Rate and Power Allocation for Multiuser OFDM: An Effective Heuristic Verified by Branch-and-Bound

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***Abstract*— The present correspondence deals with the rate and power allocation problem in multiuser orthogonal frequency division multiple (OFDM) access systems. We first derive the solution of the single user OFDM power allocation problem explicitly for a class of general rate-power functions by means of directional derivatives. This solution is employed for both designing a new heuristic and obtaining bounds in a branch-and-bound algorithm for allocating power to subcarriers. The branch-and-bound algorithm is used for performance evaluation of our new and two known power allocation heuristics by computing the exact optimum, given the number of allocated subcarriers per user.**

***Index Terms*— Multiuser channels, generalized water-filling, directional derivatives.**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) is expected to be the transmission technology of next generation mobile radio networks. The advantages of OFDM are flexibility of allocating subcarriers to users, adaptive power allocation, high spectral efficiency, low receiver complexity and simple implementation by the inverse fast Fourier transform (IFFT) and FFT, see [1], [2]. OFDM can also be integrated with multiple-input multiple-output (MIMO) techniques to raise the diversity gain and increase capacity, see [1], [3]. Further, OFDM has the advantage of mitigating the effect of inter-symbol interference (ISI) in high speed wireless communications.

One of the crucial problems is rate and power allocation of users to subcarriers in the available frequency band. Various studies have devised resource allocation algorithms. In [4], a greedy bit-removing algorithm is proposed and proven to be optimal. However, it has a fairly high complexity and is almost inapplicable for practical applications. Fast heuristic approaches are suggested in [5], [6], [7], [8]. Greedy algorithms are used both to determine how many and which subcarriers are assigned to each user. However, the result is often not unique and sometimes unstable. The branch-and-bound principle is applied in [9] to tackle two classes of problems, the minimization of the total transmit power and the maximization of data throughput. The computational complexity compared to exhaustive search is significantly reduced, however, computational results are presented only for small problem sizes up to six channels and four users.

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In this paper, we aim at finding a rate allocation to subcarriers in a multiuser OFDM system which minimizes the total power consumption under the constraints that each user receives the required minimum transmission rate. A single transmitter and K receivers are considered. A new heuristic, named SUSI (successive user integration), and a branch-and-bound algorithm are developed which utilize perfect channel state information (CSI) for assigning power to subcarriers.

As a preparatory step, the single user OFDM problem is identified by help of directional derivatives for a general class of rate-power functions as generalized water-filling (cf. [10]), weighted by the derivative of the rate-power function. This solution is used as a building block in our new heuristic and, furthermore, to obtain bounds in the branch-and-bound algorithm.

The material in this contribution is organized as follows. We start with a precise problem formulation in Section II. The single user OFDM optimal rate allocation problem is solved in Section III by help of directional derivatives. Section IV introduces the new heuristic SUSI. Section V deals with the design of a branch-and-bound algorithm, which is tested and numerically evaluated on different problems in Section VI.

II. System Model and Problem Formulation

We consider a multiuser OFDM system with N subcarriers and K users. Each user $k \in \{1, \dots, K\}$ has a requirement of R_k bits per OFDM symbol, where R_k is a nonnegative real number. Each subcarrier can be used by only one user at any given time.

Perfect channel state information is assumed to be available during transmission. Let $h_{k,n}$ denote the known channel gain of subcarrier n for user k, and $\sigma^2_{k,n}$ the according noise power. Hence, $u_{k,n} = h_{k,n}/\sigma^2_{k,n}$ is the unit signal-to-noise ratio (SNR). If power $p_{k,n}$ is expended on subcarrier n for the transmission to user k, then

$$p_{k,n}\, \frac{h_{k,n}}{\sigma^2_{k,n}} = p_{k,n}\, u_{k,n}$$

is the received power.

The interrelation between power and rate is described by the nonnegative rate-power function $\psi : \mathbb{R}^+ \to \mathbb{R}^+$. Hence, $\psi(r_{k,n})$ denotes the received power which is needed to transmit at rate $r_{k,n}$ over subcarrier n. It is quite natural to assume that ψ is monotone increasing and convex with $\psi(0) = 0$, for a conclusive motivation of these properties see [9], [11]. Function ψ depends on the minimum bit error rate (BER) that can be tolerated and the modulation and coding scheme.

