Uplink Interference-based Call Admission Control for W-CDMA Mobile Communication Systems

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Abstract—In this paper, we analyze a call admission control algorithm (CAC) where decisions are based on the total interference at base stations. The proposed CAC scheme aims at guaranteeing minimal requirements of users. It utilizes information available in the network without exchanging any further information between mobiles and base stations. The basic strategy is to make an early admission decision in most cases. Rarely the algorithm has to run several steps before admitting or rejecting a new user. The convergence of the proposed algorithm is shown analytically. Numerical experiments evince the performance of our CAC scheme.

I. INTRODUCTION

Wide-band code division multiple access (W-CDMA) is widely used for third generation mobile communication systems to meet the ever growing capacity demand. Smart radio resource management is an essential building block to ensure high system performance. This can be achieved by efficient call admission control (CAC) policies by adjusting the number of admitted calls to the present interference conditions. An overview of existing CAC algorithms can be found in [1].

A particular challenge when implementing admission control for W-CDMA systems is the limited available information. Existing distributed CAC schemes rely on local interference measurements, cf. [2] and [3]. Each mobile decides autonomously how to adjust its power based on locally collected information, which necessitates interaction between mobiles and base stations. The admission decision is based on the downlink in most of the CAC schemes proposed in literature.

In this work, we propose an interference based CAC scheme for the uplink in W-CDMA systems. The scheme is carried out by the base stations using information only from handover by methods from [4]. Computations by mobiles are not needed. Furthermore, the algorithm is independent of the underlying power control scheme. When a new user arrives, we express the interference at the base stations in terms of the interference before the user has entered the system, which is known to the system, and information collected by himself. This information turns out to be sufficient for making an early admission decision in most cases.

This paper is organized as follows. In Section II we present the system model and use a technique presented in [5] to reduce the dimensionality of a system of equations necessary for obtaining a feasible power allocation. Admissibility conditions for a new user and the proposed decentralized CAC algorithm are formulated in Section III. Numerical results are given in Section IV. We present an overview of the results and discuss future work in Section V.

II. SYSTEM MODEL

We consider the uplink of a W-CDMA system. This is the restricting factor from a capacity point of view, regarding symmetric real-time traffic like voice or video telephony, see [5], [6]. These services are sensitive against reduced transmission rates contrary to packet based data services like web browsing or file transfer.

In the following we assume a fixed allocation of n mobiles to K base stations, expressed by an assignment function

$$c: \{1, \ldots, n\} \to \{1, \ldots, K\}: i \mapsto k_i$$

such that k_i denotes *i*'s serving base station. The set of mobiles allocated to base station k is denoted by $C(k) = \{i \mid k_i = k\}, k = 1, ..., K$. Hence, the sets C(1), ..., C(K) form a partition of the set $\{1, ..., n\}$.

Let p_i denote the transmit power of mobile i, and $A_{ik} \in [0, 1]$ the transmission gain from mobile i to base station k. We assume that $A_{ik} > 0$ for all $i \in C(k)$, which is obvious to avoid meaningless assignments. In our model A_{ik} is subject to slow fading effects which are assumed to be known to the transmitter. Fast fading effects are not included, as CAC decisions must not rely on effects changing on a time scale of milliseconds. The signal-to-interference ratio of user i is then given as

$$\left(\frac{E_c}{N_0}\right)_i = \operatorname{SIR}_i = \frac{A_{ik_i}p_i}{\sum_{j \neq i} A_{jk_i}p_j + \tau_{k_i}^0},\tag{1}$$

where $\tau_{k_i}^0$ denotes the general background and thermal receiver noise at base station k_i . The numerator $A_{ik_i}p_i$ represents the received power of mobile *i* at the connecting base station k_i , $\sum_{j \neq i} A_{jk_i}p_j$ collects the received interference from all other mobiles.

Let γ_i be the minimum required SIR_i of user *i*. We require that

$$SIR_i \ge \gamma_i$$
 (2)

for all mobile stations $1 \le i \le n$.

The problem now is to determine the minimum transmit power for mobiles such that (2) is satisfied. Since the numerator of (1) is increasing in p_i and the denominator is increasing in p_j , $j \neq i$, it is clear that the minimum is attained at a boundary point such that a solution $p = (p_i)_{1 \leq i \leq n}$ of the system

$$\frac{A_{ik_i}p_i}{\sum_{j\neq i}A_{jk_i}p_j + \tau_{k_i}^0} = \gamma_i, \quad 1 \le i \le n$$
(3)

is needed. Equation (3) is easily converted into the following system of linear equations,

$$\frac{1}{\gamma_i} A_{ik_i} p_i - \sum_{j \neq i} A_{jk_i} p_j = \tau_{k_i}^0, \quad i = 1, \dots, n.$$
(4)

The number of mobiles is usually large such that several thousand equations may be involved. Dimensionality reduction is an important issue, also addressed in two related papers [7] and [8]. In [5] a similar approach is used to characterize the non-existence of any feasible power allocation.

