Distributed estimation and Power Allocation for Passive Radar Sensor Networks with Imperfect CSI

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***Abstract*—In this paper we address the design of a distributed passive radar sensor network system, including the power allocation among sensors, as well as linear signal fusion, considering the channel state information (CSI) estimation error. Analytical solutions are obtained for each case, achieving an optimum performance. In particular, it is assumed that both sensing and communication wireless channels are estimated erroneously, however, with a known error covariance. The performance of the proposed methods is numerically evaluated over different levels of thermal noise and CSI error, where a superior performance is observed compared to the available designs which assume the availability of perfect CSI.**

***Index Terms*—CSI estimation error, sensor networks, passive radar**

I. Introduction

The goal of a distributed passive radar is to provide a reliable estimation of a source signal, by collecting and combining the individual passive observations from a network of sensor nodes (SN) in a fusion center (FC). See [1] for some related applications. In this respect, problems regarding the optimal power allocation among the SNs and the energy-aware system design are of interest, due to the weak, and usually low power budget sensor systems, see [1]–[4], and the references therein. In [2] an optimal power allocation and fusion strategy is analytically obtained, assuming the availability of perfect network information. However, this assumption is not practical in many related applications, due to the unreliable and noisy nature of SNs. In [5] a linear system design is proposed considering the occasional node failure in the network, assuming that the statistics of SN failure can be obtained at the FC. In [6] the sensitivity of the proposed design in [2] is evaluated under the impact of channel estimation error. However, in all of the aforementioned designs, and the further related works [7]–[12], the wireless channels are assumed to be perfectly known in the design process.

Building on the proposed designs in [2], and the insights obtained from [6], we revisit the optimal linear system design, in terms of signal fusion at FC, as well as the power allocation at SNs, taking into account the impact of channel estimation error. Analytical expressions are obtained in both cases, achieving an optimum performance. The performance of the proposed methods are then evaluated via numerical simulations. A notable gain is observed compared to the case where CSI error is not considered in the design process, for a system with a large signal strength, or with a low channel estimation accuracy.

II. System Model

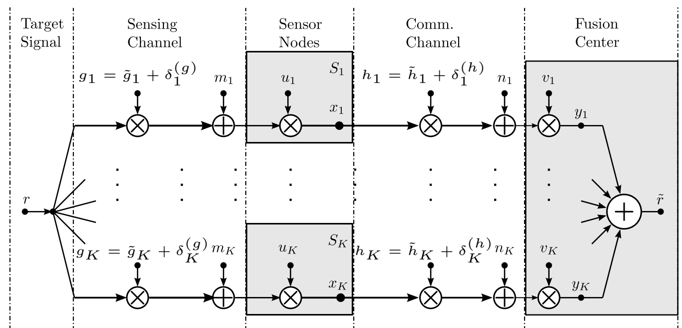

Figure 1: System model of the distributed sensor network.

In this section, we consider a network of K amplify and forward (AF) passive sensor nodes which cooperate to attain a single observation of a target signal $r \in \mathbb{C}$ using a FC as shown in Fig. 1. The target signal is sensed by the SNs through the sensing channels, and re-transmitted to the FC through the communication channels. All the channels, i.e., both sensing and communication channels, are assumed to be wireless, frequency-flat, and static during the observation process. The index $k \in \mathbb{K}$ represents different SNs, where set $\mathbb{K}$ represent the index set of all SNs.

A. Imperfection in CSI

We consider an imperfect channel state information of both sensing and communication channels. The sensing channel g_k can be written as

$$g_k = \tilde{g}_k + \delta_k^{(g)}, \quad k \in \mathbb{K}, \tag{1}$$

where $\tilde{g}_k$ and $\delta_k^{(g)}$ are the estimated channel coefficient and estimation error for the sensing channel g_k of the k-th sensor, respectively. The channel error model is similar to one used in [13], [14]. The sensing channel estimation error $\delta_k^{(g)}$ is assumed to be zero mean, statistically independent and with variance $\Delta_k^{(g)}$.