The objective now is to find a subcarrier assignment of minimum total transmit power such that each user receives the required data rate. In mathematical terms this reads as

$$\min \sum_{k=1}^{K} \sum_{n=1}^{N} \frac{\psi(r_{k,n})}{u_{k,n}} \tag{1}$$

such that

$$\sum_{n=1}^{N} r_{k,n} = R_k, \quad k = 1, \ldots, K$$

$$\sum_{n=1}^{N} r_{k,n}\, r_{\ell,n} = 0, \quad k, \ell = 1, \ldots, K,\ k \neq \ell$$

$$r_{k,n} \geq 0, \quad k = 1, \ldots, K,\ n = 1, \ldots, N$$

Problem (1) is a complicated mixed continuous and combinatorial optimization problem since a joint decision on subcarrier and power allocation has to be made.

In case that there is only one user ($K = 1$) the problem becomes much easier. In the following, we give an explicit solution to the single user OFDM problem, which will be employed later as a building block for a new effective heuristic, and moreover for deriving bounds in the branch-and-bound algorithm on the full OFDM assignment problem. Directional derivatives turn out to be a powerful tool for this purpose.

III. Single User OFDM

We use the directional derivative $Df(\hat{x}, x)$ at $\hat{x}$ in the direction of x for optimizing convex functions f over a convex domain $\mathcal{C}$, for a definition see, e.g., [12]. The minimum of some convex function f is attained at $\hat{x}$ if and only if $Df(\hat{x}, x) \geq 0$ for all $x \in \mathcal{C}$, for a proof see [12]. The single user power assignment problem is embedded into the above general framework by setting

$$\mathcal{R} = \Big\{ \boldsymbol{r} = (r_1, \ldots, r_N) \mid r_i \geq 0, \textstyle\sum_{i=1}^{N} r_i = R \Big\},$$

the admissible rate region. The allocation problem then reads as

$$\min_{\boldsymbol{r} \in \mathcal{R}} \sum_{i=1}^{N} \frac{\psi(r_i)}{u_i}. \tag{2}$$

Since rate-power function $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ is assumed to be nonnegative, monotone increasing and convex on $\mathbb{R}^+$, the function $f(\boldsymbol{r}) = \sum_{i=1}^{N} \frac{\psi(r_i)}{u_i}$ is convex as well. Hence, the directional derivative of f exists, and for differentiable ψ it is easily determined as

$$Df(\hat{\boldsymbol{r}}, \boldsymbol{r}) = \sum_{i=1}^{N} \frac{\psi'(\hat{r}_i)}{u_i}\, (r_i - \hat{r}_i). \tag{3}$$

Now, some rate allocation $\hat{\boldsymbol{r}} \in \mathcal{R}$ is optimal if and only if $Df(\hat{\boldsymbol{r}}, \boldsymbol{r}) \geq 0$ for all $\boldsymbol{r} \in \mathcal{R}$. Applying this condition to (3) yields that $\hat{\boldsymbol{r}} \in \mathcal{R}$ is optimal if and only if $\min_{\boldsymbol{r} \in \mathcal{R}} \sum_{i=1}^{N} \frac{\psi'(\hat{r}_i)}{u_i}\, r_i$ is attained at $\hat{\boldsymbol{r}}$. The minimum amounts to $R \cdot \min_{1 \leq j \leq N} \psi'(\hat{r}_j)/u_j$ such that $\hat{\boldsymbol{r}} \in \mathcal{R}$ is optimal if and only if

$$\sum_{i=1}^{N} \frac{\psi'(\hat{r}_i)}{u_i}\, \hat{r}_i = R \min_{1 \leq j \leq N} \frac{\psi'(\hat{r}_j)}{u_j}. \tag{4}$$

Condition (4) holds if and only if the minimum on the right hand side is attained for all addends on the left with positive $\hat{r}_i$. In summary, we have shown that rate allocation $\hat{\boldsymbol{r}} \in \mathcal{R}$ is optimal if and only if

$$\frac{\psi'(\hat{r}_i)}{u_i} = \min_{1 \leq j \leq N} \frac{\psi'(\hat{r}_j)}{u_j} \text{ for all } i \text{ with } \hat{r}_i > 0. \tag{5}$$

In order to construct an explicit solution, equation (5) is further evaluated. Let

$$\begin{aligned} \hat{r}_i &= \begin{cases} \psi'^{-1}(\lambda u_i), & \text{if } \lambda u_i > \psi'(0) \\ 0, & \text{otherwise} \end{cases}, \\ \lambda &\text{ such that } \sum_{i=1}^{N} \hat{r}_i = R. \end{aligned} \tag{6}$$

