_k, which describes the instantaneous transmission rate in our setting. The effective capacity characterizes the maximum constant arrival rate that can be supported by the link with a random service process while satisfying (4).

[1]Note that while the source and relay nodes can have separate power sources, jointly deciding on the power levels subject to a total power constraint improves the throughput performance along with the power efficiency of the relay network.

In this work, we adopt the effective capacity formulation to obtain the QoS-constrained throughput in the considered AF relaying network with FBL codes.

III. AF relaying with QoS constraints: FBL throughput and optimal power allocation

A. Throughput of AF relaying with target error probability

Note that we consider an AF relaying network with queuing and reliability constraints described by a QoS exponent θ and a target error probability ε, respectively. First of all, the coding rate should be adapted according to instantaneous channel fading to guarantee the target error probability ε. According to the results in [9], the relationship between the coding rate and error probability is given by

$$r = \mathcal{R}\left(\gamma, \varepsilon, m\right) = \log(1+\gamma) - \log e \sqrt{\frac{\gamma(\gamma+2)}{(\gamma+1)^2 m}} Q^{-1}\left(\varepsilon\right) + \frac{\log m}{m} + \frac{o(1)}{m}, \tag{7}$$

where $Q\left(x\right) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ is the Gaussian Q-function. In other words, the instantaneous departure rate in bits/block from the source buffer of each frame is rm with probability $1-\varepsilon$, while with probability ε the departure rate is zero. Then, the LMGF of the random departure is given by $\ln\left\{\mathbb{E}_{z_1,z_2}\left[e^{-\theta m r}(1-\varepsilon)+\varepsilon\right]\right\}$. Finally, the QoS-constrained FBL throughput (i.e., effective capacity) in bits/symbol in this scenario is given by

$$\begin{aligned} R_{\mathrm{E}} &= -\frac{1}{2m}\frac{\Lambda_c\left(-\theta\right)}{\theta} \\ &= -\frac{1}{2m\theta}\ln\left\{\mathbb{E}_{z_1,z_2}\left[e^{-\theta m r}(1-\varepsilon)+\varepsilon\right]\right\}. \end{aligned} \tag{8}$$

Obviously, both the coding rate r and throughput R_{E} are influenced by the choice of power allocation p_1 and p_2.

B. Power allocation problem statement

Recall that we consider an AF relaying network with QoS constraints, i.e., the delay QoS exponent θ and target error probability ε. Our objective is to maximize the throughput R_{E} in bits/symbol. In addition, the service to the destination requires a basic guarantee that the channel states of the two hops are sufficiently good. This requirement in terms of received SNR at the destination is reflected by the condition that $\gamma_2 \geq \gamma_{\mathrm{th}} \geq 0$ dB, while the equivalent requirement in terms of coding rate is $r \geq \mathcal{R}\left(\gamma_{\mathrm{th}}, \varepsilon, m\right)$. On the other hand, due to the randomness of the channel fading, it is possible that the maximal SNR over power allocation is still not enough, i.e., $\gamma_2^\circ = \max\limits_{p_1+p_2=p_{\mathrm{tot}}} \gamma_2 \leq \gamma_{\mathrm{th}}$. In such a case, we simply skip this frame, i.e., allocate zero power for both the relay and the source in this frame.

Based on the above analysis, the problem of optimally allocating power among the source and the AF relay is stated as follows:

$$\begin{aligned} \max_{(p_1,p_2)\in\Omega} \quad & R_{\mathrm{E}}(p_1,p_2) \\ s.t.: \quad & p_1 + p_2 = p_{\mathrm{tot}}, \end{aligned} \tag{9}$$

where Ω is the feasible set of (p_1,p_2), given by

$$\Omega = \begin{cases} (p_1,p_2) = (0,0) & \text{case } \gamma_2^\circ < \gamma_{\mathrm{th}}, \\ (p_1,p_2) \in (0,p_{\mathrm{tot}})^2 & \text{case } \gamma_2^\circ \geq \gamma_{\mathrm{th}}. \end{cases} \tag{10}$$

IV. Optimal power allocation

In this section, we characterize the an optimal power allocation policy by solving the optimization problem in (9). In the following, we show the convexity of Problem (9). We start with a proposition regarding the coding rate.

