RSS-based Positioning of Multiple Co-Channel Targets with Unknown Transmit Power in a Log-Normal Shadowing Scenario

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***Abstract*—This paper tackles the problem of target localization of multiple co-channel targets based on the sum of the received signal strength of all targets at different sensor nodes. The problem is hard, since the targets can transmit at the same time on the same channel. The propagation channel is assumed to be log-normal shadowing model. We propose an unbiased estimator. The underlying complicated optimization problem is then solved by a heuristic based on mixed-integer programming, which presents a low-complexity. The performance of the estimator is justified to be good using simulations.**

***Index Terms*—multi-source localization, compressed sensing, mixed-integer programming, internet of things**

I. Introduction

It is envisaged that the majority of applications in the context of internet of things and 5G mobile networks depend on the location awareness to deliver better services. Therefore, the old topic of localization is not yet obsolete. In the literature a variety of techniques have been exploited to solve such a problem. We here stick to received signal strength (RSS)-based technique due to its simplicity and lower cost compared to time difference of arrival (TDOA) or angle of arrival (AoA), [1]. Despite its vulnerability against uncertainties of path-loss model, RSS localization is beneficial whenever the precision can be somewhat compromised for price. The RSS-based localization for a single target with unknown transmit power is studied in many publications, such as [2], where the by dividing the RSS of two different receivers the transmit power cancels out. The remaining of the problem is a standard multilateration problem. This technique is known as differential or ratio of RSS which is not applicable in our case, since we consider that there are more then one transmitter on the same channel which causes co-channel interference problem.

The work [3] also considers a multi-target scenario, but it does not assume co-channel interference since each receiver knows the RSS of each transmitter, separately. To the best of our knowledge, the only papers which consider the multi-user case with co-channel interference is [4], [5]. Nevertheless, [4] does not exploit the explicit path-loss model but performs fingerprinting, instead. On the one hand, fingerprinting avoids the model uncertainty of the path-loss model. It, on the other hand, involves the difficulty of building the radio maps which can be costly and time consuming. The built radio map can also be different in reality, when moving humans or objects vary the propagation profile of the environment. The paper deals with the problem through ℓ_1-minimization which needs a sufficiently enough number of observation. This translates into a radio map with high granularity. To surmount this problem, they use not only the RSS but also cross-correlation of the received signal at different sensors. This improves performance, but at the cost of more expensive sensors as well as higher power consumption [5]. We, here, assume a log-normal shadowing path-loss model. To solve the intractable underlying mathematical problem, we resort to a grid based solution and techniques of mixed integer programming (MIP). Also, to maintain a low-complexity we keep the number of grid points (GPs) low and adapt the GPs, iteratively. An interesting application for our scenario is finding the position of the illegitimate secondary user(s) with unknown transmit power. Note such interfering users decrease the throughput of the primary user or even cause link failure due to strong interference.

The organization of this paper is as follows: the system model is described in Sec. II and statistical behavior of RSS at the sensor receiver in Sec. III. We propose in Sec. IV a mixed integer quadratic programming (MIQP) formulation, assuming the targets are located only at GPs. An adaptive scheme is proposed in Sec. V to refine the GPs to overcome the problem of off-grid targets. The performance of the presented solutions will be justified by means of computer simulations in Sec. VI. The Sec. VII concludes the paper.

Notations: All mathematical notations, symbols and variables of this paper are summarized in Tab. I and Tab. II.

Table I: Summary of general mathematical notations

Notation	Description
$\mathbb{N}$	set of all integer positive and non-zero numbers
$\mathbb{R}$	set of all real numbers
$\mathbb{R}_+$	set of all non-negative real numbers
δ_{lm}	Kronecker delta function, i.e., $\delta_{lm} = \begin{cases} 1, & l = m, \\ 0, & l \neq m. \end{cases}$

Table II: Summary of systems symbols and variables

Symbol	Description
K	number of sensor nodes, $K \in \mathbb{N}$
N	number of active targets, $N \in \mathbb{N}$
M	number of GPs given $\sqrt{M} \in \mathbb{N}$
$\mathbb{F}_{\mathrm{L}}$	the index-set $\mathbb{F}_{\mathrm{L}} = \{1, \cdots, L\}$ for any given number $L \in \mathbb{N}$