$\psi'(0)$ is understood as the right sided derivative of the convex rate-power function ψ at 0, which exists. Any $\hat{\boldsymbol{r}}$ satisfying (6) is optimal since $\psi'(\hat{r}_i) = \lambda u_i$, i.e., $\psi'(\hat{r}_i)/u_i = \lambda$ is constant for all i with $\hat{r}_i > 0$. Otherwise, $\psi'(0)/u_i \geq \lambda$ holds such that $\lambda = \min_{1 \leq j \leq N} \psi'(r_j)/u_j$ follows. Hence, condition (4) is fulfilled, which shows optimality.

The following example visualizes the above principles. Let the rate-power function, its derivative and inverse be given as (cf. [9])

$$\begin{aligned} \psi(r) &= a\big(2^r - 1\big), \\ \psi'(r) &= a(\ln 2)2^r, \\ \psi'^{-1}(y) &= \log_2\big(y/(a \ln 2)\big). \end{aligned} \tag{7}$$

Parameter $a > 0$ depends on the coding and modulation scheme and the minimum acceptable bit error rate. Note that the location of the minimum is independent of a.

The subsequent procedure determines water-filling solution (6) for rate-power functions of type (7). Let $U^m = \{j_1, \ldots, j_m\}$ denote the set of indices corresponding to the m largest SNRs $u_{j_1} \geq \cdots \geq u_{j_m}$ and $\boldsymbol{u}^m = (u_{j_1}, \ldots, u_{j_m})$. $\mathrm{gMean}(\boldsymbol{u}^m)$ and $\mathrm{hMean}(\boldsymbol{u}^m)$ denote the geometric and harmonic mean, respectively, of the components of $\boldsymbol{u}^m$. Let $\hat{m}$ be the largest $m \leq N$ such that $\log_2\left(\frac{u_{j_m}}{\mathrm{gMean}(\boldsymbol{u}^m)} 2^{R/m}\right)$ is positive. Then $\hat{m}$ represents the number of positive rates from the optimum solution (6), which is given as

$$\begin{aligned} r_i &= \begin{cases} \log_2\left(\frac{u_i}{\mathrm{gMean}(\boldsymbol{u}^{\hat{m}})} 2^{R/\hat{m}}\right), & \text{if } i \in U^{\hat{m}} \\ 0, & \text{otherwise} \end{cases}, \\ \lambda &= \frac{a \ln(2)}{\mathrm{gMean}(\boldsymbol{u}^{\hat{m}})} 2^{R/\hat{m}} \end{aligned} \tag{8}$$

with minimal power

$$\sum_{i \in U^{\hat{m}}} \frac{\psi(r_i)}{u_i} = a\hat{m} \left[\frac{1}{\mathrm{gMean}(\boldsymbol{u}^{\hat{m}})} 2^{\frac{R}{\hat{m}}} - \frac{1}{\mathrm{hMean}(\boldsymbol{u}^{\hat{m}})} \right]. \tag{9}$$

IV. A Successive User Integration Algorithm (SUSI)

Once subcarriers are assigned to users, the optimum rate and power allocation is easily determined by applying the results of Section III. Solution (6), with special case (9), is now used as a building block for a heuristic algorithm which first affiliates users into the system and then locally exchanges user subcarriers to improve power consumption.

Algorithm 1 SUSI()

```
INITIALIZE(A_1, ..., A_K) { ⋃̇_{k∈{1,...,K}} A_k ⊂ {1, ..., N}}
repeat
  havechanged ←false
  for k = 1 ... K do
    repeat
      n_min ←
      argmin_{1≤n≤N:n∉A_k} POW(A_1, ..., A_K, n, k)
      ΔP ←
      POW(A_1, ..., A_K, n_min, k) - POW(A_1, ..., A_K)
      if ΔP < 0 then
        ASSIGNSUBCARRTOUSER(A_1, ..., A_K, n_min, k)
        havechanged ←true
      end if
    until ΔP ≥ 0
  end for
until havechanged = false
```

Algorithm 2 BB(Subcarrier n)

```
for each k ∈ {1, ..., K} do
  if n_k < m_k then
    Assign[n] ←k
    n_k ←n_k + 1
    if POWERBOUND(Assign) < bestPower then
      if n > 1 then
        BB(n − 1)
      else
        bestPower ←POWERBOUND(Assign)
        bestAssign ←Assign
      end if
    end if
    n_k ←n_k − 1
  end if
end for
```