Proposition 1. *Consider AF relaying with target error probability $\varepsilon \geq 10^{-27}$ and blocklength $m \geq 100$. Under the SNR constraint $\gamma_2 \geq 0$ dB and a given power constraint p_{tot}, i.e., $p_1 + p_2 = p_{\mathrm{tot}}$, the instantaneous coding rate r is strictly increasing and concave in γ_2 and concave in p_2.*

Proof: Let $\varphi = Q^{-1}\left(\varepsilon\right)\sqrt{\frac{1}{m}}$. Then, according to (7), we have the first order derivative of r with respect to the SNR γ_2 given as

$$\frac{\partial r}{\partial \gamma_2} = \frac{\log e}{1+\gamma_2} - \frac{\varepsilon \log e}{\sqrt{\left(1 - \frac{1}{(1+\gamma_2)^2}\right)}} \frac{1}{(1+\gamma_2)^3}. \tag{11}$$

It clearly holds that $\frac{\partial r}{\partial \gamma_2} > 0$ when $\gamma_2 \geq 1 = 0$ dB. Then, the second order derivative is given by

$$\begin{aligned}
\frac{\partial^2 r}{\partial {\gamma_2}^2} &= -\frac{\log e}{(1+\gamma_2)^2} + \frac{\varphi \log e}{2\left(1-\frac{1}{(1+\gamma_2)^2}\right)^{\frac{3}{2}}} \frac{1}{(1+\gamma_2)^6} + \frac{\varphi \log e}{\sqrt{\left(1-\frac{1}{(1+\gamma_2)^2}\right)}} \frac{3}{(1+\gamma_2)^4} \\
&= -\frac{\log e}{(1+\gamma_2)^2} + \frac{\varphi \log e}{2\left((1+\gamma_2)^2-1\right)^{\frac{3}{2}}} \frac{1}{(1+\gamma_2)^3} + \frac{\varphi \log e}{\sqrt{(1+\gamma_2)^2-1}} \frac{3}{(1+\gamma_2)^3} \\
&= \frac{\log e}{(1+\gamma_2)^3} \left\{ -\left(1+\gamma_2\right) + \frac{\varphi}{2({\gamma_2}^2+2\gamma_2)^{\frac{3}{2}}} + \frac{3\varphi}{\sqrt{{\gamma_2}^2+2\gamma_2}} \right\}.
\end{aligned} \tag{12}$$

When $\varepsilon \geq 0.5$, we have $\varphi \leq 0$. Then $\frac{\partial^2 r}{\partial {\gamma_2}^2} < 0$. On the other hand, when $\varepsilon < 0.5$, $\frac{\partial^2 r}{\partial {\gamma_2}^2}$ is increasing in φ and therefore decreasing in ε and m. For an extreme scenario where $m = 100, \varepsilon = 10^{-27}$, we have $\varphi = 1.1058$. Then, $\frac{\partial^2 r}{\partial {\gamma_2}^2} \leq 0$ if $\phi\left(\gamma_2\right) \leq 0$, where $\phi\left(\gamma_2\right) = -\left(1+\gamma_2\right) + \frac{\varphi}{2({\gamma_2}^2+2\gamma)^{\frac{3}{2}}} + \frac{3\varphi}{\sqrt{{\gamma_2}^2+2\gamma_2}}$. Obviously, $\phi\left(\gamma_2\right)$ is decreasing in γ_2 for $\varphi > 0$. In particular, we have $\phi\left(1\right) = -0.0164$ for $\varphi = 1.1058$. Hence, $\phi\left(\gamma_2\right) < 0$ for $\gamma_2 \geq 1 = 0$ dB. Therefore, r is concave in γ_2 under the constraint guaranteeing $\gamma_2 \geq 0$ dB while the values of the target error probability and blocklength are within practical interest, i.e., $m \geq 100, \varepsilon \geq 10^{-27}$.

