Topology Generation and Power Assignment in IR-UWB Networks

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Abstract— **In this paper, two different power assignment strategies for an ultra-wideband impulse radio (IR-UWB) multi-user system are presented and compared in terms of computational complexity. The algorithms aim at finding the lowest possible transmission power for each node in the network under the constraint that individual QoS demands of all nodes are met. The first direct approach results in a high computational complexity contradicting the intention of building low complexity IR-UWB transceivers. We show how the complexity of the power assignment problem can be substantially lowered by analytical means if the network topology is taken into consideration. Additionally, an algorithm creating the necessary network topology is discussed.**

I. Introduction

UWB radio systems offer the attractive possibility of achieving high data rates with low transmission power. In IR-UWB short duration pulses with an ultra wide bandwidth of several gigahertz are transmitted (see [1]). Since no mixer and due to low transmission power no power amplifier might be necessary in the transceiver, a very low complexity hardware design with low power consumption is possible. These properties are especially interesting for wireless sensor networks relying on long battery lifetime and cheap low complexity hardware. To avoid the effort of maintaining the entire network synchronised, interference from other nodes is usually accepted. It can however be diminished by the introduction of pseudo random time hopping codes (compare [2][3]). Due to the interference limitation of IR-UWB networks questions of a fair sharing of the radio resources considering individual quality-of-service (QoS) demands of all nodes arise. This issue can be handled by an appropriate power assignment to the nodes in the network. In interference limited systems the signal-to-interference and noise ratio (SINR) is used to derive algorithms that determine the necessary transmission power to fulfill the QoS requirements. For IR-UWB this was done e.g. in [4] and [5].

In this paper, two approaches to determine minimal transmission power to maintain the QoS requirements for all nodes are described. The considered scenario is especially suited for wireless sensor networks. The first approach results in a system of linear equations with a dimensionality of the number of transmitting nodes in the network. In wireless sensor networks usually a large number of transmitting nodes is assumed leading to a high computational complexity of this approach and preventing the intended low complexity transceiver design. This flaw is omitted by the second algorithm that lowers the computational complexity by an additional consideration of the topology of the network.

The topology corresponds to a subdivision of the network into several smaller sub-networks with the goal of reducing transmission power and the administration effort like synchronisation for the coordinator nodes. In the literature, several algorithms for network topology creation with different objectives are suggested. Usually the network throughput or the connectivity of the nodes is optimised (see e.g. [6]). In this paper, a simple method based on ideas of a heuristic clustering algorithm from [7] is presented. It aims at detecting clusters in the spatial node distribution and forms a subnetwork for each cluster of nodes to decrease the aggregate transmission power of the network. Based on this topology a second algorithm for determining the optimal power assignment is described. It generalises the analytical methods described in [8] for IR-UWB and results in a reduced dimensionality of the system of equations describing the problem.

The system model for the considered impulse radio transmission scheme and the network topology is described in section II. The optimal power assignment for all nodes in the network to meet required SINRs is derived analytically in section III. In section IV the algorithm that constructs the topology of the network is discussed. The resulting network topology is used in section V to derive an algorithm for the optimal power assignment with a lowered computational complexity. The paper concludes with an overview of the results in section VI.

II. System model

We begin with a description of the system model. The notation is summarised in Table I. An ultra-wideband impulse radio network with a set $\mathcal{K}$ of nodes is considered. The network is subdivided into M so called piconets, each containing one piconet coordinator (PNC) from the set $\mathcal{M}$, serving as an administration node for several client nodes. The set of client nodes in the network is $\mathcal{L} = \mathcal{K} \setminus \mathcal{M}$ with cardinality $L = |\mathcal{L}|$.

TABLE I
NOTATION USED IN THIS PAPER

$\mathcal{K}$	Set of all nodes in the network
$\mathcal{M}$	Set of piconet coordinators
$\mathcal{L}$	Set of client nodes ($\mathcal{L} = \mathcal{K} \setminus \mathcal{M}$)
m_i	Piconet coordinator of node i
$\mathcal{C}(m) \subseteq \mathcal{L}$	Set of nodes allocated to piconet coordinator m
T_f	Pulse repetition time
T_c	Time shift step of the time hopping code
N_i	Number of pulse repetitions per information bit
N_h	Number of time hopping slots in one frame T_f
γ_i	Target SINR level of node i
P_{im}	Transmission power of node i to piconet coordinator m
g_{im}	Pathgain between node i and piconet coordinator m
η_m	Background noise energy at piconet coordinator m
$c_j^{(k)}$	Symbol j of the k-th nodes time hopping code
A	Amplitude of the transmitted pulses
$d_i^{(k)}$	Transmitted binary symbol i of node k
δ	Modulation index for pulse position modulation
$g(t)$	Pulse shaping function
m_p	Correlation of a pulse with the transceiver waveform
σ_a^2	Expected cross correlation between one interfering pulse and the receiver waveform