II. System Model

The system of consideration consists of $N \in \mathbb{N}$ active targets with unknown position and $K \in \mathbb{N}$ passive sensor nodes (SNs) with known positions. Each target $n \in \mathbb{F}_N$ transmits a signal with the unknown power p_n. We know that that transmit power of each target is bounded as follows

$$\underline{P} \leq p_n \leq \bar{P}\,,\ \forall n \in \mathbb{F}_N\,, \tag{1}$$

where $\underline{P},\ \bar{P} \in \mathbb{R}_+$ are, respectively, the lowest and highest possible values for the transmit power of an active target. The propagation channel is based on the log-normal shadowing attenuation model presented in [6]. In a multi-source scenario, the RSS r_k at sensor k is the sum of different terms corresponding to the received power of each target signal [7], [8]:

$$r_k = \sum_{n \in \mathbb{F}_N} c_0\, p_n\, d_{kn}^{-\alpha}\, 10^{\frac{\zeta_{kn}}{10}}\,, \tag{2}$$

where d_{kn} is the distance between sensor k and n^{th} target, α is the path-loss exponent and $\zeta_{kn} \sim \mathcal{N}(0, \sigma_{kn}^2)$ is a zero-mean Gaussian random variable with power of σ_{kn}^2. It models the log-normal shadowing between each pair of sensor and target nodes and is assumed to be identically and independently distributed (iid). The coefficient c_0 is given by [6],

$$c_0 := \frac{G_t G_r \lambda^2}{(4\pi)^2}\,,$$

where G_t and G_r are the gains of transmit and receive antennae, respectively. The wavelength is denoted by λ. We assume that c_0 is known and without loss of generality and for the sake of simplicity, $c_0 = 1$.

Here, we have neglected the thermal additive noise due to the fact that shadowing has much stronger effect on RSS compared to the thermal noise [9]–[11]. The main reason for such assumption is that the receivers have considerably higher detection threshold than sensitivity. Also, the RSS measurements are usually performed after correct decoding of the information data out of the received data packets [9]. Besides, the effect of additive noise can be somewhat compensated using methods of blind estimation of noise power , e.g., [12], [13].

The area of observation is assumed to be an square in the range of $[-w, w]$, $w \in \mathbb{R}_+$ in both x- and y- axes, in the Cartesian coordinate system. The targets and sensors are randomly distributed within the area. The ordered pair $(\check{x}_k, \check{y}_k)$ stands for the coordinate of k^{th} sensor node, while target n is located at the unknown position (x_n, y_n). Assuming that fusion center (FC) acquires the values of RSS r_k of the k^{th} sensor error-freely upon successful communication from SN, it has to solve the following system of nonlinear and non-convex equation to find the position (x_n, y_n) of each target:

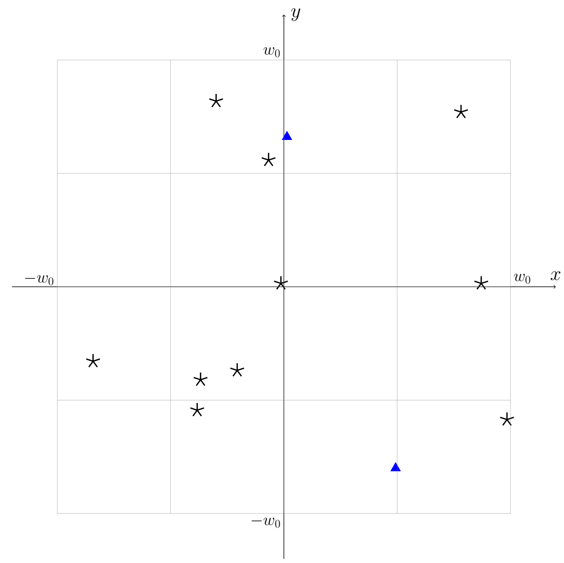

Figure 1. A wireless sensor network consisting of $K = 10$ passive sensors ($\star$) and $N = 2$ targets ($\blacktriangle$) . The grid granularity is $G = \sqrt{M} = 5$, which means the area of interest is divided to $(\sqrt{M} - 1)^2 = 16$ smaller sugars. This leads to $M = 25$ GPs, around which we look for the targets. The size of each grid square is $\Delta_g = \frac{w}{2}$.

$$r_k = \sum_{n \in \mathbb{F}_N} \frac{p_n\, 10^{\frac{\zeta_{kn}}{10}}}{\left(\sqrt{(x_n - \check{x}_k)^2 + (y_n - \check{y}_k)^2}\right)^{\alpha}}\,. \tag{3}$$

III. Sum of log normal random variables

The sum of log-normal ($\mathcal{LN}$) random variables (r.vs) has an unknown probability distribution function (pdf) [14], even for sum of two r.vs. The underlying reason is that the moment generating function of the $\mathcal{LN}$ distribution is not defined, [15]. In general, $\mathcal{LN}$ distribution has a bad behavior and cannot be described by its moments. This holds, consequently, for sum log-normal ($\mathcal{SLN}$) distribution. On the other hand, all the moments of the $\mathcal{LN}$ distribution exist in closed-form. In the literature there are several works trying to approximate $\mathcal{SLN}$, cf. [16] and references therein, based on either $\mathcal{LN}$ approximation, e.g., [14], or numerical methods. Since our goal is to come up with a good location estimator, we are interested in a closed-form approximation which does not need a prior knowledge of the cumulative distribution function (cdf) of the RSS at each sensor. Therefore, we choose the Fenton-Wilkinson, [14], method which approximates $\mathcal{SLN}$ by a $\mathcal{LN}$ and matching the first and second moments. Though, this is not very accurate in low values of the r.v, but is suitable for our purpose due to its simplicity and due to the fact the all the moments of $\mathcal{SLN}$ distribution exist in closed-form. We will see in what follows, that even this simple approximation leads to a very complicated optimization problem for finding the position of the targets.