On the other hand, by inserting $p_1 = p_{\rm tot} - p_2$ into (3), we have

$$\begin{aligned}
\gamma_2 = \frac{\left(p_{\rm tot} - p_2\right) p_2 z_1 z_2}{\sigma^2\left(1+p_2 z_2\right)} &= \frac{z_1}{\sigma^2} \frac{P_{\rm tot} z_2 p_2 - p_2^2 z_2}{1+p_2 z_2} \\
&= \frac{z_1}{\sigma^2} \frac{P_{\rm tot} z_2 p_2 + \frac{1}{z_2} + 2p_2 - \left(p_2^2 z_2 + 2p_2 + \frac{1}{z_2}\right)}{1+p_2 z_2} \\
&= \frac{z_1}{\sigma^2} \frac{p_{\rm tot} z_2 p_2 + \frac{1}{z_2} + 2p_2 - \frac{1}{z_2}\left(p_2 z_2 + 1\right)^2}{1+p_2 z_2} \\
&= \frac{z_1}{\sigma^2 z_2} \frac{\left(p_{\rm tot} z_2 + 2\right) p_2 z_2 + 1}{1+p_2 z_2} - \frac{z_1}{z_2 \sigma^2}\left(p_2 z_2 + 1\right) \\
&= \frac{z_1}{\sigma^2 z_2}\left(p_{\rm tot} z_2 + 2\right) - \frac{z_1}{\sigma^2 z_2} \frac{p_{\rm tot} z_2 + 1}{1+p_2 z_2} - \frac{z_1}{z_2 \sigma^2}\left(p_2 z_2 + 1\right).
\end{aligned} \tag{13}$$

Then, we have the first and second order derivatives of γ_2 with respect to p_2 as

$$\frac{\partial \gamma_2}{\partial p_2} = \frac{z_1}{\sigma^2} \frac{p_{\rm tot} z_2 + 1}{\left(1+p_2 z_2\right)^2} - \frac{z_1}{\sigma^2}, \tag{14}$$

$$\frac{\partial^2 \gamma_2}{\partial p_2^2} = -\frac{2 z_1 z_2}{\sigma^2} \frac{p_{\rm tot} z_2 + 1}{\left(1+p_2 z_2\right)^3} \leq 0. \tag{15}$$

Finally, under the constraint guaranteeing $\gamma_2 \geq 0$ dB while the target error probability and blocklength have values within practical interest, i.e., $m \geq 100, \varepsilon \geq 10^{-27}$, we have

$$\frac{\partial^2 r}{\partial p_2^2} = \underbrace{\frac{\partial^2 r}{\partial \gamma_2^2}}_{\leq 0} \underbrace{\left(\frac{\partial \gamma_2}{\partial p_2}\right)^2}_{\geq 0} + \underbrace{\frac{\partial r}{\partial \gamma_2}}_{\geq 0} \underbrace{\frac{\partial^2 \gamma_2}{\partial p_2^2}}_{\leq 0} \leq 0. \tag{16}$$

■

Based on the above statements, we have the following characterization of Problem (9)

Proposition 2. *Problem* (9) *is a convex optimization problem, when the target error probability and coding blocklength satisfy* $\varepsilon \geq 10^{-27}$ *and* $m \geq 100$*.*

Proof: According to (8), we have

$$\frac{\partial R_{\rm E}}{\partial r} = \frac{e^{-\theta m r}(1-\varepsilon)}{2\mathbb{E}\left[e^{-\theta m r}(1-\varepsilon)+\varepsilon\right]} \geq 0, \tag{17}$$

$$\frac{\partial^2 R_{\rm E}}{\partial r^2} = \frac{-\theta m e^{-\theta m r}(1-\varepsilon)\varepsilon}{2\mathbb{E}[e^{-\theta m r}(1-\varepsilon)+\varepsilon]^2} \leq 0. \tag{18}$$

By replacing p_1 by $p_{\rm tot} - p_2$, Problem (9) can be reformulated by

$$\max_{(p_1,p_2)\in\Omega} \quad R_{\rm E}(p_2) \tag{19}$$

Then, we have the second order derivatives of R_{E} with respect to p_2 given as follows:

$$\frac{\partial^2 R_{\mathrm{E}}}{\partial {p_2}^2} = \underbrace{\frac{\partial^2 R_{\mathrm{E}}}{\partial r^2}}_{\leq 0} \underbrace{\left(\frac{\partial r}{\partial p_2}\right)^2}_{\leq 0} + \underbrace{\frac{\partial R_{\mathrm{E}}}{\partial r}}_{\geq 0} \underbrace{\frac{\partial^2 r}{\partial {p_2}^2}}_{\leq 0} \leq 0. \tag{20}$$

We have used in the above result that under the constraint guaranteeing $\gamma \geq 0$ dB, we have $\frac{\partial^2 r}{\partial p_i^2} \leq 0$ according to Proposition 1. Hence, R_{E} is concave in p_2 when $\gamma_2 \geq 0$ dB. As a result, Problem (19) as well as the original problem in (9) are convex optimization problems. ■

So far, we have shown the convexity of the considered power allocation problem. In the following, we further study the optimal power allocation policy.