The network topology is described by an assignment function mapping each client node i to its PNC m_i

$$c : \mathcal{L} \to \mathcal{M} : i \mapsto m_i. \qquad (1)$$

The resulting piconets are denoted by the sets $\mathcal{C}(m)$. The topology generation is conducted in a preliminary step and is described in more detail in section IV. Communication is always accomplished from client nodes to the corresponding PNC. As multiple access scheme a pseudo random time hopping code is used to lower the probability of repeated pulse collisions from two transmitters. Binary pulse position modulation (2-PPM) is assumed as modulation scheme for all nodes. The transmitted signal of a node can then be written as

$$s(t) = A \sum_{j=-\infty}^{\infty} g(t - jT_f - c_j T_c - \delta d_{\lfloor j/N \rfloor}). \qquad (2)$$

Here A is the amplitude of the transmitted pulse, T_f is the pulse repetition time interval, c_j denotes the time hopping code of the node, T_c the corresponding time shift in the code and finally d_i is the i-th transmitted binary symbol. Each data bit is transmitted by N identical pulses to enhance the quality of reception. The function $g(t)$ shapes the impulses, e.g., to adhere to regulatory constraints. Some of the above parameters are illustrated in Figure 1.

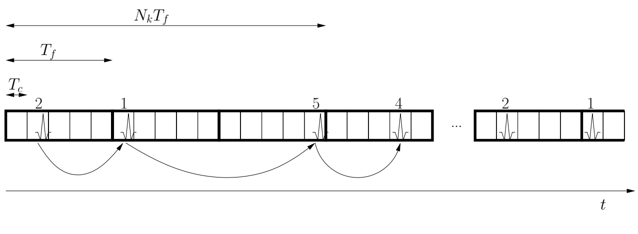

Fig. 1. Illustration of some parameters used in the system model. In the example $c^{(k)} = (2, 1, 5, 4)$, $d_1 = 1$ and $d_2 = 0$.

A coherent correlation receiver is assumed, which is the optimal receiver for the transmitted signal from (2) in AWGN channels, see [9]. This receiver correlates the received signal with a template signal $v(t)$ which is in an AWGN channel of the form

$$v(t) = g(t) - g(t - \delta). \qquad (3)$$

The output of the correlator is continuously sampled and the estimation of the transmitted signal is performed by comparing the sampled value with a given threshold.

Each transceiver node has an individual demand of the transmitted data rate and on the bit error rate for its link to the PNC. This results in a minimal target SINR γ_i for each node.

To derive the SINR after the correlator for the considered receiver additional variables have to be introduced.

$$m_p = \int_{-\infty}^{\infty} g(t)v(t)dt \qquad (4)$$

denotes the correlation of a normalised pulse with the receiver template and

$$\sigma_a^2 = \frac{1}{T_f} \int_{-\infty}^{\infty} \Big(\int_{-\infty}^{\infty} g(t-s)v(t)dt \Big)^2 ds \qquad (5)$$

represents the expected value of the cross correlation of a normalised and shifted pulse from another transmitter with the receiver template. The time shift of the pulse is assumed to be a random variable with uniform distribution between 0 and T_f. With these variables and also taking into account the pulse amplitudes and pathgains the SINR of node i can be written as

$$\text{SINR}_i = N_i \frac{(A_i m_p)^2 g_{i m_i}}{\sigma_a^2 \sum_{j \neq i} A_j^2 g_{j m_i} + \eta'_{m_i}},$$

where η'_{m_i} is the energy of the additve noise after the correlator at the receiving PNC m_i. The number of repeated pulses N_i

can be seen as a kind of processing gain. With the energy of one pulse

$$E = \int_0^{T_f} g^2(t)dt$$

the mean value of the power P can be written as

$$P = \frac{E}{T_f} = \frac{E}{N_h T_c}$$

where N_h is the number of time hopping slots ($= \max(c_j^{(i)})$) in one frame of length T_f. Controlling this value is an easy way to vary the transmission power [4]. In summary the SINR for platform i reads as

$$\text{SINR}_i = N_i \frac{g_{im_i} P_i}{\sigma^2 \sum_{j \neq i} g_{jm_i} P_j + \frac{1}{T_f} \eta_{m_i}}, \tag{6}$$

whith $\sigma^2 = \frac{\sigma_a^2}{m_p^2}$ and $\eta_{m_i} = \frac{\eta'_{m_i}}{m_p^2}$. We assume that the interference caused by other IR-UWB nodes can be modelled approximately by white Gaussian noise (standard Gaussian approximation (compare [3])). Although the noise distribution converges only very slowly to the normal distribution as the number of nodes in the network increases, this assumption is justified for the present work since the algorithms in this paper are especially designed for scenarios with a very large number of transceiver nodes.