Let R_{kn} be the r.v that represents the value of RSS at sensor k due the transmit signal from n^{th} target, i.e, $r_{kn} = p_n\, d_{kn}^{-\alpha}\, 10^{\frac{\zeta_{kn}}{10}}$, then the l^{th} moment of R_{kn} is given by

$$\mathcal{E}\left(R_{kn}^l\right) = \left(p_n\, d_{kn}^{-\alpha}\right)^l \beta_{kn}^{l^2}, \tag{4}$$

where $\beta_{kn} = e^{\frac{(\ln 10)^2 \sigma_{kn}^2}{200}}$. Obviously, the mean and variance of R_{kn} read

$$\mathcal{E}(R_{kn}) = p_n\, d_{kn}^{-\alpha} \beta_{kn}\,, \tag{5}$$

$$\mathrm{Var}(R_{kn}) = (p_n\, d_{kn}^{-\alpha})^2 (\beta_{kn}^2 - 1)\beta_{kn}^2\,. \tag{6}$$

Let us assume that all σ_{kn} are equal, then define $\beta = \beta_{kn}$ for all pair of sensors and targets. A scenario that such assumption does not hold is multi-floor indoor environments [7]. We further assume that the value of β is known. Since, all the r.vs R_{kn} are pairwise independent the mean M_k and variance V_k of the r.v $R_k \coloneqq \sum\limits_{n\in\mathbb{F}_N} R_{kn}$ is given by

$$M_k = \beta g_k\,,\ g_k \coloneqq \sum_{n\in\mathbb{F}_N} p_n\, d_{kn}^{-\alpha}\,, \tag{7a}$$

$$V_k = (\beta^2 - 1)\beta^2 h_k\,,\ h_k \coloneqq \sum_{n\in\mathbb{F}_N} (p_n\, d_{kn}^{-\alpha})^2\,. \tag{7b}$$

Now, similar to [14] we approximate the r.v R_k by a $\mathcal{LN}$:

$$R_k \approx e^{\mu_k + \sigma_k X}\,, \tag{8}$$

where X is a zero-mean normal r.v with variance 1. We find the values of μ_k and σ_k such that the mean and variance of R_k equate with the ones of the r.v $e^{\mu_k+\sigma_k X}$:

$$M_k = e^{\mu_k + \frac{\sigma_k^2}{2}}\,, \tag{9}$$

$$V_k = e^{2\mu_k + 2\sigma_k^2}\,, \tag{10}$$

which results in

$$\mu_k = 2\ln(M_k) - \frac{1}{2}\ln(M_k^2 + V_k)\,, \tag{11a}$$

$$\sigma_k^2 = \ln(M_k^2 + V_k) - 2\ln M_k\,. \tag{11b}$$

Finally, we state that the $\ln r_k$ is approximately a normal r.v:

$$\ln r_k \approx \mu_k + \sigma_k X\,. \tag{12}$$

Deriving classical estimators, e.g., maximum likelihood (ML) or minimum variance unbiased (MVU) is not possible, since the exact pdf of the RSS is unknown. We are rather interested in an estimator which is unbiased and has minimal mean square error (MSE). This is different from the classical MVU estimator, since we want to solve the following optimization

$$\min_{\substack{x_n, y_n, p_n\\ n\in\mathbb{F}_N}} \quad \sum_{k\in\mathbb{F}_K} \sigma_k^2 \tag{13a}$$

$$\text{s.t.}\quad \mu_k = \ln r_k\,, \tag{13b}$$

$$\underline{P} \le p_n \le \bar{P}\,,\ \forall n\in\mathbb{F}_N\,, \tag{13c}$$

$$-\,w \le x_n \le w\,,\ \forall n\in\mathbb{F}_N\,, \tag{13d}$$

$$-\,w \le y_n \le w\,,\ \forall n\in\mathbb{F}_N\,. \tag{13e}$$

The constraint (13b) guarantees that the estimator is unbiased. Note, μ_k and σ_k both are functions of the variables x_n, y_n and p_n, $n \in \mathbb{F}_N$. Unfortunately, this problem is mathematically intractable. We, hence, propose a heuristic to solve the problem. The heuristic is based on MIQP. For so doing, we need to discretize the area by a grid of granularity of $\sqrt{M} \in \mathbb{N}$ which means M GPs in total. Let $\mathcal{G}_M^w(x, y)$ define the grid set centered at the point (x, y) of width $2w \in \mathbb{R}_+$ and M members. Indeed, it is the set of the equidistant GPs