Let $\frac{\partial \gamma_2}{\partial p_2}$ be equal to zero. Then, we have

$$\begin{aligned} &\frac{\partial \gamma_2}{\partial p_2} = \frac{z_1}{\sigma^2} \frac{p_{\mathrm{tot}} z_2 + 1}{(1 + p_2 z_2)^2} - \frac{z_1}{\sigma^2} = 0 \\ &\Leftrightarrow p_{\mathrm{tot}} z_2 + 1 = (1 + p_2 z_2)^2 \\ &\Leftrightarrow \sqrt{p_{\mathrm{tot}} z_2 + 1} - 1 = p_2 z_2 \\ &\Leftrightarrow p_2^{\circ} = \frac{\sqrt{p_{\mathrm{tot}} z_2 + 1} - 1}{z_2}. \end{aligned} \tag{21}$$

Note that $\frac{\partial^2 \gamma_2}{\partial p_2^2} \leq 0$. Then, the maximum achievable SNR optimized over power allocation is given by

$$\begin{aligned} \gamma_2^{\circ} &= \max_{p_1 + p_2 = p_{\mathrm{tot}}} \gamma_2 = \gamma_2|_{p_2 = p_2^{\circ}} \\ &= \frac{\left(z_2 p_{\mathrm{tot}} - \sqrt{p_{\mathrm{tot}} z_2 + 1} + 1\right)\left(\sqrt{p_{\mathrm{tot}} z_2 + 1} - 1\right) z_1}{z_2 \sigma^2 \sqrt{p_{\mathrm{tot}} z_2 + 1}}. \end{aligned} \tag{22}$$

According to Proposition 1, when $\gamma_2^{\circ} \geq \gamma_{\mathrm{th}}$, $\frac{\partial r}{\partial \gamma_2} > 0$ holds. Hence, $\frac{\partial r}{\partial p_2} = \frac{\partial r}{\partial \gamma_2} \frac{\partial \gamma_2}{\partial p_2} = 0 \Leftrightarrow \frac{\partial \gamma_2}{\partial p_2} = 0$. The optimal solution of Problem (19) is given by $(p_1^*, p_2^*) = (p_{\mathrm{tot}} - p_2^{\circ}, p_2^{\circ})$. On the other hand, if $\gamma_2^{\circ} < \gamma_{\mathrm{th}}$, according to the feasible set provided in (10) the optimal solution is $(p_1^*, p_2^*) = (0, 0)$.

Based on the above analysis, the analytical solution of Problem (9) can be summarized as follows

Proposition 3. *The solution of Problem* (9) *is given by*

$$\begin{cases} (p_1^*, p_2^*) = (0, 0) & \text{case } \gamma_2^{\circ} < \gamma_{\mathrm{th}}, \\ (p_1^*, p_2^*) = (p_{\mathrm{tot}} - p_2^{\circ}, p_2^{\circ}) & \text{case } \gamma_2^{\circ} \geq \gamma_{\mathrm{th}}, \end{cases} \tag{23}$$

where p_2° and γ_2° are provided in (21) *and* (22)*, respectively.*

V. Simulation Results

In this section, we provide our simulation results to validate our analytical model and evaluate the AF relaying performance. In the simulations, we consider the following parameterization: First, we set the power constraint to 20 dBm and the path-losses of both hops individually to 10 dB. In addition, we assume independent Rayleigh fading channel for the two links, where the exponentially distributed fading power has a mean value of $\mathbb{E}\{z\} = 1$.