The algorithm starts with a given initial assignment $(A_1, \ldots, A_K)$, where A_k denotes the set of subcarriers assigned to user k, and processes users cyclically in an arbitrary but fixed order. Given a current assignment, the algorithm deals with user k as follows. Each subcarrier n not yet assigned to user k is considered as a candidate for being allocated to user k. The total power consumption for this candidate assignment is evaluated. The subcarrier n_{min} which minimizes the total power consumption is then allocated to user k provided that the new total power consumption is less than the current one. This subcarrier assignment is iterated until no further improvement can be found for user k. Thereafter, the next user is treated accordingly. The algorithm stops if no further subcarrier can be allocated to any user. As the successive power values are strictly decreasing and the number of different assignments is limited by K^N, algorithm SUSI converges to a local minimum.

Algorithm 1 contains the pseudocode of SUSI which is briefly commented in the following. The minimum power consumption for each user is determined by evaluating (9). If no subcarrier is assigned, the power consumption is set to an appropriate very large value. The total power consumption of assignment $(A_1, \ldots, A_K)$ is returned by procedure POWER$(A_1, \ldots, A_K)$. Function POWER$(A_1, \ldots, A_K, n, k)$ returns the total power consumption if subcarrier n is newly assigned to user k. Note that only the values for user k and l from $n \in A_l$ must be updated.

For a brief complexity analysis we assume that the number of repetitions of the outer loop is constant. Obviously, there are at most K iterations of the FOR-loop. The inner loops evaluating argmin and (9) have at most N repetitions each. Executing ASSIGNSUBCARRIERTOUSER() needs at most $2\log(N)$ steps for inserting and deleting subcarrier n to and from sorted lists of subcarriers, respectively. Hence, the overall complexity is bounded by $O(KN(N^2 + \log(N))) = O(KN^3)$. However, in practice the number of subcarriers per user is far less than N. If the number of subcarriers per user is bounded by $\hat{N}$, the complexity reduces to $O(KN\hat{N}^2)$. Additionally, if all of the subcarriers are used, evaluation of (9) can be done in constant time by caching the harmonic and geometric means and updating only if the subcarrier assignment is changed. In this case the complexity reduces further to $O(KN\hat{N})$.

V. A Branch-and-Bound Algorithm

For benchmarking purposes we devise a branch-and-bound algorithm which assigns subcarriers to users in an optimal way once the maximum numbers of subcarriers per user are fixed. Its pseudocode is represented in Algorithm 2. The maximum numbers of subcarriers $(m_1, \ldots, m_K)$ and an initial solution are computed by a heuristic in advance. This solution has power *bestPower* and its assignment is denoted by *bestAssign* with the meaning *bestAssign*$[n] = k$ whenever $n \in A_k$. Each level of the underlying tree represents a subcarrier and branches correspond to the users of that subcarrier. After setting the current number of assigned subcarriers n_k to zero for all customers k the branch-and-bound algorithm (BB) is started by invoking BB(N). In the FOR-loop, current subcarrier n is assigned to each user k that may accommodate additional subcarriers. Procedure POWERBOUND() computes the minimal power needed, if every user k utilizes his n_k currently assigned subcarriers and his $\min\{m_k - n_k, n - 1\}$ best of the remaining subcarriers $1, \ldots, n-1$, which is obviously a lower bound for the total power. If this bound exceeds the current best value *bestPower*, the branch is cut. Otherwise the branch-and-bound algorithm is either called for the next subcarrier $n-1$, if $n > 1$ holds, or we have found a new best solution. Obviously some subcarriers may be used for the power bound more than once. Thus the power bound is not tight, in particular if $\sum_{k=1}^{K} m_k \gg N$ holds. Nevertheless it works quite well (cf. Section VI), if $\sum_{k=1}^{K} m_k = N$ holds. Note that the performance of the algorithm strongly depends on the initial bound and the order of users applied in the FOR-loop.

VI. Channel Simulation and Numerical Experiments

The multipath fading model in [13] is used to generate fading profiles for testing our algorithm BB(N). The Fourier

TABLE I

PARAMETERS FOR CHANNEL SIMULATIONS AND NUMERICAL EVALUATIONS.