$$\begin{aligned}\mathcal{G}_M^w(x,y) \coloneqq \big\{&(x_i, y_j)\,|\, x_i = x - w + (i-1)\Delta_g\\ &y - w + (j-1)\Delta_g), i, j \in \mathbb{F}_G\big\}\,,\end{aligned} \tag{14}$$

where $G = \sqrt{M}$ and $\Delta_g = \frac{2w}{\sqrt{M}-1}$ is the width of one grid square. Then, we discretize the observation area into the GPs $(\tilde{x}_m, \tilde{y}_m) \in \mathcal{G}_M^w(0,0)$, $m \in \mathbb{F}_M$. Fig. 1 depicts an example of the described network where the grid is given by $\mathcal{G}_{25}^w(0,0)$.

IV. MIQP

Let us momentarily assume that the N targets are located exactly at N GPs, while the other $M-N$ GPs are not occupied by any target. This assumption is not realistic, since the targets can be anywhere within the monitoring area. We, nevertheless, tackle the problem of off-grid targets later in Sec. V. Let $\tilde{d}_{km}$ denote the distance between sensor $k \in \mathbb{F}_K$ and the m^{th} GP

$$\tilde{d}_{km} = \sqrt{(\tilde{x}_m - \check{x}_k)^2 + (\tilde{y}_m - \check{y}_k)^2}\,,\ m \in \mathbb{F}_M\,. \tag{15}$$

which results in RSS $\tilde{r}_k$ at the SN k:

$$\tilde{r}_k = \sum_{m\in\mathbb{F}_M} \tilde{p}_m \tilde{d}_{km}^{-\alpha} 10^{\frac{\zeta_{kn}}{10}}\,. \tag{16}$$

Then, it is correct to write

$$\ln \tilde{r}_k = \tilde{\mu}_k + \tilde{\sigma}_k X\,, \tag{17}$$

where the variable $\tilde{p}_m$ is zero when the GP is not occupied by any target. It is, otherwise, equal to the transmit power of the corresponding target, i.e.,

$$\tilde{p}_m = \begin{cases} 0\,, & \nexists\, n \in \mathbb{F}_N \mid (x_n, y_n) = (\tilde{x}_m, \tilde{y}_m) \\ p_n\,, & \exists\, n \in \mathbb{F}_N \mid (x_n, y_n) = (\tilde{x}_m, \tilde{y}_m) \end{cases}. \tag{18}$$

Using (7) and (11), we can derive the values of $\tilde{\mu}_k$ and $\tilde{\sigma}_k^2$ as

$$\tilde{\mu}_k = \ln\beta + 2\ln g_k - \frac{1}{2}\ln\left(g_k^2 + (\beta^2 - 1)h_k\right)\,, \tag{19}$$

$$\tilde{\sigma}_k^2 = \ln\left(g_k^2 + (\beta^2 - 1)h_k\right) - 2\ln g_k\,, \tag{20}$$

where

$$\tilde{g}_k = \sum_{m\in\mathbb{F}_M} \tilde{p}_m\, \tilde{d}_{km}^{-\alpha}\,, \tag{21}$$

$$\tilde{h}_k = \sum_{m\in\mathbb{F}_M} \left(\tilde{p}_m\, \tilde{d}_{km}^{-\alpha}\right)^2\,. \tag{22}$$

It is ideal to minimize the MSE $\sum\limits_{k\in\mathbb{F}_K} \tilde{\sigma}_k^2$ subject to $\mathcal{E}(\ln \tilde{r}_k) = \tilde{\mu}_k = \ln r_k$ for the localization. The constraint imposes the estimator to be unbiased. Since both $\tilde{\mu}_k$ and $\tilde{\sigma}_k^2$ are complicated functions of the variables $\tilde{x}_m$, $\tilde{y}_m$ and $\tilde{p}_m$, we rather try to minimize $(\ln r_k - \tilde{\mu}_k)^2$ via the following optimization problem:

$$\min_{\substack{s_m, \tilde{p}_m, \\ m \in \mathbb{F}_M}} \sum_{k \in \mathbb{F}_K} (\ln r_k - \tilde{\mu}_k)^2 \tag{23a}$$
$$\text{s.t.} \quad \tilde{p}_m \in \mathbb{R}_+ \,, \; \forall m \in \mathbb{F}_M \,, \tag{23b}$$
$$s_m \, \underline{P} \leq \tilde{p}_m \leq s_m \, \bar{P} \,, \; \forall m \in \mathbb{F}_M \,, \tag{23c}$$
$$s_m \in \{0, 1\} \,, \; \forall m \in \mathbb{F}_M \,, \tag{23d}$$
$$\sum_{m \in \mathbb{F}_M} s_m = N \,. \tag{23e}$$

Note that r_k is the RSS reading at k^{th} sensor. The constraint (23b) and (23c) take our knowledge of the upper and lower bound of the transmit power, i.e., eq. (1), into consideration. Also, constraints (23d) and (23e) make sure that exactly N GPs are chosen as the location of the targets.