For this purpose select some base station $k \in \{1, ..., K\}$ and rewrite (4) as

$$\frac{1}{\gamma_i}A_{ik}p_i - \sum_{j \in \mathcal{C}(k) \setminus \{i\}} A_{jk}p_j = \tau_k, \quad i \in \mathcal{C}(k),$$
(5)

where $\tau_k = \tau_k^0 + \sum_{j \notin C(k)} A_{jk} p_j$ is the interference at base station k, composed of the background noise and the interference from mobiles in other cells, called external base station interference.

Proposition 1: If a solution of system (5) exists, it is given by

$$p_i = \frac{\tau_k}{A_{ik}(\frac{1}{\gamma_i} + 1)\left(1 - \sum_{j \in \mathcal{C}(k)} \frac{\gamma_j}{1 + \gamma_j}\right)} = \xi_i(k) \,\varphi_k \,\tau_k, \quad (6)$$

for all $i \in \mathcal{C}(k)$ where

$$\xi_i(k) = \left(A_{ik}\left(\frac{1}{\gamma_i}+1\right)\right)^{-1},$$

$$\varphi_k = \left(1 - \sum_{j \in \mathcal{C}(k)} \frac{\gamma_j}{1+\gamma_j}\right)^{-1}.$$

The proof of Proposition 1 can be found in [5].

The factors multiplying τ_k in (6) depend on the minimum required SIR_i and the attenuation A_{ik} only. It is hence determined by fixed system parameters, and is independent of the variables p_i .

From the third factor in the denominator of (6) it follows that there is no feasible power allocation if there exists some base station $k \in \{1, ..., K\}$ such that

$$\sum_{j \in \mathcal{C}(k)} \frac{\gamma_j}{1 + \gamma_j} \ge 1. \tag{7}$$

In the following we assume that $\sum_{j \in \mathcal{C}(k)} \frac{\gamma_j}{1+\gamma_j} < 1$ for all $k = 1, \ldots, K$ and briefly describe how the dimensionality reduction works.

With the solutions p_i from (6), $i \in \mathcal{C}(k)$, τ_k may be written as

$$\tau_k = \tau_k^0 + \sum_{\substack{j \notin \mathcal{C}(k)}} A_{jk} p_j$$

= $\tau_k^0 + \sum_{\substack{m \neq k}} \left(\varphi_m \sum_{\substack{j \in \mathcal{C}(m)}} A_{jk} \xi_j(m) \tau_m \right)$
= $\tau_k^0 + \sum_{\substack{m \neq k}} c_{mk} \tau_m, \quad k = 1, \dots, K$ (8)

with quantities $c_{mk} = \varphi_m \sum_{j \in \mathcal{C}(m)} A_{jk} \xi_j(m)$, again independent of p_i .

In order to obtain a compact representation of system (8) we define the nonnegative $K \times K$ matrix $C = (c_{mk}\bar{\delta}_{mk})_{m,k=1,...,K}$, where $\bar{\delta}_{mk} = 1 - \delta_{mk}$ denotes the complementary Kronecker delta such that C has diagonal entries 0 and non-diagonal entries c_{mk} . Then (8) reads as

$$(\boldsymbol{I} - \boldsymbol{C}')\boldsymbol{\tau} = \boldsymbol{\tau}^0 \tag{9}$$

with the obvious notation $\tau = (\tau_1, \ldots, \tau_K)'$ and $\tau^0 = (\tau_1^0, \ldots, \tau_K^0)'$. The mobile's power can be computed by (6) whenever τ is available, which in turn is obtained from the algorithm presented in Section III.

The uplink transmission gain A_{ik} from mobile *i* to base station *k* is usually not available directly. However, each mobile *i* measures the downlink transmission gain A_{ik}^m from its neighboring base stations for handover purposes. If the frequency used for up- and downlink are close-by, the path gains will essentially differ in fast fading effects only. Averaging A_{ik}^m at mobiles over sufficiently many values then leads to a reliable estimate \hat{A}_{ik} of A_{ik} . The estimated values \hat{A}_{ik} are reported to the base stations and used as uplink path gains in the following admission control algorithm.

