Location Tracking of Mobiles in Cellular Radio Networks

Martin Hellebrandt and Rudolf Mathar

***Abstract*— Some useful services in cellular radio networks and also a class of handover algorithms require knowledge of the present position and velocity of mobiles. This paper deals with a method to track mobiles by on-line monitoring of field strength data of surrounding base stations at successive time points. Such data is available in present global system for mobile communication (GSM) systems each 0.48 s and also in code-division multiple-access (CDMA) systems for transmission control. Because of strong random fluctuations of the signals, appropriate smoothing is the key point of the procedure. We develop a locally linear prediction model of successive positions as a basis for Kalman filtering. This approach turns out to be extremely successful, achieving average mislocations of 70 m in simulated test runs. Further improvement is possible by using external geographical information.**

***Index Terms*— Field strength, GSM standard, Kalman–Bucy filter, least-squares estimation, smoothing.**

I. Introduction

FOR REASONS of efficient network control and for offering very useful additional services in cellular radio networks, it is necessary to know the position and velocity of mobiles. Particularly in hierarchical network structures with umbrella cells and small microcells below, allocation and handover algorithms should take account of the mobility of stations. Fast sources, with speed above a certain threshold, should be assigned to large cells, while nearly stationary transmitters can be served by microcells. This strategy reduces the number of handoffs, and, hence, the traffic load in the fixed network.

If the location of a mobile is known, additional services can be offered to subscribers. Knowing the position of vehicles in a transport system, e.g., allows for an efficient planning and use of resources. Also, in case of a car breakdown or an emergency call, automatic monitoring of the position would be of great help for immediate assistance. Many further applications can be imagined.

Two quantities can be used to obtain distance and speed information: signal strength of different base stations measured at a mobile and corresponding propagation times. Both parameters are subject to strong irregular variations caused by short-term fading, shadowing and reflections. In this paper, we focus on signal strength measurements and present a method how to obtain usable distances from noisy data. First, a locally linear model of a mobile's motion is developed. Minimum least-squares estimation in this model leads to recursive Kalman filtering, which yields very reliable estimates. This improves our previously suggested smoothing procedure by linear regression (see [6]). The same method could also be used in code-division multiple-access (CDMA) systems, where accurate field strength measurement are available for transmission control.

The following is dedicated to a short review of the literature on mobile tracking methods. First attempts to monitor the position of vehicles arose from the need of knowing the disposition and status of vehicles in transport systems (see [15] and [16] for an overview). Reference [3] describes a system where the signal strength of a mobile's transmitter is measured on a statistical basis by a set of base stations. From *a priori* information of the corresponding contours, the most probable location of the mobile is determined. Concerning the basic idea, this approach is related to what we will develop in our paper. However, in [3] no feasible search procedure for realistic scenarios is offered.

Using elementary geometric considerations, and a least-squares estimate to smooth measurement errors, [17] develops a trilateration method based on radio frequency travel time measurements between the vehicle and fixed sensors located at the edges of a square. This method was further pursued by [4] for channel allocation in cellular networks. Refined trilateration techniques for road environments, using time delays between reception of a mobile's transmission at different nodes as input data, are investigated by [19].

Signal strength measurements are used in [5] to assign a mobile to a certain base station coverage zone. A channel allocation algorithm is introduced which uses this information as a basic ingredient. In [2], the area of interest is divided into small subareas which are (not uniquely) characterized by a list of discrete signal power values from different base stations. However, a nonsatisfying behavior of the system is reported, if complicated shadowing environments are considered.

Recently, in [10] and [11] adaptive schemes based on hidden Markov models, neural networks, and pattern recognition methods have been employed to estimate the position of mobiles.

Some work has also been devoted to the estimation of velocity only. The elapsed time until a cell handoff in a picocell occurs, yields a rough estimate of a mobile's speed and forms the basis for a handover algorithm in [18]. Reference [13] uses the number of level crossings of the average signal level to estimate the velocity. Reference [9] develops a method