III. Power Control for IR-UWB Networks

Power control is a means to fulfill the QoS requirements of clients in IR-UWB networks. Higher transmission power for one node provides a higher QoS for this node, but simultaneously deteriorates the SINR of all other nodes due to the increase of interference in the denominator of (6). The objective of power control algorithms is to determine minimal transmission power for each node, such that the QoS demands of all nodes are fulfilled, i.e., finding the minimal element of the power region

$$\mathcal{P}(\boldsymbol{\gamma}) = \{\boldsymbol{P} \geq 0 \mid \text{SINR}_i(\boldsymbol{P}) \geq \gamma_i,\ i = 1, ..., L\}. \tag{7}$$

Region (7) is described by a system of L linear inequalities which after some algebra may be rewritten in matrix form as

$$[\boldsymbol{I} - \boldsymbol{\Gamma} \boldsymbol{N}^{-1} \boldsymbol{B}]\boldsymbol{P} \geq \boldsymbol{\tau}, \quad \boldsymbol{\tau} > 0. \tag{8}$$

$\boldsymbol{\Gamma}$ and $\boldsymbol{N}$ are diagonal matrices with the k-th entry containing the target SINR γ_k and the number of pulse repetitions for one data bit N_k of the k-th node. The transmission powers of the nodes are combined in the vector $\boldsymbol{P}$. The entries b_{ij} of matrix $\boldsymbol{B}$ are

$$b_{ij} = \begin{cases} \sigma^2 g_{jm_i}/g_{im_i}, & i \neq j \\ 0, & i = j \end{cases}$$

and the i-th element of vector $\boldsymbol{\tau}$ contains the entry $\tau_i = \frac{\eta_{m_i}\gamma_i}{T_f N_i g_{im_i}}$. To avoid unnecessary waste of transmission energy and to minimise the interference on other systems the minimal power to achieve the target SINR should be used. Hence, the system of inequalities becomes a system of linear equations

$$[\boldsymbol{I} - \boldsymbol{\Gamma} \boldsymbol{N}^{-1} \boldsymbol{B}]\boldsymbol{P}^* = \boldsymbol{\tau}. \tag{9}$$

Here the vector $\boldsymbol{P}^*$ contains the minimal transmission power of the nodes. Consequently all entries of this vector have to be positive since negative powers can not be assigned. According to a generalisation of Perron-Frobenius theory in [10], this system of linear equations has a positive solution $\boldsymbol{P}^* > 0$, iff the spectral radius of the matrix $\boldsymbol{\Gamma} \boldsymbol{N}^{-1} \boldsymbol{B}$ is less than one. The spectral radius of a matrix is the maximal absolute value of its eigenvalues. Hence the optimal power assignment for the complete network is

$$\boldsymbol{P}^* = [\boldsymbol{I} - \boldsymbol{\Gamma} \boldsymbol{N}^{-1} \boldsymbol{B}]^{-1}\boldsymbol{\tau}, \tag{10}$$

iff the spectral radius is less than one. To compute the vector $\boldsymbol{P}^*$ the inversion of the $L \times L$ matrix $[\boldsymbol{I} - \boldsymbol{\Gamma} \boldsymbol{N}^{-1} \boldsymbol{B}]$ is necessary resulting in complexity of O(L^3). Hence, for networks with hundreds or thousands of nodes, as are considered here, this direct approach is computationally infeasible. In the next section an algorithm is presented which creates a network topology as the basis for the reducing the computational complexity of the power control problem.

IV. Topology Generation in IR-UWB Networks

Topology generation means to subdivide the whole network into several smaller ones. In the context of IEEE standards 802.15.3a and 802.15.4a these networks are called *piconets*. They are introduced to simplify the administration effort, as only the nodes in the smaller piconet instead of the complete network have to be administrated by a controller.