$$\begin{aligned} \mathcal{E}\{|g_k|^2\} &= \mathcal{E}\{|\tilde{g}_k + \delta_k^{(g)}|^2\}, \\ &= |\tilde{g}_k|^2 + \Delta_k^{(g)}, \end{aligned} \tag{2}$$

where $\mathcal{E}\{.\}$ is the expectation operator. Here, the expectation operator is computed over distribution of $\delta_k^{(g)}$. Similarly, the communication channel h_k can be written as

$$h_k = \tilde{h}_k + \delta_k^{(h)}, \ \ k \in \mathbb{K}, \tag{3}$$

where $\tilde{h}_k$ and $\delta_k^{(h)}$ are the estimated channel coefficient and estimation error for the communication channel h_k of the k-th sensor, respectively. The communication channel estimation error $\delta_k^{(h)}$ is also assumed to be zero mean, statistically independent and with variance $\Delta_k^{(h)}$.

$$\begin{aligned} \mathcal{E}\{|h_k|^2\} &= \mathcal{E}\{|\tilde{h}_k + \delta_k^{(h)}|^2\}, \\ &= |\tilde{h}_k|^2 + \Delta_k^{(h)}. \end{aligned} \tag{4}$$

where the expectation operator is computed over distribution of $\delta_k^{(h)}$.

B. Operation of SNs

If a target signal $r \in \mathbb{C}$ is present, each SN receives and amplifies the incoming signal using an amplification coefficient $u_k \in \mathbb{C}, \ k \in \mathbb{K}$. The communication with FC is performed by using orthogonal waveforms for each sensor so that the signal from different sensors can be separated and processed in FC. The process of each SN can be hence described as

$$\begin{aligned} x_k &= u_k \left(g_k r + m_k\right), \ \ k \in \mathbb{K}, \\ &= u_k \left(\left(\tilde{g}_k + \delta_k^{(g)}\right) r + m_k\right), \ \ k \in \mathbb{K}, \end{aligned} \tag{5}$$

where x_k is the transmitted signal from the k-th sensor and $m_k \in \mathbb{C}$ is an additive white Gaussian noise (AWGN) component at the sensor k, with variance M_k. The power of the transmitted signal x_k can be written as

$$X_k = \mathcal{E}\{|x_k|^2\} = |u_k|^2 \left(R\left(|\tilde{g}_k|^2 + \Delta_k^{(g)}\right) + M_k\right), \tag{6}$$

where the expectation operator is computed over distribution of $\delta_k^{(g)}$, r and m_k. Correspondingly, the power of the target signal r can be expressed as

$$R = \mathcal{E}\{|r|^2\}. \tag{7}$$

In [2], the detailed function of each SN is discussed. Furthermore, it is assumed that the power consumption of k-th SN is limited by $X_{\text{max},k}$, while the total power consumption of the network is limited by X_{tot}. This is formulated as

$$X_k \leq X_{\text{max},k}, \ \ k \in \mathbb{K}, \tag{8}$$

and

$$\sum_{k \in \mathbb{K}} X_k \leq X_{\text{tot}}. \tag{9}$$

C. Fusion Center

The transmitted signal from the SNs passes through the communication channel, with coefficient $h_k \in \mathbb{C}$, and arrive at the FC combined with an AWGN component $n_k \in \mathbb{C}$, with variance N_k. This is described as

$$y_k = n_k + h_k x_k. \tag{10}$$

The estimated target signal at the FC can be hence written as

$$\begin{aligned} \tilde{r} &= \sum_{k \in \mathbb{K}} v_k y_k \\ &= \sum_{k \in \mathbb{K}} v_k \left(n_k + \left(\tilde{h}_k + \delta_k^{(h)}\right) u_k \left(\tilde{g}_k + \delta_k^{(g)}\right) r\right) \\ &+ \sum_{k \in \mathbb{K}} v_k \left(\left(\tilde{h}_k + \delta_k^{(h)}\right) u_k m_k\right), \end{aligned} \tag{11}$$

where $v_k \in \mathbb{C}$ represents the applied fusion weight at the received signal from the k-th sensor.