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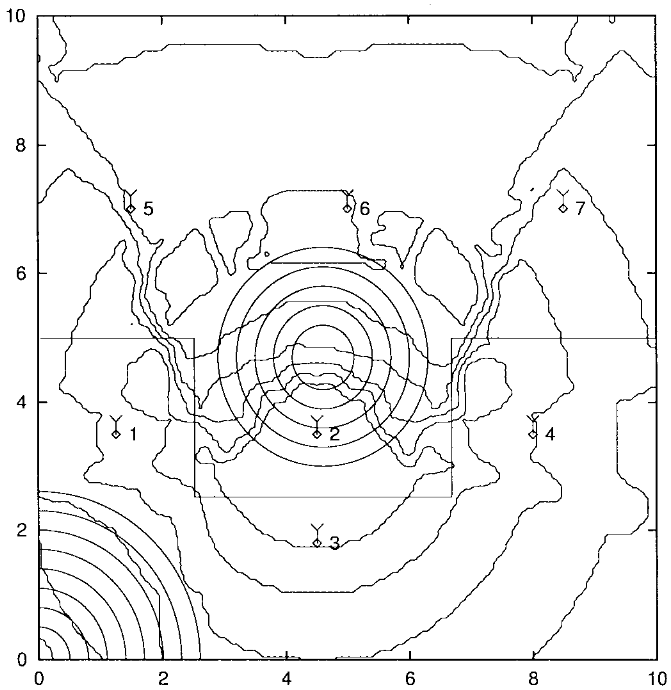

Fig. 1. Base stations, the mobile's track, and isoclines of base station 2.

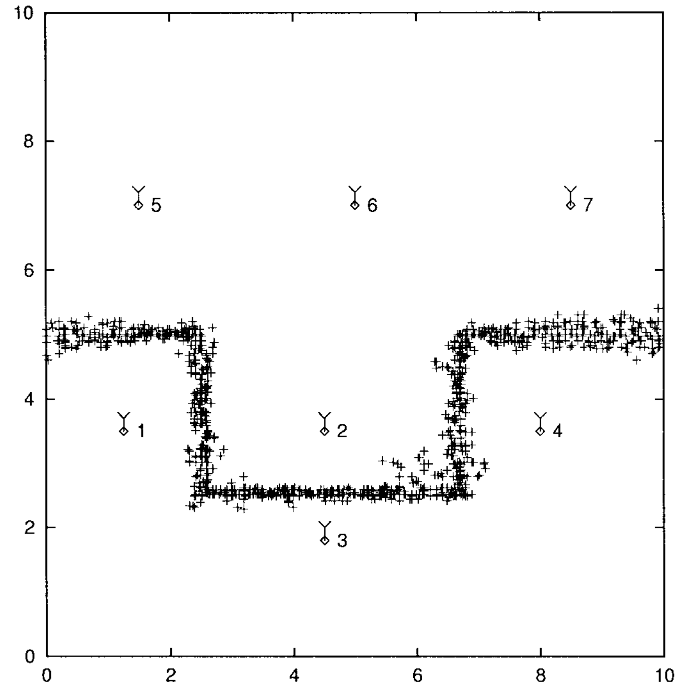

Fig. 2. Unsmoothed successive estimated positions $\mathbf{y}(t_k)$.

for velocity estimation if diversity reception is available. The authors determine the speed of a mobile via the estimated expected diversity branch switching rate between two diversity branches using the Doppler effect.

It should be mentioned that Kalman filtering is also used for aircraft tracking by radar, for an overview, cf. the recent book [1].

II. Getting Distance Information from Field Strength Data

We assume that there are n base stations in the relevant area $A \subset R^2$. The average signal power $s_i(\mathbf{z})$ is supposed to be known for each base station $i = 1, \ldots, n$ and each location $\mathbf{z} \in A$.

There exist planning tools which predict the average signal power quite accurately also in complicated environments [7], [12]. Even a three-dimensional (3-D) model is available in [14]. These tools may be used to determine the average signal strength at any position in certain scenarios. Moreover, such data can also be scanned in a field test while moving through the area of interest with a monitoring equipment.

Let $\gamma_i(t)$, $i = 1, \ldots, n$ denote the measured signal strength of base station i at a certain position. To estimate this position by a least-squares approach, the following problem has to be solved:

$$\text{minimize} \quad g(\mathbf{z}) = \sum_{i=1}^{n} (s_i(\mathbf{z}) - \gamma_i(t))^2 \quad \text{over} \quad \mathbf{z} \in A. \qquad (1)$$

A corresponding solution $\mathbf{y}(t) = (y_1(t), y_2(t))'$ estimates the position of a mobile at time t.