Identifier	Numerical value
Base frequency	$f = 1.5$ GHz
Bandwidth of each Subcarrier	8 kHz
Power delay constant	$\gamma = 286 * 10^{-9}$
Average number of rays	$\lambda = \frac{1.}{51*10-9}$
Power delay for cluster	$\Gamma = 336 * 10^{-9}$
Average number of rays per cluster	$\Lambda = \frac{1}{168*10^{-9}}$
Exponent of radial attenuation	$\alpha = 2.5$
Maximal distance user and BS [m]	$d_{max} = 100$
Video rate [Bits/s]	$VR = 128000$
Audio rate [Bits/s]	$AR = 32000$
Mean data rate [Bits/s]	$MDR = 64000$
Video proportion	$VP = 0.1$
Audio proportion	$AP = 0.4$
Data proportion	$DP = 0.5$
Antenna Gains	$G_t = G_r = 1$

transform of the channel low-pass impulse response is given by

$$H(f) = \sum_{l=0}^{\infty}\sum_{k=0}^{\infty} \beta_{k,l} e^{j(\theta_{k,l}+2\pi f(T_l+\tau_{k,l}))},$$

where l labels the clusters, in which a burst of rays occurs, and k enumerates the rays in a cluster. The phases $\theta_{k,l}$ are independent uniformly distributed over $[0, 2\pi)$. The random variables $\beta_{k,l}$ are Rayleigh distributed with mean square values $\bar{\beta}^2_{k,l} = \bar{\beta}^2(0,0)\, e^{-\frac{T_l}{\Gamma}} e^{-\frac{\tau_{k,l}}{\gamma}}$, and the cluster and ray arrival times T_l and $\tau_{k,l}$ are i.i.d. with exponential inter-arrival times and parameters Λ and λ. The amplitude depends on $\bar{\beta}^2(0,0) = \frac{G_r G_t (\frac{c}{f4\pi})^2}{\gamma\lambda}\, d^{-\alpha}$, where G_r and G_t are the antenna gains of the receiver and the transmitter, c denotes the speed of light and $d^{-\alpha}$ gives the radial attenuation depending on the distance d between user and transmitter.

In our simulations, K users are uniformly distributed in a disk of radius d_{max}, which determines the distance d from the transmitter located at the center. The channel profiles are statistically independent generated amongst users. Each user is assigned either a fixed video- or audio-rate or an exponentially distributed data rate. The concrete parameters used in our simulations are collectively shown in Table I.

In the following we compare the performance of algorithm SUSI and two heuristics from the work [5]. As a benchmark algorithm BB is used. In our study, the preassigned maximum number of subcarriers for BB is determined by both heuristics BABS (bandwidth assignment based on SNR) from [5] and the new algorithm SUSI, the better solution is used as benchmark. Note that SUSI yields not only the number of subcarriers but also a complete solution consisting of subcarrier and power assignment.

The number allocation by BABS is then completed to a full power assignment by the heuristic procedures ACG (amplitude craving greedy) and RC-3 (rate craving with 3 stages), both suggested in [5]. The ACG algorithm depends on the particular ordering of the subcarriers. Hence, ACG is started 2000 times with independently generated orderings of the subcarriers. Note that the RC-3 algorithm needs an estimate for rates $r_{k,n}$, which is calculated using equation (8) with $R = R_k$, $\boldsymbol{u}^{\hat{m}} = \boldsymbol{u}_k = (u_{k,1}, \ldots, u_{k,N})$ and $\hat{m} = N$.

TABLE II

PERCENTAGE VALUES BY WHICH THE HEURISTICS EXCEED THE SOLUTION BY BB.

Scen.	Number of	ACG		RC-3		SUSI	
K/N	Samples	avg	max	avg	max	avg	max
3/20	1000	2.04	41.45	0.83	27.47	0.44	5.89
4/20	1000	2.44	25.26	0.80	10.07	0.90	7.42
5/20	1000	2.74	31.96	0.86	14.57	1.32	8.42
3/30	500	2.34	19.56	1.08	15.32	0.19	3.53
4/30	500	2.83	23.57	1.07	9.45	0.46	3.73
5/30	200	3.34	17.03	1.25	9.80	0.81	11.55
6/30	200	1.66	19.19	0.47	6.22	0.60	12.27
4/40	300	2.98	19.35	1.27	11.39	0.19	2.46

TABLE III

PERCENTAGE OF VISITED NODES AND RUNNING TIMES OF BABS-BB.