Because of the binary variables s_m, the problem (23) is non-convex. It involves the continuous variables $\tilde{p}_m$ which makes it an MIQP problem. Such a problem is $\mathcal{NP}$-hard [17]. Yet, because of its quadratic objective it could be solved fast and efficiently using branch and bound method, [18], for low number of GPs. This hands in an accurate solution in case targets are located exactly at assumed GPs. Since, the targets are not at grid points, in general, the number of gird points needs to be very big for an accurate positioning. Unfortunately, values of M in the order of 100 is already a big number which blows up the complexity. For this reason, in the literature such problems are tackled based on ℓ_1-minimization, e.g., [3], [4], and [19]. Such techniques impose a big number of observations, i.e., number of sensor nodes in our case, compared to the combinatorial solution. In the next section, we instead propose a heuristic based on MIQP with small value of M that adapts the GPs. The advantage of such an approach is that it avoids increasing the number of sensor nodes. More sensor nodes means increased communication overhead towards the FC, which in turn means higher hardware costs and power consumption.

V. Adaptive Grid Refinement

In order to combat with problem of off-grid targets with a low number of GPs, we need to devise a smart solution. Thus, our proposed heuristic introduces the set of new variables $d\tilde{x}_m$, $d\tilde{y}_m$ which, in each iteration, let the selected GP $(\tilde{x}_m, \tilde{y}_m)$ be adapted to $(\tilde{x}_m + d\tilde{x}_m, \tilde{y}_m + d\tilde{y}_m)$. Also, the transmit power $\tilde{p}_m$ is updated to $\tilde{p}_m + d\tilde{p}_m$ in each iteration. Let us start doing this by deriving the first order Taylor series expansion of the function $f_k(\tilde{p}_1, \cdots, \tilde{p}_M, \tilde{x}_1, \cdots, \tilde{x}_M, \tilde{y}_1, \cdots, \tilde{y}_M)$ or in short form f_k,

$$f_k \coloneqq \ln r_k - \tilde{\mu}_k = \ln r_k - \ln \beta - 2 \ln g_k + \frac{1}{2} \ln \left(g_k^2 + (\beta^2 - 1) h_k \right) \,, \tag{24}$$

w.r.t θ that stands for any of the variables $\tilde{x}_m$, $\tilde{y}_m$ and $\tilde{p}_m$

$$\frac{\partial f_k}{\partial \theta} = -\frac{2}{g_k} \frac{\partial g_k}{\partial \theta} + \frac{\frac{\partial g_k}{\partial \theta} g_k + \frac{1}{2}(\beta^2 - 1) \frac{\partial h_k}{\partial \theta}}{g_k^2 + (\beta^2 - 1) h_k} \,,$$

where

$$\frac{\partial g_k}{\partial \tilde{x}_m} = \frac{\alpha \, \tilde{p}_m \, (\check{x}_k - \tilde{x}_m)}{\tilde{d}_{km}^{\alpha+2}}, \frac{\partial h_k}{\partial \tilde{x}_m} = \frac{2\alpha \, \tilde{p}_m^2 \, (\check{x}_k - \tilde{x}_m)}{\tilde{d}_{km}^{2\alpha+2}},$$

$$\frac{\partial g_k}{\partial \tilde{y}_m} = \frac{\alpha \, \tilde{p}_m \, (\check{y}_k - \tilde{y}_m)}{\tilde{d}_{km}^{\alpha+2}}, \frac{\partial h_k}{\partial \tilde{y}_m} = \frac{2\alpha \, \tilde{p}_m^2 \, (\check{y}_k - \tilde{y}_m)}{\tilde{d}_{km}^{2\alpha+2}},$$

$$\frac{\partial g_k}{\partial \tilde{p}_m} = \tilde{d}_{km}^{-\alpha} \,, \frac{\partial h_k}{\partial \tilde{p}_m} = 2 \, \tilde{p}_m \, \tilde{d}_{km}^{-\alpha} \,.$$