III. A DECENTRALIZED CAC ALGORITHM

In what follows we assume that there exists a feasible power allocation for n users before trying to admit a new user n + 1. This means that there exists a power allocation which minimizes the transmitted power subject to fulfilling individual quality demands for each mobile. As explained in Section II, a feasible power allocation corresponds to a feasible external base station interference vector. Let τ^{old} be the feasible base station interference vector prior to admitting mobile n + 1. Then by (9), τ^{old} fulfills the following system of equations

$$(\boldsymbol{I} - \boldsymbol{C}')\boldsymbol{\tau}^{\text{old}} = \boldsymbol{\tau}^0.$$
(10)

The base station interference vector $\boldsymbol{\tau}^{\text{old}} = (\tau_1^{\text{old}}, \dots, \tau_K^{\text{old}})'$ can be calculated at the base station as follows. The total interference $\tau_k^{\text{tot}} = \tau_k^0 + \sum_{i=1}^n A_{ik}p_i$ can be measured directly at base station k. Additionally

$$\mathrm{SIR}_i = \frac{A_{ik}p_i}{\tau_k^0 - A_{ik}p_i}$$

for each $i \in C(k)$ is accessible by direct measurements or can be determined from the bit error rate of the received signal. Thus,

$$A_{ik}p_i = \frac{\mathrm{SIR}_i \tau_k^{\mathrm{tot}}}{1 - \mathrm{SIR}_i}$$

and hence

$$\tau_k^{\text{old}} = \tau_k^{\text{tot}} - \sum_{i \in \mathcal{C}(k)} A_{ik} p_i$$

In summary, au^{old} is accessible at the base stations.

Without loss of generality, let user n + 1 be assigned to base station K, that means $n + 1 \in \mathcal{C}(K)$, otherwise relabel the base stations. Let $\tilde{C}, \tilde{C}(k), \tilde{\xi}_i(k), \tilde{\varphi}_k, \tilde{c}_{mk}$ be defined as above but user n + 1 included. We assume that τ^0 with user n + 1 included is the same as in the case of n users. To obtain a feasible power allocation, according to Section II, it is sufficient to solve the following system of equations

$$(I - \tilde{C}')\tau = \tau^0.$$
(11)

Let τ^{new} be the solution to (11), provided there exists any. The vector τ^{new} is the external base station interference vector in the case of n+1 users. Our aim is to use the known base station interference vector τ^{old} to compute the unknown vector τ^{new} . Therefor we transform (11) to

$$(I - C'B - \Delta)\tau = \tau^0, \qquad (12)$$

where C' denotes the transpose of matrix C, and furthermore

$$\boldsymbol{B} = \operatorname{diag}(1, \dots, 1, \frac{\varphi_K}{\varphi_K}),$$
$$\boldsymbol{\Delta} = (d_{ij})_{1 \le i,j \le K}, \text{ with}$$
$$d_{ij} = \begin{cases} 0, & \text{if } j \ne K \text{ or } i = j = K\\ \tilde{\varphi}_K \tilde{\xi}_{n+1}(K) A_{n+1,i}, & \text{if } j = K \text{ and } i \ne K \end{cases}.$$

The $K \times K$ matrix C depends on the path gains and the spreading gains. It can be computed by the base stations, however, our CAC scheme does not require precise knowledge of C in the majority of cases. Matrix B depends on the spreading gains only and Δ on the spreading gains and the path gains from mobile n + 1 to each base station. This information is available at each base station for performing handover control.

Let $\rho(A)$ denote the spectral radius of a matrix $A \in \mathbb{R}^{K \times K}$. We resume the results so far in the subsequent

Proposition 2: The following statements are equivalent.

- 1) Mobile n + 1 is admissible.
- 2) $(I C'B \Delta)\tau = \tau^0$ has a positive solution.
- 3) $\rho(C'B + \Delta) < 1.$
- 4) $\lim_{l\to\infty} (C'B + \Delta)^l = 0.$

A proof of the equivalence of the last three statements is given in [4, pp. 618, 619]

The following iteration establishes the basis of our CAC scheme.

$$\boldsymbol{\tau}(l) = (\boldsymbol{C}'\boldsymbol{B} + \boldsymbol{\Delta})\,\boldsymbol{\tau}(l-1) + \boldsymbol{\tau}^0, \quad l = 1, 2, \dots \quad (13)$$

This iteration converges to τ^{new} if there exists a positive solution to (11).

Proposition 3: Assume $\rho(C'B + \Delta) < 1$ and let $\tau^{\text{new}} > 0$ be the solution to (11). Then it holds that

$$\lim_{l\to\infty} \boldsymbol{\tau}(l) = \boldsymbol{\tau}^{\text{new}}$$

for any initial value $\tau(0)$.