To start with, we consider within a single frame the impact of power allocation on the coding rate. The results are provided in Fig. 2. The figure confirms our analytical results in Proposition 1 that the coding rate is concave in the power allocation between the source and the relay for different setups of blocklength and target error probability. More importantly, we observe that the optimal values of these curves (with different m and ε) are achieved by the same choice of p_2 (recall that p_{tot} is fixed). This actually agrees with Equation (21) that the optimal amount of power allocated to the relay to maximize γ_2 or r is only influenced by the power constraint and the instantaneous channel gain z_2. In addition, the figure also indicates that a longer blocklength can support a higher coding rate. Moreover, guaranteeing a lower ε results in a loss in the coding rate. In particular, this loss is more significant for the short blocklength case.

Next, we analyze the QoS-constrained throughput of the considered AF relaying network. In particular, we study the relation between the maximal throughput achieved by power allocation with the QoS exponent θ. The results are shown in Fig. 3, where the solid/dotted curves are obtained by the proposed optimal solution in Proposition 3 while the green bubble dots present the corresponding maximal throughputs over power allocation obtained via exhaustive search. First, we note that the performance determined with exhaustive search matches the performance of the proposed solution, and confirms our analysis. In addition, as expected a high θ (corresponding to strict QoS requirements) results in a low throughput. However, it is interesting that as θ increases these throughputs curves decrease differently. In particular, a long blocklength generally supports a higher throughput when the QoS requirements are loose (low θ). On the other hand, when the QoS requirements become strict, a short blocklength is preferred. In addition, for the long blocklength ($m = 1000$) case a high error probability introduces a slightly higher throughput, while the opposite holds for the short blocklength ($m = 100$) case.

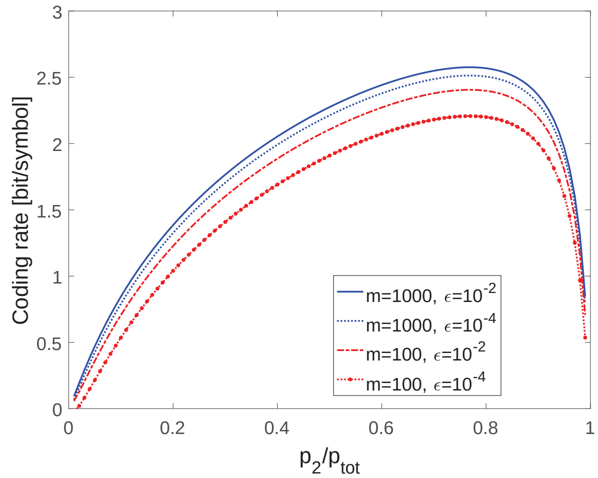

Fig. 2. The impact of power allocation on the coding rate within a transmission frame.

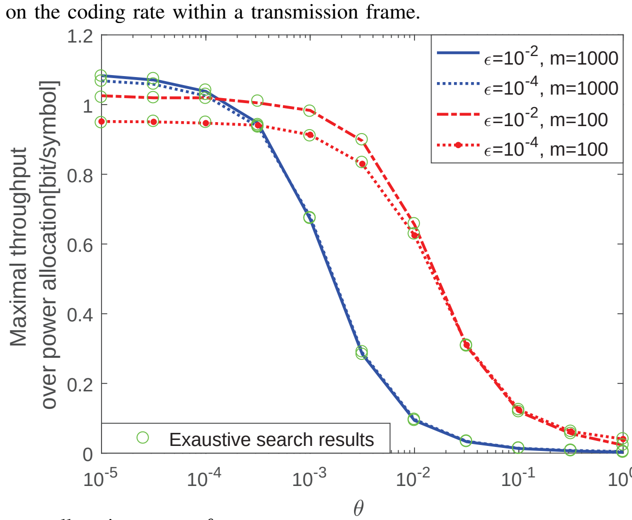

Fig. 3. The maximal throughput over power allocation versus θ.