Assume at the iteration i the vector $\boldsymbol{\theta}$ is given, $\boldsymbol{\theta} \coloneqq (\tilde{p}_1^{i-1}, \cdots, \tilde{p}_M^{i-1}, \tilde{x}_1^{i-1}, \cdots, \tilde{x}_M^{i-1}, \tilde{y}_1^{i-1}, \cdots, \tilde{y}_M^{i-1})$. Also, let us define a_{km}^{i-1}, b_{km}^{i-1} and c_{km}^{i-1} as, respectively, $\frac{\partial f_k}{\partial \tilde{x}_m}(\boldsymbol{\theta})$, $\frac{\partial f_k}{\partial \tilde{y}_m}(\boldsymbol{\theta})$, $\frac{\partial f_k}{\partial \tilde{p}_m}(\boldsymbol{\theta})$, Thus, f_k at i^{th} iteration can be approximated by its first order Taylor series expansion:

$$f_k \approx f_k^{i-1} + \sum_{m \in \mathbb{F}_M} a_{km}^{i-1} \mathrm{d}\tilde{x}_m + b_{km}^{i-1} \mathrm{d}\tilde{y}_m + c_{km}^{i-1} \mathrm{d}\tilde{p}_m \,,$$

where $f_k^{i-1} \coloneqq f(\boldsymbol{\theta})$ and $d\tilde{x}_m$, $d\tilde{y}_m$ and $d\tilde{p}_m$ are our optimization variables and will be explained, shortly. At the i^{th} iteration, we apply the following update rules

$$\tilde{x}_m^i = \tilde{x}_m^{i-1} + \mathrm{d}\tilde{x}_m \,, \tag{25a}$$
$$\tilde{y}_m^i = \tilde{y}_m^{i-1} + \mathrm{d}\tilde{y}_m \,, \tag{25b}$$
$$\tilde{p}_m^i = \tilde{p}_m^{i-1} + \mathrm{d}\tilde{p}_m \,. \tag{25c}$$

Then, at i^{th} iteration, given the values of $\tilde{x}_m^{i-1}$, $\tilde{y}_m^{i-1}$, $\tilde{p}_m^{i-1}$ and thus f_k^{i-1}, we solve the following problem, instead of (23)

$$\min_{\substack{s_m, \mathrm{d}\tilde{x}_m, \\ \mathrm{d}\tilde{y}_m, \mathrm{d}\tilde{p}_m \\ m \in \mathbb{F}_M}} \sum_{k \in \mathbb{F}_K} (f_k^{i-1} + \sum_{m \in \mathbb{F}_M} a_{km}^{i-1} \mathrm{d}\tilde{x}_m + b_{km}^{i-1} \mathrm{d}\tilde{y}_m + c_{km}^{i-1} \mathrm{d}\tilde{p}_m)^2 \tag{26a}$$

s.t.

$$\mathrm{d}\tilde{x}_m, \, \mathrm{d}\tilde{y}_m, \, \mathrm{d}\tilde{p}_m \in \mathbb{R} \,, \tag{26b}$$
$$s_m \, \underline{P} - p_m^{i-1} \leq \mathrm{d}\tilde{p}_m \leq s_m \, \bar{P} - p_m^{i-1} \,, \tag{26c}$$
$$- s_m \, (w + x_m^{i-1}) \leq \mathrm{d}\tilde{x}_m \leq s_m \, (w - x_m^{i-1}) \,, \tag{26d}$$
$$- s_m \, (w + y_m^{i-1}) \leq \mathrm{d}\tilde{y}_m \leq s_m \, (w - y_m^{i-1}) \,, \tag{26e}$$
$$- s_m \, \delta \leq \mathrm{d}\tilde{x}_m \leq s_m \, \delta \,, \tag{26f}$$
$$- s_m \, \delta \leq \mathrm{d}\tilde{y}_m \leq s_m \, \delta \,, \tag{26g}$$
$$s_m \in \{0, 1\} \,, \tag{26h}$$
$$\sum_{m \in \mathbb{F}_M} s_m = N \,. \tag{26i}$$

The constraints (26c)-(26e) guarantee that $\tilde{p}_m$ and $(\tilde{x}_m, \tilde{y}_m)$ are always in their admissible ranges, i.e., $[\underline{P}, \bar{P}]$ and $[-w, w]$, respectively. On the other hand, the constraints (26f) and (26g) imposes the update points x_m^i and y_m^i to lie in a square of width $\delta = \frac{w}{\sqrt{M} - 1}$ centered at the m^{th} GP.

Hopefully, defining the variables $\mathrm{d}\tilde{x}_m$ and $\mathrm{d}\tilde{y}_m$ makes it possible to come up with off-grid points as the position of the targets. To overcome the complexity associated with the MIQP nature of (26), we choose a small M and adapt the GPs iteratively via (25). Alg. 1 shows our heuristic, elaborately.