To solve (1), complete knowledge of the irregular average power surfaces $s_i(\mathbf{z}) : A \to R^+$ is required. For practical purposes it is sufficient to solve (1) over $\mathbf{z}$ on a grid $\mathcal{L} \subset A$ with grid constant of 25 m, say, in both coordinates. Then (1) is a finite optimization problem. After having determined a reliable estimator $\mathbf{y}(t_\ell)$, subsequent minima must be searched only over a local area of grid values in $\mathcal{L}$ surrounding $\mathbf{y}(t_\ell)$, since the mobile's distance from $\mathbf{y}(t_\ell)$ is bounded by the radius $v \cdot \Delta t$, v the maximum velocity, and Δt the time between subsequent measurements. The minimization can be carried out on line by complete enumeration and yields a fast method to trace the mobile by estimates $\mathbf{y}(t_k) \in \mathcal{L}$, $k = 0, \ldots, m$.

To test the above described procedure, we consider a relevant area of 10 × 10 km with seven base stations, whose positions are depicted in Fig. 1. Moreover, the average signal strength isoclines of base station 2 are included, deflected by two hills located at coordinates (0,0) and (4.8,4.8). The hills are indicated by circles. A mobile is moving from the left to the right margin with constant speed of 100 km/h on a track containing a quarter right turn, two quarter left turns, and again a quarter right turn. The mobile can be observed for 9 min in the relevant area, and its way is represented by a solid line in Fig. 1. Corresponding field strength data were simulated by GOOSE, a simulation tool developed at ComNets, Aachen University of Technology. This is the same scenario we used in [6].

Successive positions $\mathbf{y}(t_k)$, $k = 0, \ldots, 1120$ were estimated by solving the discrete optimization problem described above. Corresponding values are depicted in Fig. 2, which show strong irregular variations. An appropriate smoothing procedure is the key point to achieve an accurate estimate of the mobile's track.

III. Smoothing by Kalman Filtering

We first introduce a stochastic model to describe the random nature of the measurements $\mathbf{y}(t_k) = (y_1(t_k), y_2(t_k))$ obtained from (1). In the following, $'$ denotes the transpose of a vector or a matrix. Define a four-dimensional (4-D) stochastic process

$$\mathbf{X}(t) = (X_1(t), X_2(t), V_1(t), V_2(t))', \qquad t \in R.$$

$X_1(t), X_2(t)$ denote the x and y coordinates of a mobile's random position and $V_1(t)$, $V_2(t)$ the x and y coordinates of the velocity vector at time t. Observations are taken at discrete time points $t_k = t_0 + \Delta t \cdot k$, $k \in N_0$. We assume that $\mathbf{X}(t_k)$ satisfies the discrete linear recursion

$$\mathbf{X}(t_{k+1}) = \mathbf{\Phi X}(t_k) + \mathbf{\Gamma W}(t_k), \qquad k \in N_0 \tag{2}$$

where $\mathbf{\Phi}$ and $\mathbf{\Gamma}$ are the following matrices:

$$\mathbf{\Phi} = \begin{pmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \mathbf{\Gamma} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \Delta t & 0 \\ 0 & \Delta t \end{pmatrix}.$$

$\mathbf{W}(t_k) = (W_1(t_k), W_2(t_k))'$, $k \in N_0$ are stochastically independent random errors, two-dimensional (2-D) normally distributed with expectation $\mathbf{0}$ and covariance matrix $\mathbf{Q} = \sigma^2 \mathbf{I}_2$, denoted by $\mathbf{W}(t_k) \sim N(\mathbf{0}, \mathbf{Q})$. $\mathbf{I}_\ell$ denotes the identity matrix of order ℓ.

The interpretation of (2) is as follows. If the mobile station is located at $(X_1(t_k), X_2(t_k))'$ with velocity vector $(V_1(t_k), V_2(t_k))'$ at time t_k, then after time Δt it has moved to position $(X_1(t_k), X_2(t_k))' + \Delta t \cdot (V_1(t_k), V_2(t_k))'$. The components of the actual velocity vector are now changed by a random amount $\Delta t \cdot (W_1(t_k), W_2(t_k))'$.