III. MMSE Design of Network Parameters Under imperfect channel state information

In this section we propose an MMSE design of the network parameters for unbiased class of estimators. In the first step, we observe that the unbiasedness property can be enforced as

$$\mathcal{E}\left\{\tilde{r} - r\right\} = 0 \ \ \Rightarrow \ \ \sum_{k \in \mathbb{K}} v_k \tilde{h}_k u_k \tilde{g}_k = 1, \tag{12}$$

following the identity (11) and the fact that all noise terms are zero mean and independent, i.e., in Equation (11) the terms with $\delta_k^{(g)}$,$\delta_k^{(h)}$, m_k , and n_k becomes zero. Here, the expectation operator is computed over distribution of $\delta_k^{(g)}$, $\delta_k^{(h)}$, r, m_k and n_k. Furthermore, the mean squared error of the estimation can be written as

$$\begin{aligned} V :&= \mathcal{E}\{|\tilde{r} - r|^2\} \\ &= \sum_{k \in \mathbb{K}} |v_k|^2 N_k + \sum_{k \in \mathbb{K}} |v_k|^2 |u_k|^2 \left(|\tilde{h}_k|^2 + \Delta_k^{(h)}\right) M_k \\ &+ R \sum_{k \in \mathbb{K}} |v_k|^2 |u_k|^2 \left(\left(|\tilde{h}_k|^2 + \Delta_k^{(h)}\right) \Delta_k^{(g)} + |\tilde{g}_k|^2 \Delta_k^{(h)}\right), \\ &= \sum_{k \in \mathbb{K}} |v_k|^2 N_k + \sum_{k \in \mathbb{K}} |v_k|^2 |u_k|^2 a_k, \end{aligned} \tag{13}$$

where

$$\begin{aligned} a_k :=& R\left(\left(|\tilde{h}_k|^2 + \Delta_k^{(h)}\right) \Delta_k^{(g)} + |\tilde{g}_k|^2 \Delta_k^{(h)}\right) \\ &+ \left(|\tilde{h}_k|^2 + \Delta_k^{(h)}\right) M_k. \end{aligned} \tag{14}$$

The Equation (13) follows via the application of (12), and the fact that all the noise terms are mutually independent and zero-mean. As a result, the MSE minimization problem can be formulated for an unbiased class of estimates which satisfy the defined power constraints as

$$\begin{aligned} \underset{u_k, v_k, \ k \in \mathbb{K}}{\text{minimize}} \ \ & V \\ \text{s.t.} \ \ & (8), (9), (12). \end{aligned} \tag{15}$$

Unfortunately, (15) is not a jointly convex optimization problem. Nevertheless it is separately convex over the fusion weights, i.e., $v_k,\ k \in \mathbb{K}$. In the following part we obtain a set of optimal fusion weights for a fixed set of amplification coefficients.

A. Optimal Linear Fusion

The power constraints in (8), (9) are invariant to the choice of fusion weights. The corresponding optimization problem over the fusion weights, when the amplification coefficients are fixed, is formulated as

$$\underset{v_k,\, k\in\mathbb{K}}{\text{minimize}} \quad V \tag{16a}$$

$$\text{s.t.} \quad \sum_{k\in\mathbb{K}} v_k \tilde{h}_k u_k \tilde{g}_k = 1. \tag{16b}$$

As a first step, using the following lemma, we provide information about the phase of the system parameters at optimum point:

Lemma 1. *For any optimal choice of system parameters* $(v_k, u_k, \forall k \in \mathbb{K})$*, the following update is feasible and does not degrade (increase) the objective value in* (16)*:* $\forall k \in \mathbb{K}$ *:*

$$v_{k,new} = |v_k| \frac{\left(\tilde{h}_k\tilde{g}_k\right)^*}{|\tilde{h}_k\tilde{g}_k| \left(\sum_{k\in\mathbb{K}} |v_k u_k \tilde{h}_k \tilde{g}_k|\right)}, u_{k,new} = |u_k|, \tag{17}$$

where $(\cdot)^*$ *represents conjugation.*

Proof. The new parameter updates do not violate the power constraints (8) and (9) as absolute value of amplification factor u_k and X_k are kept constant. The unbaised condition (12) also holds:

$$\sum_{k\in\mathbb{K}} v_{k,new} u_{k,new} \tilde{h}_k \tilde{g}_k = \frac{\sum_{k\in\mathbb{K}} |v_k u_k \tilde{h}_k \tilde{g}_k|}{\sum_{k\in\mathbb{K}} |v_k u_k \tilde{h}_k \tilde{g}_k|} = 1. \tag{18}$$

Consequently, using (12) and triangular inequality , we have:

$$\sum_{k\in\mathbb{K}} |v_k u_k \tilde{h}_k \tilde{g}_k| \geq \sum_{k\in\mathbb{K}} v_k u_k \tilde{h}_k \tilde{g}_k = 1, \tag{19}$$

which conclude that the variable update does not increase the norms of v_k and u_k and hence does not increase the objective (13) as a_k is also real-valued. ☐

The above lemma provides us some useful information about the phase of the system parameters. It shows that the real-valued assumption of $u_k,\ k \in \mathbb{K}$ does not reduce the optimality. It also provides us with an optimal choice of $\angle v_k$ and simplifies our optimization problem into calculating $|u_k|$, $|v_k|$, $k \in \mathbb{K}$ by assuming

$$u_k \in \mathbb{R}^+, v_k = |v_k| \angle(\tilde{h}_k \tilde{g}_k)^*, \tag{20}$$

where $\angle(\cdot)$ represents the phase and $\mathbb{R}^+$ is the set of positive real numbers. Hence our simplified optimization problem can be written as

$$\underset{|v_k|,\, k\in\mathbb{K}}{\text{minimize}} \quad V \quad \text{s.t.} \quad \sum_{k\in\mathbb{K}} |v_k| u_k |\tilde{h}_k \tilde{g}_k| = 1. \tag{21}$$

which is a convex optimization problem. The corresponding Lagrangian function to (21) can be subsequently written as

$$\mathcal{L}\left(|v_k|, \lambda\right) = \lambda \left(1 - \sum_{k\in\mathbb{K}} |v_k| u_k |\tilde{h}_k \tilde{g}_k|\right) + \sum_{k\in\mathbb{K}} |v_k|^2 N_k + \sum_{k\in\mathbb{K}} |v_k|^2 |u_k|^2 a_k. \tag{22}$$

For any optimal solution to (21), the derivative of the Lagrangian should vanish with respect to $|v_k|$. This is written as

$$\frac{\partial \mathcal{L}}{\partial |v_k|} = 0 \quad \Leftrightarrow \quad 2|v_k| \left(N_k + |u_k|^2 a_k\right) - \lambda u_k |\tilde{h}_k \tilde{g}_k| = 0. \tag{23}$$

Accordingly, the absolute value of v_k can be stated as

$$|v_k| = \frac{\lambda}{2} \frac{u_k |\tilde{h}_k \tilde{g}_k|}{N_k + |u_k|^2 a_k}. \tag{24}$$

Following the identity

$$\sum_{k\in\mathbb{K}} |v_k| \frac{\partial \mathcal{L}}{\partial |v_k|} = 0, \tag{25}$$

we obtain $\lambda = 2V$. The absolute value of v_k in terms of MMSE (V) can be obtained as

$$|v_k| = \frac{u_k |\tilde{h}_k \tilde{g}_k| V}{N_k + |u_k|^2 a_k}. \tag{26}$$

Furthermore, the derivative of the Lagrangian with respect to λ also vanishes at optimality and can be stated as