Proof: Successive application of (13) yields

$$\boldsymbol{\tau}(l) = (\boldsymbol{C}'\boldsymbol{B} + \boldsymbol{\Delta})^{l}\boldsymbol{\tau}(0) + \left[\sum_{i=0}^{l-1} (\boldsymbol{C}'\boldsymbol{B} + \boldsymbol{\Delta})^{i}\right]\boldsymbol{\tau}^{0}.$$
 (14)

As $\rho(C'B + \Delta) < 1$ by assumption, Proposition 2 yields

$$(\boldsymbol{I} - (\boldsymbol{C}'\boldsymbol{B} + \boldsymbol{\Delta}))^{-1} = \sum_{i=0}^{\infty} (\boldsymbol{C}'\boldsymbol{B} + \boldsymbol{\Delta})^{i}.$$
 (15)

In particular, it holds that

$$\lim_{l\to\infty} (\boldsymbol{C}^\top \boldsymbol{B} + \boldsymbol{\Delta})^l = 0$$

as the series in (15) converges, and

$$\lim_{l\to\infty} \left[\sum_{i=0}^{l-1} (\boldsymbol{C}'\boldsymbol{B} + \boldsymbol{\Delta})^i\right] \boldsymbol{\tau}^0 = \boldsymbol{\tau}^{\text{new}},$$

using (12) and (15). Thus, we get

$$\begin{split} &\lim_{l \to \infty} \boldsymbol{\tau}(l) \\ &= \lim_{l \to \infty} (\boldsymbol{C}' \boldsymbol{B} + \boldsymbol{\Delta})^l \boldsymbol{\tau}(0) + \left[\lim_{l \to \infty} \sum_{i=0}^{l-1} (\boldsymbol{C}' \boldsymbol{B} + \boldsymbol{\Delta})^i \right] \boldsymbol{\tau}^0 \\ &= 0 \cdot \boldsymbol{\tau}(0) + \boldsymbol{\tau}^{\text{new}} = \boldsymbol{\tau}^{\text{new}}. \end{split}$$

which concludes the proof.

Speed of convergence can be improved and good approximations at early terminations can be achieved by choosing tailored initial values. In what follows, a reasonable initial value is derived.

Proposition 4: Any iteration value $\tau(l), l \in \mathbb{N}$, is lower bounded by

$$\boldsymbol{\tau}(l) \ge \sum_{j=0}^{l} \boldsymbol{\Delta}^{j} \, \boldsymbol{\tau}^{\text{old}}.$$
 (16)

Moreover

$$\boldsymbol{\tau}^{\text{new}} \ge (\boldsymbol{I} - \boldsymbol{\Delta})^{-1} \, \boldsymbol{\tau}^{\text{old}}.$$
 (17)

Proof: The first result is proven by induction. Consider the case l = 1,

$$\boldsymbol{\tau}(1) = \left(\boldsymbol{C}'\boldsymbol{B} + \boldsymbol{\Delta}\boldsymbol{B}\right)\boldsymbol{\tau}^{\text{old}} + \boldsymbol{\tau}^{0}.$$

As B is componentwise nonnegative, we get

where (10) is used in the last equation. Assume that for some $l \in \mathbb{N}$ (16) holds. Then it follows

Since B is nonnegative we obtain

$$egin{aligned} oldsymbol{ au}(l+1) &\geq & (oldsymbol{C}'+oldsymbol{\Delta})\sum_{j=0}^loldsymbol{\Delta}^joldsymbol{ au}^{ ext{old}} \ &= & oldsymbol{C}'oldsymbol{ au}^{ ext{old}}+oldsymbol{C}'\sum_{j=1}^loldsymbol{\Delta}^joldsymbol{ au}^{ ext{old}}+\ && \sum_{j=1}^{l+1}oldsymbol{\Delta}^joldsymbol{ au}^{ ext{old}}+oldsymbol{ au}^0. \end{aligned}$$

As $C' \sum_{j=1}^{l} \Delta^{j} \tau^{\text{old}}$ is componentwise nonnegative too, we get from (10)

$$egin{array}{lll} m{ au}(l+1) &\geq & -(m{I}-C')m{ au}^{\mathrm{old}} + \sum_{j=0}^{l+1}m{\Delta}^j\,m{ au}^{\mathrm{old}} + m{ au}^0 \ & = & \sum_{j=0}^{l+1}m{\Delta}^j\,m{ au}^{\mathrm{old}} \end{array}$$

which proves (16). As $\rho(\boldsymbol{\Delta}) = 0 < 1$ it holds that

$$\sum_{j=0}^{\infty} \Delta^j = (I - \Delta)^{-1}$$

and thus (17) follows from

$$oldsymbol{ au}^{ ext{new}} = \lim_{l o \infty} oldsymbol{ au}(l) \geq \lim_{l o \infty} \sum_{j=0}^l oldsymbol{\Delta}^j oldsymbol{ au}^{ ext{old}} = (oldsymbol{I} - oldsymbol{\Delta})^{-1} oldsymbol{ au}^{ ext{old}}.$$