The (random) acceleration from time t_k to t_{k+1} is given by

$$A(t_k) = \|(W_1(t_k), W_2(t_k))'\| = \left(W_1^2(t_k) + W_2^2(t_k)\right)^{1/2}.$$

It is well known that $A(t_k)$ is Rayleigh distributed with parameter σ^2 and expectation

$$E(A(t_k)) = \sigma\sqrt{\pi/2}. \tag{3}$$

This allows for estimating σ^2 from an estimator $\bar{a}$ of $E(A(t_k))$ by setting $\hat{\sigma}^2 = 2\bar{a}^2/\pi$.

$\mathbf{X}(t_k)$ cannot be observed directly, due to the random disturbances by fading and shadowing. To take these effects into account the estimated positions $(y_1(t_k), y_2(t_k))'$ are modeled by independent additive random errors as

$$\mathbf{Y}(t_k) = \mathbf{M}\,\mathbf{X}(t_k) + \mathbf{U}_k, \qquad k \in N_0 \tag{4}$$

where $\mathbf{M} = \left(\begin{smallmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{smallmatrix}\right)$ and $\mathbf{U}_k \sim N(\mathbf{0}, \mathbf{R})$. If $\mathbf{x}(t_k)$ are known positions in a test scenario, $\mathbf{R}$ can be estimated from observed values $\mathbf{y}(t_k)$ in the usual way.

Equations (2) and (4) form a discrete linear difference equation with white Gaussian noise. The state at time t_k is estimated by the variance minimal conditional expectation of $\mathbf{X}(t_k)$ given previous observations $\tilde{\mathbf{Y}}(t_k) = (\mathbf{Y}(t_0), \ldots, \mathbf{Y}(t_k))$ as

$$\hat{\mathbf{X}}_k(t_k) = E[\mathbf{X}(t_k) \mid \tilde{\mathbf{Y}}(t_k)]$$

and the predicted value at time t_k as

$$\begin{aligned} \hat{\mathbf{X}}_{k-1}(t_k) &= E[\mathbf{X}(t_k) \mid \tilde{\mathbf{Y}}(t_{k-1})] \\ &= E[\mathbf{\Phi X}(t_{k-1}) + \mathbf{\Gamma W}(t_{k-1}) \mid \tilde{\mathbf{Y}}(t_{k-1})] \\ &= \mathbf{\Phi} E[\mathbf{X}(t_{k-1} \mid \tilde{\mathbf{Y}}(t_{k-1})] \\ &= \mathbf{\Phi}\hat{\mathbf{X}}_{k-1}(t_{k-1}). \end{aligned} \tag{5}$$

Corresponding covariance matrices are denoted by

$$\mathbf{C}_k(t_k) = \mathrm{Cov}[\mathbf{X}(t_k) \mid \tilde{\mathbf{Y}}(t_k)]$$

and

$$\begin{aligned} \mathbf{C}_{k-1}(t_k) &= \mathrm{Cov}[\mathbf{X}(t_k) \mid \tilde{\mathbf{Y}}(t_{k-1})] \\ &= E[(\mathbf{X}(t_k) - \hat{\mathbf{X}}_{k-1}(t_k))(\mathbf{X}(t_k) \\ &\quad - \hat{\mathbf{X}}_{k-1}(t_k))' \mid \tilde{\mathbf{Y}}(t_{k-1})] \\ &= E[\mathbf{\Phi}(\mathbf{X}(t_{k-1}) - \hat{\mathbf{X}}_{k-1}(t_{k-1}))(\mathbf{X}(t_{k-1}) \\ &\quad - \hat{\mathbf{X}}_{k-1}(t_{k-1}))'\mathbf{\Phi}' \mid \tilde{\mathbf{Y}}(t_{k-1})] + \mathbf{\Gamma Q \Gamma}' \\ &= \mathbf{\Phi}\,\mathrm{Cov}[\mathbf{X}(t_{k-1}) \mid \tilde{\mathbf{Y}}(t_{k-1})]\mathbf{\Phi}' + \mathbf{\Gamma Q \Gamma}' \\ &= \mathbf{\Phi C}_{k-1}(t_{k-1})\,\mathbf{\Phi}' + \mathbf{\Gamma Q \Gamma}'. \end{aligned} \tag{6}$$

Optimal recursive estimators of minimal variance are obtained by the Kalman–Bucy filter in the following theorem (cf., [8], p. 201).