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \quad \Leftrightarrow \quad \left(1 - \sum_{k\in\mathbb{K}} |v_k| u_k |\tilde{h}_k \tilde{g}_k|\right) = 0, \tag{27}$$

and using (26), we obtain the objective function as

$$V = \left(\sum_{k\in\mathbb{K}} \frac{|\tilde{h}_k \tilde{g}_k|^2 u_k^2}{u_k^2 a_k + N_k}\right)^{-1} = \left(\sum_{k\in\mathbb{K}} \frac{U_k}{U_k \frac{a_k}{|\tilde{h}_k \tilde{g}_k|^2} + \frac{N_k}{|\tilde{h}_k \tilde{g}_k|^2}}\right)^{-1} = \left(\sum_{k\in\mathbb{K}} \frac{U_k}{U_k \alpha_k + \beta_k}\right)^{-1}, \tag{28}$$

where $U_k = u_k^2$, $\alpha_k = \frac{a_k}{|\tilde{h}_k \tilde{g}_k|^2}$ and $\beta_k = \frac{N_k}{|\tilde{h}_k \tilde{g}_k|^2}$, which consequently from (24) results in

$$|v_k| = \left(\sum_{k\in\mathbb{K}} \frac{U_k}{U_k \alpha_k + \beta_k}\right)^{-1} \times \frac{u_k |\tilde{h}_k \tilde{g}_k|}{N_k + |u_k|^2 a_k}, \tag{29}$$

where $|v_k|$ represents the absolute value of the optimal fusion weight, corresponding to the k-th sensor. The optimal fusion weights can be obtained by applying (29) in (17). The optimal U_k can be obtained via the optimization problem,

$$\min_{U_k,\, k\in\mathbb{K}} \quad -\sum_{k\in\mathbb{K}} \frac{U_k}{U_k\alpha_k+\beta_k} \tag{30a}$$

$$\text{s.t.} \quad \sum_{k\in\mathbb{K}} c_k U_k \leq X_{\text{tot}}, \tag{30b}$$

$$c_k U_k \leq X_{\text{max}}, \quad U_k \geq 0, \tag{30c}$$

where $c_k = \left(R\left(|\tilde{g}_k|^2+\Delta_k^{(g)}\right)+M_k\right)$. Similar optimization problem is already studied in [5]. Using similar steps to [5], i.e., solving the optimization problem by using the KKT conditions and water filling algorithm, we get the optimal weights at the sensor nodes as,

$$U_k^\star = \frac{1}{\alpha_k}\sqrt{\frac{\beta_k/c_k}{\lambda^\star}} - \frac{\beta_k}{\alpha_k} \tag{31}$$

and

$$\lambda^\star = \left(\frac{X_{tot}-\sum_{k\in\mathbb{K}_{sat}} X_{\max,k}+\frac{\beta_k c_k}{\alpha_k}}{\sum_{k\in\mathbb{K}}\frac{\sqrt{\beta_k c_k}}{\alpha_k}}\right)^{-2}. \tag{32}$$

Optimal values for u_k and v_k can be respectively obtained as $u_k^\star = \sqrt{U_k^\star}$ and considering the corresponding equations.

IV. Simulation Results

In this section, we investigate the performance of the proposed algorithm using numerical simulations. We simulate a network with $K = 1000$ SNs, where all the estimated sensing and communication channel and the corresponding estimation errors are zero mean and follow a Gaussian distribution with variance ${\sigma_g}^2$, ${\sigma_h}^2$, Δ_g , and Δ_h respectively. The values given in Table I are used as the default simulated network parameters. For each set of channel realizations, i.e. g_k, h_k, 1000 realizations of r, n_k, m_k, are generated to evaluate the network performance. The resulting network is then averaged over 1000 channel realizations.

In Fig 2, the performance of the network, in terms of MSE, is plotted with respect to variance of the estimation error $\Delta_g = \Delta_h$ on the sensing and communication channels, for different noise levels at the SNs and FC. We assume the variance of channel estimation error is same for all the sensor nodes. It can be observed that the resulting MSE increases as the noise intensity increases, or the variance of the channel estimation error increases. We can also observe that the robust algorithm that considers the imperfection in CSI, outperforms the one that does not consider it for all the noise and estimation error values. The performance gain increases as $\Delta_g = \Delta_h$ increases and also for small amount of noise. We can also notice that the analytical results matches with the numerical results.