As shown in Proposition 4, the new external base station interference vector τ^{new} is bounded below by $(I - \Delta)^{-1} \tau^{\text{old}}$. This lower estimate also forms a good initial value, as simulations reveal, cf. Section IV. Thus, we suggest the initial value

$$\boldsymbol{\tau}(0) = (\boldsymbol{I} - \boldsymbol{\Delta})^{-1} \boldsymbol{\tau}^{\text{old}}$$

As mentioned in Section II, there is no feasible power allocation if there exists some base station $k \in \{1, ..., K\}$ such that (7) is fulfilled. Therefor, the first step of our algorithm is to check if (7) holds.

Let $\tau_i^{\min}, \tau_i^{\max} \in \mathbb{R}$ be admission threshold values, $1 \le i \le K$, and $\tau^{\min} = (\tau_1^{\min}, \ldots, \tau_K^{\min})', \tau^{\max} = (\tau_1^{\max}, \ldots, \tau_K^{\max})'$. Expedient values τ^{\min} and τ^{\max} are determined by simulation in Section IV.

Based on the above we propose the following CAC algorithm, see Figure 1 for the flowchart.

1) The path gains from mobile n + 1 to each base station are reported to the designated base station.



Fig. 1. CAC algorithm

- 2) If $\sum_{j \in \mathcal{C}(k)} \frac{\gamma_j}{1+\gamma_j} \ge 1$ for some $k \in \{1, \dots, K\}$, the new mobile is rejected and the algorithm stops.
- 3) Neighboring base stations transmit τ_i^{old} and the admission threshold value τ_i^{\min} and τ_i^{\max} .
- 4) If $\tau(0) < \tau^{\min}$, the new mobile is admitted and the algorithm stops.
- 5) If $\tau(0) > \tau^{\max}$, the new mobile is rejected and the algorithm stops.
- 6) m iterations of (13) are performed.
- 7) The mobile is admitted if $\tau(m)$ is smaller than τ^{\max} .

If matrix C is not available, the algorithm can be implemented with m = 0.

IV. SIMULATION RESULTS

Numerical simulations are used to evaluate the performance of the proposed CAC scheme. We investigated a scenario consisting of 19 base stations located on a hexagonal grid. The mobiles are placed randomly, according to a uniform distribution. Network and transmission parameters are set to values corresponding to current third generation W-CDMA networks.

Let $t(0) = \max_k \{\tau(0)_k\}$ denote the maximal component of $\tau(0)$. Due to the random nature of the distribution of the



Fig. 2. Empirical cumulative distribution function (ecdf) of T(0) for the first user that could not be admitted n + 1, the last admissible user n and the n - 5th user.

users position, t(0) can be interpreted as a realizations of a random variable T(0). The empirical cumulative distribution function $F_{T(0)}$ is displayed in Figures 2(a) and 2(b) for 12kb voice and 64kb data connections respectively. Comparing the graph for user n - 5, which is the fifth last user that can be admitted, with the graph for user n + 1 indicates that they can be clearly separated. Choosing an appropriate threshold τ_{\min} allows for an very early admission decision even for high system load. With high probability, no iterations of (13) are necessary.

Our simulations suggest to choose the value of $\tau_k^{\max} = \tau_{\max}$ independent of the users traffic type and independent of k. The value of $\tau_k^{\min} = \tau_{\min}$ depends on the traffic type. However, scaling τ_{\min} with the traffic type dependent factor $\frac{\gamma_i}{\gamma_{i+1}}$ leads to a lower threshold, which is again independent of the traffic type and independent of k.

For the few cases where iterations of (13) are necessary, either a fast convergence to values well below τ_{max} or divergence, resulting in values above the threshold τ_{max} , can be observed in the simulations.

V. CONCLUSION

In this paper, we have introduced a new CAC policy, which is easy to implement and essentially depends on information which happens to be available at the base station for handover purposes. It is furthermore shown that the full iteration scheme converges to the optimal power allocation.

The simulation results indicate that in most cases after the initial step no further iterations are necessary for a final decision. Hence, early termination of the access control scheme induces only small errors. The computational load for admission control can thus be kept small while almost maintaining the full network capacity. This recommends the proposed algorithm as a fast and efficient candidate for call admission control.

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