To provide a clear insight on the impact of error probability and blocklength, we therefore further investigate the throughput performance while varying error probability in Fig. 4 and varying the blocklength in Fig. 5. Again, we observe a perfect match

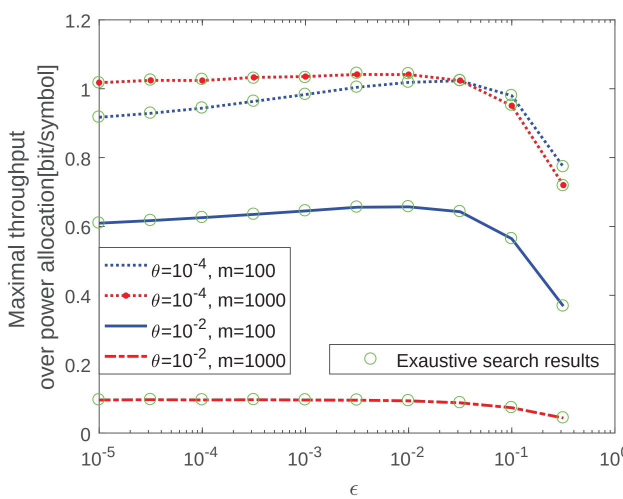

Fig. 4. The maximal throughput over power allocation versus ϵ.

between the proposed optimal solution and the result with the exhaustive search in the figures. Fig. 4 in general indicates that the throughputs are concave or quasi-concave in the error probability ε, while the optimal choice of ε is different for different setups, i.e., for different coding blocklength and QoS constraints. Finally, from Fig. 5 we observe that when θ is relatively large, the throughput curves are decreasing in the blocklength. On the other hand, with a relatively small θ, the throughputs are observed to be concave in the blocklength.

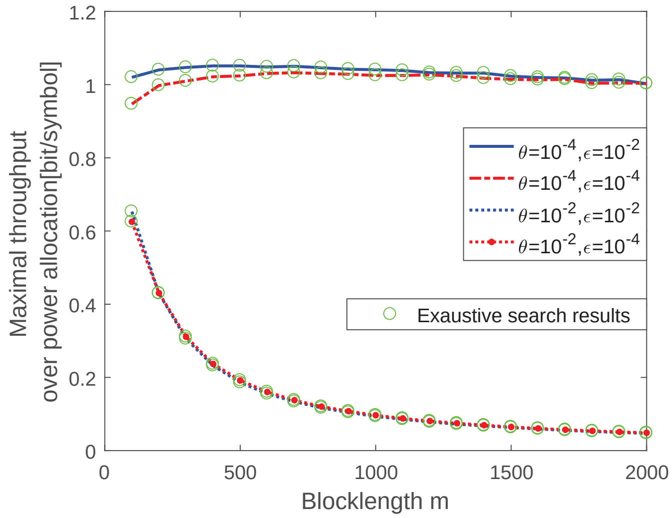

Fig. 5. The maximal throughput over power allocation versus ε.

VI. Conclusion

In this work, we have studied a two-hop AF relaying network in the FBL regime and derived the QoS-constrained throughput of the network. Moreover, we have proposed an optimal power allocation policy for the network to maximize the throughput. Via simulations, first we have shown that the proposed power allocation policy has the same performance as that determined via exhaustive search. In addition, it is observed that the throughputs are concave in the error probability and are decreasing in the QoS exponent. Moreover, we have also observed that when the QoS exponent is relatively large, the throughput curves are decreasing in the blocklength, while they are observed to be concave in the blocklength when QoS exponent is small.

References

[1] J. Laneman, D. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[2] S. Karmakar and M. Varanasi, “The diversity-multiplexing tradeoff of the dynamic decode-and-forward protocol on a MIMO half-duplex relay channel,” *IEEE Trans. Inf. Theory*, vol. 57, no. 10, pp. 6569–6590, Oct. 2011.

[3] J. Li, B. Makki and T. Svensson, "Performance analysis and cooperation mode switch in HARQ-based relaying." in *IEEE Globecom Workshops*. Austin, TX, Dec. 2014.

[4] Y. Hu, J. Gross and A. Schmeink, “QoS-Constrained energy efficiency of cooperative ARQ in multiple DF relay systems,” *IEEE Trans. Veh.Technol.*, vol. 65, no. 2, pp.848–859, Feb. 2016

[5] J. Tang and X. Zhang, "Cross-layer resource allocation over wireless relay networks for quality of service provisioning," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 4, pp. 645-656, May 2007.

[6] T. Q. S. Quek, H. Shin and M. Z. Win, "Robust wireless relay networks: Slow power allocation with guaranteed QoS," *IEEE J. Sel. Topics Signal Process.* , vol. 1, no. 4, pp. 700-713, Dec. 2007.