Piconets consist of one piconet controller and several client nodes. The network administration is performed by the piconet controller. In this paper we assume that all communication in the piconets is conducted to the PNC. This assumption makes the PNC play a similar role as a base station in a cellular communication network with the difference that the role of the PNC may be exchanged between nodes.

Several algorithms have been suggested to create a network topology. The most simple ones use a random election of the PNCs. This approach neglects the localisation capabilities of IR-UWB and by this the spatial distribution of the nodes. To take advantage of this additionally available information clustering algorithms may be used. We suggest a simple heuristic clustering approach computing both the number of piconets and the allocation of the client nodes to the piconets. The approach is based on ideas in [7]. A formal description of the algorithm is given in Algorithm 1.

It starts with an initialisation of the already introduced sets $\mathcal{M}, \mathcal{L}$ und $\mathcal{K}$. Initially the set $\mathcal{H}$ is empty, and $\mathcal{S}$ contains all nodes. The parameter $d_{\max}$ is chosen as the maximal transmission range of a transmitter. In the main loop of the algorithm the element of $\mathcal{S}$ with minimal mean distance to all other nodes within the transmission range is chosen as PNC. Afterwards all neighbouring nodes of the new PNC being in $\mathcal{S}$ are deleted from $\mathcal{S}$. If $\mathcal{S}$ is nonempty the process

Algorithm 1 Algorithm for topology generation

Initialise:
$d_{ij} \leftarrow \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \ \forall\, i,j \in \mathcal{K}$
$(\mathcal{M} = \mathcal{H}) \leftarrow \emptyset,\ (\mathcal{S} = \mathcal{L}) \leftarrow \mathcal{K},\ d_{\max}$
$\overline{d_k} \leftarrow \frac{1}{K'} \sum_{i=1}^{K'} d_{ki} \ \forall\ \{k \in \mathcal{K} | d_{ki} < d_{\max}\}$
while $\mathcal{S} \neq \emptyset$ **do**
  $k^* \leftarrow \{k \in \mathcal{S} | \overline{d_k^*} < \overline{d_k}\}$;
  $\mathcal{M} \leftarrow \mathcal{M} \cup k^*$;
  $\mathcal{L} \leftarrow \mathcal{L} \setminus k^*$;
  $\mathcal{H} \leftarrow \{k_i \in \mathcal{S} | d_{k^*i} < d_{\max}\}$;
  $\mathcal{S} \leftarrow \mathcal{S} \setminus \mathcal{H}$;
end while
$\mathcal{C}(m) \leftarrow \{n \in \mathcal{L} | d_{nm} < d_{nl}\ l \in \mathcal{M}\} \ \forall\ m \in \mathcal{M}$;
$\mathrm{CG}(\mathcal{C}(m)) \leftarrow \begin{pmatrix} \sum_{i=1}^{|\mathcal{C}(m)|} \frac{\sqrt{(x_i - x_m)^2}}{|\mathcal{C}(m)|} \\ \sum_{i=1}^{|\mathcal{C}(m)|} \frac{\sqrt{(y_i - y_m)^2}}{|\mathcal{C}(m)|} \end{pmatrix} \ \forall\ m \in \mathcal{M}$;
$i^* \leftarrow \{i \in \mathcal{C}(m) | d_{i^*CG} < d_{iCG}\} \ \forall\ m \in \mathcal{M}$;
$m \leftarrow i^* \ \forall\ m \in \mathcal{M}$;

returns to the beginning of the loop and selects the next PNC. If the loop is finished and the PNCs have been selected all client nodes are associated to the spatially nearest PNC. This algorithm however, often leads to PNCs not in the center of the piconets but close to the border. To avoid this counterintuitive drawback, in the next step the centres of gravity CG are determined for all piconets and the node nearest to the centre of gravity takes over the role of the PNC. An output of the algorithm for an exemplary scenario is visualised in Figure 2.

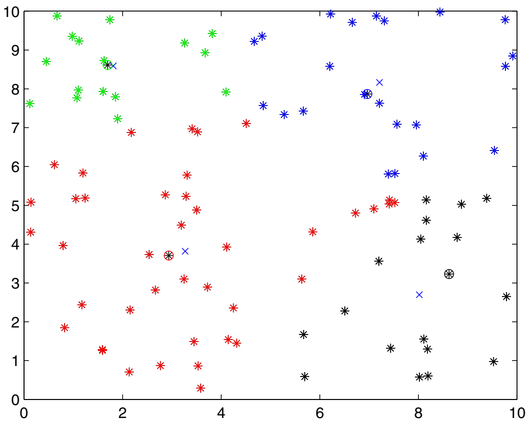

Fig. 2. A network topology generated by the described algorithm. Different colours of the client nodes, mark different piconets. Blue crosses mark the exact centres of gravity and the circled stars the PNCs.