Scenario	Percentage of Visited Nodes			Running Time in s		
K/N	min	avg	max	min	avg	max
3/20	6.29E-08	1.40E-03	5.10E-02	0.01	1.7	875
4/20	1.33E-09	1.53E-04	1.52E-02	0.01	4.1	544
5/20	6.56E-11	3.45E-05	4.72E-03	0.01	10.4	735
3/30	4.13E-12	3.24E-04	7.13E-02	0.01	59.1	1973
4/30	9.02E-15	1.18E-05	4.59E-03	0.01	68.5	1741
5/30	3.78E-16	1.73E-06	3.02E-04	0.01	58.0	851
6/30	1.26E-17	7.88E-08	1.57E-05	0.03	122.4	1299
4/40	1.98E-19	2.52E-06	6.08E-04	0.01	76.7	959

Tables II and III show the results of an extensive study on BABS-ACG, BABS-RC-3 and SUSI with sample sizes between 200 and 1000. Algorithm SUSI is always started with an empty assignment. Examples with running times exceeding 2000 seconds were prematurely terminated and substituted by a new data set. This happened in about 3.8% of all cases. The reason for extraordinary long running times in some rare cases is that many subcarriers per user are left idle such that the cutting bounds in BB are rather imprecise.

Table II contains eight scenarios with K users and N subcarriers as indicated in the first column. The sample size and the percentage by which the result of the heuristics exceeds the conditional minimum from BB are further given. Results refer to the average and the maximum over all samples, respectively. The average iteration number of the outer loop in algorithm SUSI is about 1.8, and the maximum number of iterations is approximately 10.

Table III refers to the effectiveness of the branch-and-bound algorithm under resource allocation by BABS. The percentage numbers of visited nodes are presented. On average, a fraction of about 10^{-4} out of all nodes is visited, with an obvious tendency towards smaller values if the problem size increases. These values indicate the excellent performance of BB. Moreover, average running times of about 100 sec are quite acceptable for evaluation purposes. On average, the total number of nodes of the branch-and-bound trees ranges between 10^8 for scenario $3/20$ and $5 \cdot 10^{20}$ for scenario $4/40$. All computations were carried out on AMD Athlon XP 2600+ CPUs.

An overall picture of how algorithm SUSI performs compared to RC-3 is given in Figure 1. The relative deviations from the best known optimum in each of the 1000 samples of scenario $3/20$ are ordered and depicted in increasing manner. For example, the 800th ordered sample of SUSI is about 0.7% worse than the conditional optimum from BB, whereas RC-3

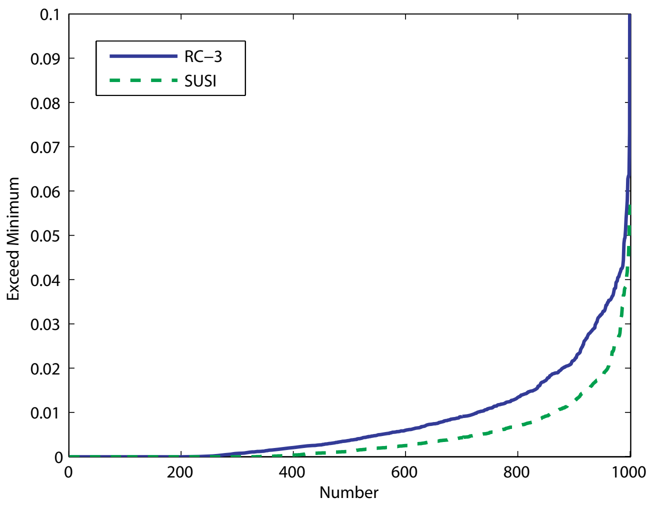

Fig. 1. Scenario 3/20: ordered values of the relative deviation from the solution by BB of algorithms RC-3 and SUSI.

is about 1.4% worse. It can be seen that SUSI roughly halves these values over the whole range, and in approximately 400 cases nearly meets the conditional optimum.

To summarize the results of the benchmarking tests, we note that the new heuristic SUSI remarkably outperforms RC-3 in cases where there are only a few users competing for subcarriers. Algorithm SUSI behaves moderately worse than RC-3 if the number of users is increased. The computational complexity of SUSI is slightly less than RC-3. The low linear complexity algorithm ACG yields results of low quality throughout the whole range of examples.

It should be mentioned that different heuristics for resource and subcarrier allocation may be combined and the minimum result may be taken as the optimum. This has been carried out by the authors and led to significant improvements.

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