[7] M. Fadel, A. El-Keyi and A. Sultan, "QoS-constrained multiuser peer-to-peer amplify-and-forward relay beamforming,"*IEEE Trans. on Signal Process.*, vol. 60, no. 3, pp. 1397-1408, March 2012.

[8] Y. Polyanskiy, H. Poor, and S. Verdu, “Channel coding rate in the finite blocklength regime,” *IEEE Trans. Inf. Theory*, vol. 56, no. 5, pp. 2307–2359, May 2010.

[9] V. Y. F. Tan and M. Tomamichel, “The third-order term in the normal approximation for the AWGN channel,” *IEEE Trans. Inf. Theory*, vol. 61, no. 5, pp. 2430–2438, May 2015.

[10] ——, “Dispersion of the Gilbert-Elliott channel,” *IEEE Trans. Inf. Theory*, vol. 57, no. 4, pp. 1829–1848, Apr. 2011.

[11] W. Yang, G. Durisi, T. Koch and Y. Polyanskiy “Quasi-static multiple-antenna fading channels at finite blocklength,” *IEEE Trans. Inf. Theory*, vol. 60, no. 7, Jul. 2014.

[12] M. C. Gursoy, “Throughput analysis of buffer-constrained wireless systems in the finite blocklength regime,” *EURASIP J. Wireless Commun.*, vol. 2013:290, Dec. 2013.

[13] G. Ozcan and M. C. Gursoy, “Throughput of cognitive radio systems with finite blocklength codes,” *IEEE J. Sel. Areas Commun.*, vol. 31, no. 11, pp. 2541-2554, Nov. 2013.

[14] P. Wu and N. Jindal, “Coding versus ARQ in fading channels: How reliable should the phy be?” *IEEE Trans. Commun.*, vol. 59, no. 12, pp. 3363–3374, Dec. 2011.

[15] B. Makki, T. Svensson, and M. Zorzi, “Finite block-length analysis of the incremental redundancy HARQ,” *IEEE Wireless Commn. Lett.*, vol. 3, no. 5, pp. 529–532, Oct. 2014.

[16] ——, “Finite block-length analysis of spectrum sharing networks using rate adaptation,” *IEEE Trans. Commun.* , vol. 63, no. 8, pp. 2823–2835, Aug 2015.

[17] S. Xu *et al.*, "Energy-efficient packet scheduling with finite blocklength codes: Convexity analysis and efficient algorithms," *IEEE Trans. Wireless. Commn.*, vol.15, no.8, pp.5527-5540, Aug. 2016.

[18] Y. Hu, M. Serror, K. Wehrle and J. Gross, "Finite blocklength performance of cooperative multi-terminal wireless industrial networks,"*IEEE Trans. Veh. Technol.*, vol. PP, no. 99, pp. 1-1.

[19] Y. Hu, M. C. Gursoy and A. Schmeink, "Relaying-Enabled Ultra-Reliable Low Latency Communications in 5G", *IEEE Network*, accepted to appear.

[20] Y. Hu, J. Gross and A. Schmeink, “On the capacity of relaying with finite blocklength,” *IEEE Trans. Veh. Technol.*, vol. 65, no. 3, pp. 1790–1794, Mar. 2016.

[21] Y. Hu, J. Gross and A. Schmeink, “On the performance advantage of relaying under the finite blocklength regime,” *IEEE Commn. Lett.*, vol. 19, no. 5, pp. 779 - 782, May 2015.

[22] Y. Hu, A. Schmeink and J. Gross "Blocklength-limited performance of relaying under quasi-static Rayleigh channels," *IEEE Trans. Wireless. Commn.*, vol. 15, no. 7, pp. 4548 - 4558, Jul. 2016.
[23] Y. Li, M. C. Gursoy and S. Velipasalar, "Throughput of two-hop wireless channels with queueing constraints and finite blocklength codes," *IEEE ISIT*, Barcelona, Spain, Jul. 2016.
[24] D. Wu and R. Negi, "Effective capacity: a wireless link model for support of quality of service," *IEEE Trans. Wireless Commun.*, vol. 2, no. 4, pp. 630–643, Jul. 2003.
[25] C.-S. Chang, "Stability, queue length, and delay of deterministic and stochastic queueing networks," *IEEE Trans. Automat. Contr.*, vol. 39, pp. 913–931, May 1994