Theorem 1: The minimum variance estimator of the state at time t_k is given by

$$\hat{\mathbf{X}}_k(t_k) = \hat{\mathbf{X}}_{k-1}(t_k) + \mathbf{K}(t_k)(\mathbf{Y}(t_k) - \mathbf{M}\hat{\mathbf{X}}_{k-1}(t_k)). \tag{7}$$

Covariance matrices are updated by

$$\mathbf{C}_k(t_k) = \mathbf{C}_{k-1}(t_k) - \mathbf{K}(t_k)\mathbf{M}\mathbf{C}_{k-1}(t_k) \tag{8}$$

where

$$\mathbf{K}(t_k) = \mathbf{C}_{k-1}(t_k)\mathbf{M}'(\mathbf{M}\mathbf{C}_{k-1}(t_k)\mathbf{M}' + \mathbf{R})^{-1} \tag{9}$$

is the Kalman gain.

With initial values $\hat{\mathbf{X}}_0(t_0)$ and $\mathbf{C}_0(t_0)$ recursion (7) can be evaluated via (5), (6), (8), and (9). The corresponding algorithm is described below in (10), and updated values are denoted by $^+$. $\mathbf{y}_k$ denotes the actual observed values.

Let $\mathbf{X}^+ = \hat{\mathbf{X}}_0(t_0)$ and $\mathbf{C}^+ = \mathbf{C}_0(t_0)$. Then iterate

$$\begin{aligned} \mathbf{X} &= \mathbf{X}^+ \\ \mathbf{C} &= \mathbf{C}^+ \\ \bar{\mathbf{C}} &= \mathbf{\Phi C \Phi}' + \mathbf{\Gamma Q \Gamma}' \\ \mathbf{K} &= \bar{\mathbf{C}}\mathbf{M}'(\mathbf{M}\bar{\mathbf{C}}\mathbf{M}' + \mathbf{R})^{-1} \\ \mathbf{C}^+ &= \bar{\mathbf{C}} - \mathbf{K}\mathbf{M}\bar{\mathbf{C}} \\ \mathbf{X}^+ &= \mathbf{\Phi X} + \mathbf{K}(\mathbf{y}_k - \mathbf{M\Phi X}). \end{aligned} \tag{10}$$

It remains to determine initial guesses of $\hat{\mathbf{X}}_0(t_0)$ and $\mathbf{C}_0(t_0)$. A reasonable choice for the first quantity is

$$\hat{\mathbf{X}}_0(t_0) = (y_0(t_0), y_1(t_0), 0, 0)'.$$

$(y_0(t_0), y_1(t_0))'$ is the first estimated position and the velocity is assumed zero initially.

The above Kalman filtering setup was applied to the scenario described in Section II with raw data represented in

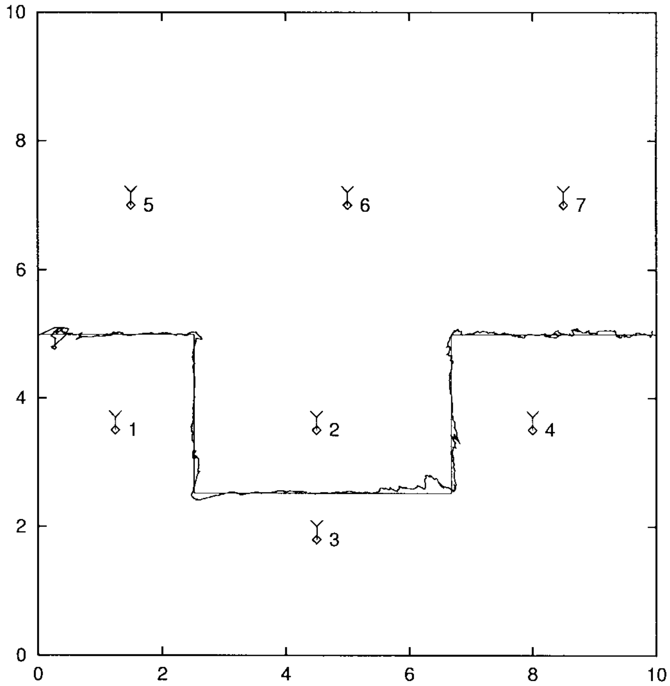

Fig. 3. Estimated track after Kalman filtering.