V. Power Control considering the Network Topology

In this section, we present a way of reducing the computational complexity of the power control algorithm by dividing the network into piconets as described in section IV.

The basic idea is a reformulation of the power control problem resulting in a system of linear equations with drastically reduced dimension. For this purpose the equations describing the minimal required SINR for the nodes of one piconet with PNC, say m, can be rewritten as

$$\frac{N_i}{\gamma_i} g_{im} P_i - \sigma^2 \sum_{j \in \mathcal{C}(m) \setminus \{i\}} g_{jm} P_j = \tau_m,\ i \in \mathcal{C}(m), \qquad (11)$$

where

$$\tau_m = \frac{1}{T_f} \eta_m + \sigma^2 \sum_{j \notin \mathcal{C}(m)} g_{jm} P_j \qquad (12)$$

agglomerates the interference at piconet controller m, composed of the background and receiver noise $\frac{1}{T_f}\eta_m$ and the interference $\sigma^2 \sum_{j \notin \mathcal{C}(m)} g_{jm} P_j$ caused by nodes from other piconets. The solution P_i of the agglomerated system (11) is given by

$$P_i = \frac{1}{g_{im}(\frac{N_i}{\gamma_i} + \sigma^2)(1 - \sigma^2 \sum_{j \in \mathcal{C}(m)} \frac{\gamma_j}{N_j + \sigma^2 \gamma_j})} \tau_m = \gamma_i(m) \tau_m, \qquad (13)$$

with $i \in \mathcal{C}(m)$. The proof of this formula is given in [8]. If τ_m is known, the optimal transmission power of the nodes in piconet m can be easily computed by this formula. In the following, $\tau = (\tau_1, ..., \tau_m)$ is determined as the solution of a system of linear equations. First, the solution $P_j = \gamma_j(m)\tau_m$ is substituted in equation (12), yielding

$$\tau_m = \frac{1}{T_f} \eta_m + \sigma^2 \sum_{n \neq m} \tau_n (\sum_{j \in \mathcal{C}(n)} g_{jm} \gamma_j(n)),\ m = 1, ..., M.$$

A compact representation is obtained by setting

$$c_{nm} = \sigma^2 \sum_{j \in \mathcal{C}(n)} g_{jm} \gamma_j(n),$$

which finally leads to

$$\tau_m = \frac{1}{T_f} \eta_m + \sum_{n \neq m} c_{nm} \tau_n,\ m = 1, ..., M. \qquad (14)$$

This system of linear equations is represented in matrix form as

$$(\boldsymbol{I} - \boldsymbol{C})\boldsymbol{\tau} = \boldsymbol{\eta}, \quad \boldsymbol{\eta} > 0, \qquad (15)$$

where the $M \times M$ matrix $\boldsymbol{C}$ is defined

$$\boldsymbol{C} = (c_{nm} \overline{\delta}_{nm})_{n,m=1,...,M}, \qquad (16)$$

with $\overline{\delta}_{nm} = 1 - \delta_{nm}$ denoting the complementary Kronecker delta, the vector $\boldsymbol{\eta}$ comprises the noise $\frac{\eta_m}{T_f}$, $m = 1, ..., M$. The solution $\boldsymbol{\tau}$ of this system of linear equations provides the optimal power assignment in the piconets via (13). Obviously, a positive solution is required. The solution of (15) is positive iff the spectral radius of the matrix $\boldsymbol{C}$ is less than one.

The complexity of computing the optimal power assignment by this approach is determined by solving the reduced system (15), which involves one inversion of the $M \times M$ matrix $\boldsymbol{C}$. This inversion requires $\mathrm{O}(M^3)$ operations, where M is the number of piconets, which is usually much lower than the number of client nodes L. Hence the computational complexity of the problem is substantially reduced.

VI. Conclusions

In this paper, analytical methods to determine the optimal power allocation of an IR-UWB network are described and compared. It is shown that by introducing piconets and corresponding coordinator nodes the computational complexity for determining the optimal power assignment is drastically reduced from $O(L^3)$ to $O(M^3)$, where L is the number of nodes and M the number of piconets. Usually, M is much less than L.

Future research will be devoted to introducing routing aspects into the system model. Additionally, we will conduct a joint optimisation of topology generation and power assignment, and aim at deriving distributed algorithms to achieve the minimal aggregate transmission power with minimal information exchange between the nodes.

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