Algorithm 1 Joint estimate of the transmit power and position of multiple co-channel targets

initialization:
- set the number of GPs $M, \sqrt{M} \in \mathbb{N}$
- define the set $\mathcal{G} \coloneqq \mathcal{G}_M^w(0,0)$, using (14)
- let $(\tilde{x}_m^0, \tilde{y}_m^0) \in \mathcal{G}$, $\forall m \in \mathbb{F}_M$
- $p_m^0 \leftarrow \frac{1}{2}(\underline{P} + \bar{P})$, $\forall m \in \mathbb{F}_M$
- set the number of iterations $I \in \mathbb{N}$
- $i \leftarrow 1$

while $i \leq I$ **do**
  find optimal values $s_m^\star$, $\mathrm{d}\tilde{x}_m^\star$, $\mathrm{d}\tilde{y}_m^\star$, $\mathrm{d}\tilde{p}_m^\star$ using (26)
  update $\tilde{x}_m^i$, $\tilde{y}_m^i$ and $\tilde{p}_m^i$ using (25)
  $i \leftarrow i + 1$
end while
$\mathcal{X} \coloneqq \left\{(\tilde{x}_m^I, \tilde{y}_m^I, \tilde{p}_m^I) \,\middle|\, s_m = 1, \forall m \in \mathbb{F}_M\right\}$
return $\mathcal{X}$

Note that the set $\mathcal{X}$ denotes the estimated position and the transmit power of the targets. In each possible combination of the combinatorial problem (26), only N out of M binary variables s_m are non-zero So, in the worst case for each possible combination a quadratic programming (QP) must be solved. The total number of possibilities is $\frac{M!}{N!(M-N)!} \propto e^{N \ln(\frac{M}{N})}$. Also, QP has a complexity of $\mathcal{O}(n^3)$, where n is the number of variables, [20]. In our case $n = 3N$, because of variables $\mathrm{d}\tilde{x}_m$, $\mathrm{d}\tilde{y}_m$ and $\mathrm{d}\tilde{p}_m$. Since the total number of iterations is I, the overall complexity of Alg. 1 is $\mathcal{O}(I \frac{M^N}{N^N})$.

Also, assume for a given N instead of using Alg. 1, we increase the granularity of the original MIQP (23) by an order of I, i.e., the same as number of iterations in Alg. 1. This means increasing the number of GPs from M to MI, then complexity will be $\mathcal{O}(I^N \frac{M^N}{N^N})$. Now, we see that Alg. 1 has a complexity reduction of order I^{N-1}. Besides, increasing M from 25 to 500, i.e., $I = 20$, does not necessarily provide us enough granularity for successful localization.

VI. Simulations

We now evaluate the performance of the proposed target localization method by means of computer simulations. In the simulation setup $\bar{P} = 1$, $\underline{P} = 0.5$ and $w = 1$Km are chosen. The results are outcome of $J = 5000$ simulation realizations, in each of which the position of sensors and realization of ζ_{kn}s are random, while the transmit power and position of targets are always the same. Let the estimated position of the n^{th} target at j^{th} realization be denoted by $(\hat{x}_n^j, \hat{y}_n^j)$. Then, positioning root mean square error (PRMSE) in meters is defined by, [4]:

$$\overline{\delta} = \sqrt{\frac{1}{JN} \sum_{j=1}^{J} \sum_{n=1}^{N} \left(\hat{x}_n^j - x_n\right)^2 + \left(\hat{y}_n^j - y_n\right)^2}, \qquad (27)$$

Let the maximum positioning error at j^{th} iteration, i.e.,

$$\delta_{\max}^j \coloneqq \max_{n \in \mathbb{F}_N} \sqrt{\left(\hat{x}_n^j - x_n\right)^2 + \left(\hat{y}_n^j - y_n\right)^2}, \qquad (28)$$

be a draw of the r.v Δ. Then, the error function P_{d_0}

$$P_{d_0} \coloneqq \Pr\left(\Delta > d_0\right) = 1 - F_\Delta(d_0), \qquad (29)$$

stands for the probability that at least one of the targets is localized with an error of more than d_0 meters. Note, F_Δ is the empirical cdf of the error Δ. Similar to (27), root mean square error (RMSE) of the transmit power is defined by

$$\overline{\rho} = \sqrt{\frac{1}{NJ} \sum_{j=1}^{J} \sum_{n \in \mathbb{F}_N} \rho_n^j}, \qquad (30)$$

where $\rho_n^j \coloneqq (p_n - \hat{p}_n^j)^2$ is the square error of the estimated power value of n^{th} target at j^{th} realization.

The simulation results for Alg. 1 are given in Fig. 2 for $N = 2$ and different values of K, G, I and σ. The parameter σ represents the strength of shadowing. We assume $\sigma =: \sigma_{kn}$ for all pairs of targets/sensors are equal. The problem (26) can be optimally solved by branch and bound method, e.g., using Gurobi [21]. Fig. 2 shows, also, the simulation result of the [5, Alg. 2]. In the legend of the figures the values of $\overline{\delta}$ and $\overline{\rho}$ are shown.