In Fig 3, the resulting network performance is depicted in terms of MSE with respect to noise at both sensors N_k and FC M_k , for different values of the channel estimation error variance $\Delta_g = \Delta_h$. Here, the resulting MSE increases as the noise at the sensors and FC increases. Here also, we can notice

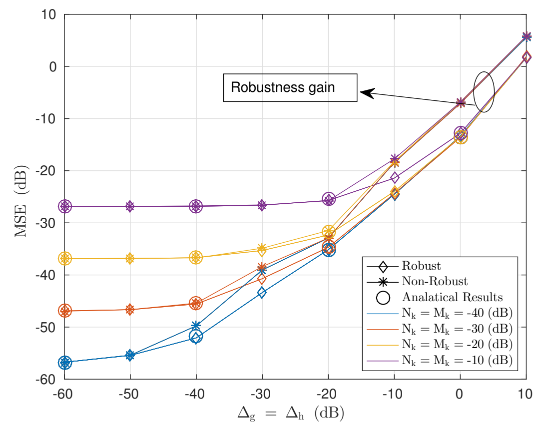

Figure 2: MSE vs Sensing Channel estimation error (dB). Robustness gain increases as channel estimation error increases.

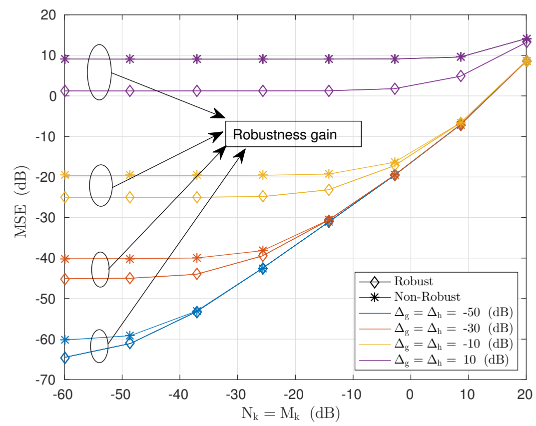

Figure 3: MSE vs Noise at both Sensor and Fusion Center (dB). More robustness gain at high signal to noise ratio region

that the performance of the robust algorithm is better for small noise at the sensors and FC, and also for the higher values of channel estimation error variance $\Delta_g = \Delta_h$.

In Fig 4, the impact of target signal power R in the performance of the network is observed. We can see that the resulting MSE increases as the target signal power increases. As expected, the robust algorithm performs better for higher values of signal power (R), .i.e., for high signal to noise ratio regime.

R	${\sigma_g}^2$	${\sigma_h}^2$	Δ_g	Δ_h	N_k	M_k	P_k	P_{tot}
10	1	1	0.1	0.1	1	1	15	250

Table I: Reference parameters

V. Conclusion

In this paper, we have addressed the problem of optimal power allocation for a distributed passive radar system, where the statistics of noise and channel estimation error are known. An optimal MMSE-based solution is presented for unbiased

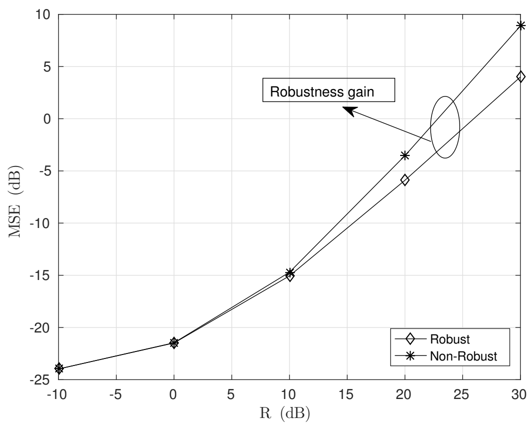

Figure 4: MSE vs Target signal power (R) (dB). More robustness gain for higher values of target signal power.

class of estimators. The numerical simulations show that the proposed robust algorithm outperforms the traditional one, for higher values of channel estimation error variance, and also for small noise at the SN and FC. We also observed that the analytical results matches with the numerical results.

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