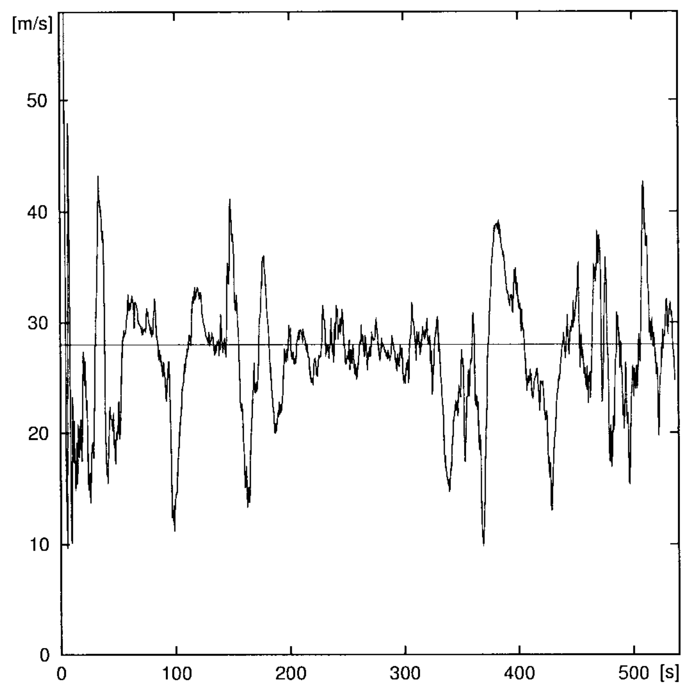

Fig. 4. Estimated velocity (m/s) after Kalman filtering.

Fig. 2. A reasonable initial value of $\mathbf{C}(t_0) = \mathrm{Cov}(\mathbf{X}(t_0) \mid \mathbf{Y}(t_0))$ is

$$\mathbf{C}(t_0) = \begin{pmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{0} & 30^2 \cdot \mathbf{I}_2 \end{pmatrix}$$

with $\mathbf{R} = 22\,000 \cdot \mathbf{I}_2$. These values are motivated by the following considerations. The variance of the deviation between the true and the estimated position was estimated as 22 000 m^2 for each coordinate. This corresponds to a standard deviation of approximately 150 m per measurement. 30^2 (m/s)^2 seems to be a reasonable upper bound for the variance of the initial velocity of a mobile (30 m/s $=$ 108 km/h).

For an estimate of $\mathbf{Q} = \sigma^2 \mathbf{I}_2$, the value of σ^2 is needed. We chose an average acceleration of 1 m/s^2 and calculated σ^2 from (3) as $\sigma^2 = 2/\pi = 0.6366$.

The results turn out to be very accurate. Fig. 3 shows the smoothed estimated track. The average deviation is about 70 m. In areas where the movement of the station is straight and at least some base stations are close enough, the estimation is nearly perfect.

It should be mentioned that the simulated sharp quarter turns are at the same speed of 100 km/h as going straight, which in reality of course is impossible. Hence, it is to be expected that the estimated track does not overswing and is closer to the true track, since mobiles slow down when turning.

Fig. 4 shows the estimated speed over time, i.e., the values

$$v(t_k) = \|(V_1(t_k), V_2(t_k))\|$$

connected by straight lines. In spite of the variations, at least after further smoothing by moving averages, e.g., the values can be employed for handover algorithms.

The Kalman filtering method used in this paper has several advantages over the regression based setup in our previous paper [6]. First, it starts from a clearly defined mobility model which reflects the true physical behavior and allows for a direct simultaneous velocity estimation. Moreover, an additional parameter controlling the influence of past measurements is not needed any more. Finally, the results are more accurate, while calculating updates in (10) is still very simple and can be carried out on line.

IV. Conclusion

In this paper, a procedure is suggested to monitor the position and velocity of mobiles in a cellular radio network from field strength data of surrounding base stations. Such data is available each 0.48 s for mobile phones according to the global system for mobile communication (GSM) standard. Also, CDMA systems provide highly accurate field strength measurements for network control which could be used. Because of fading, shadowing, and reflections individual measurements are subject to strong random disturbances. However, a large sample is available if the mobile is tracked for some time. By using a linear recursive model of mobility and by smoothing via the Kalman–Bucy filter, an accurate estimated track is achieved. The average mislocation over the whole track is about 70 m. Further work will be devoted to how to include geographical information for further improvement of a mobile's estimated position.

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