We see from the figures that increasing G and I decreases P_{d_0} at the cost of higher complexity. For instance, in case of no shadowing, the positioning error for $N = 2$ is less than 100 micro meters with a probability between $75 - 97.3\%$. Also, we see for $\gamma = 30$ and 40 the probability that the positioning error of the worst target is more than 10 meters is respectively, $8 - 18\%$ and $8 - 32\%$. These number are sufficiently good in many applications, given the fact that the area of observation is 2Km $\times$ 2Km. We see that for $N = 2$, $G = 5$ and $I = 20$ and compared to [5, Alg. 2], Alg. 1 shows an improvement of $8 - 50\%$, 9% and 16% for, respectively, no-shadowing, $\gamma = 40$ and 30. It also achieves approximately 103 m decrease in PRMSE, i.e., from 246.5 to 143.8 meters, for $\gamma \to \infty$.

In case of indoor environment, where the shadowing effect becomes stronger, we could deploy more SNs to combat with shadowing. Hopefully, RSS-based localization requires cheap and not sophisticated sensors, on the one hand and on the other hand, the proposed heuristic has low-complexity. Simulation shows that for the case $\gamma = 40$, $N = 2$, $G = 7$ and $I = 50$, increasing K from 10 to 20 and 30 decreases the probability $\Pr(\Delta > 1\text{m})$ from 70.3% to, respectively, 24%. 10.3%.

Also, to see how the complexity increase results in performance loss, Fig. 3 shows P_{d_0} against d_0 for $N = 3$ targets in case of no shadowing for different values of G, K and I. We see that for $G = 11$, $K = 20$ and $I = 50$ the error probability increases from 3.28% to 12.41% in comparison with $N = 2$.

VII. Conclusion

This paper proposes an unbiased estimator for position as well as the transmit power of multiple co-channel nodes, where their signal at SNs are superimposed. The propagation scenario is assumed to be indoor or urban outdoor areas where shadowing effect is not negligible. The problem of consideration is mathematically intractable. It is also statistically challenging, since the distribution of the sum of two or more log-normal r.vs is unknown. We, nonetheless, have proposed a heuristic to solve the underlying problem by means of MIP. The method shows a low complexity for it chooses a small number of GPs and adapts them in an iterative fashion. The numerical evaluation shows that the positioning error below 10 meters, in a ground of area of $4\mathrm{Km}^2$, is very likely to be achieved by the proposed estimator.

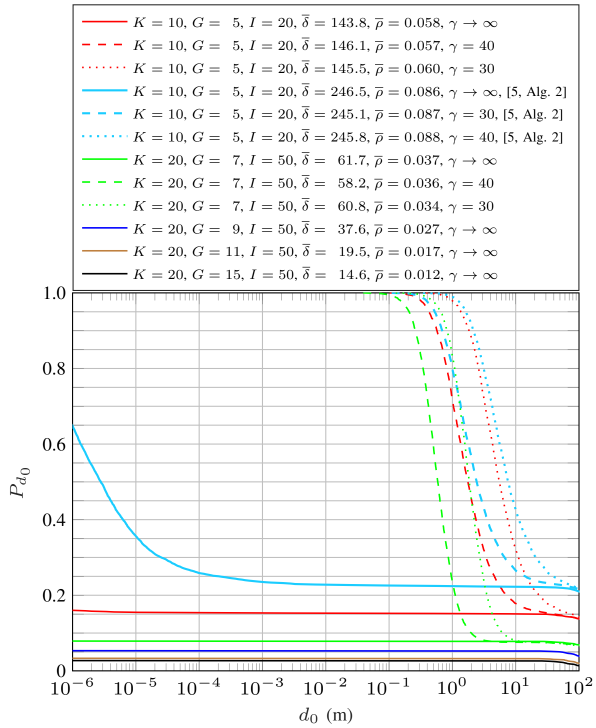

Figure 2. The error probability $\Pr(\Delta > \mathrm{d}_0)$ against positioning error d_0 for $N = 2$ targets achieved by Alg. 1 and [5, Alg. 2].

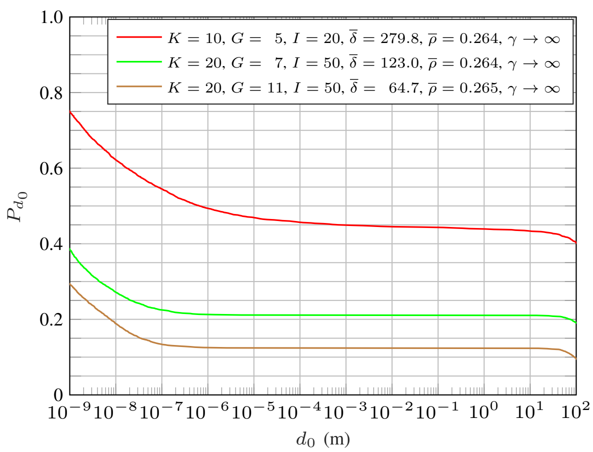

Figure 3. The error probability $\Pr(\Delta > \mathrm{d}_0)$ against positioning error d_0 for $N = 3$ targets achieved by Alg. 1.

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