Distributed Detection in Sensor Networks: Joint Optimization via Hoeffding's Inequality

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***Abstract*—This paper addresses the optimization of wireless sensor networks for distributed detection applications. In general, the jointly optimum solution for the local sensor decision rules and the fusion rule is very difficult to obtain and does not scale well with the number of sensors. In this paper, the joint optimization of the local sensor decision rules and the fusion rule is facilitated by using an upper bound on the global probability of detection error. The bound is derived using Hoeffding's inequality and allows for non-identically distributed sensor observations, multi-bit sensor output, as well as noisy communication channels between the sensors and the fusion center. By considering the problem of detecting a known signal in the presence of Gaussian noise, numerical results reveal dependencies of the obtained solutions on the prior probabilities, the total number of sensors, and the local observation SNR.**

I. Introduction

Distributed detection is one of the primary applications of wireless sensor networks and is often the first step in an overall sensing process [1]–[3]. The nodes in a sensor network typically operate on limited energy budgets and are consequently subject to communication constraints. This recommends preprocessing of measured raw data at the sensors and transmission of summary messages. In the parallel fusion network, the sensor nodes process their observations independently and make preliminary decisions about the state of the observed environment, e.g., absence or presence of a target. The sensors transmit the local decisions to a fusion center that combines the received decisions and computes the final detection result. Since the transmission channels between the battery-operated wireless sensors and the fusion center are usually subject to noise and interference, it is also necessary to take wireless channel conditions into account [4].

The problem is to design the local sensor decision rules and the fusion rule with respect to an overall performance criterion. In the general case, the jointly optimum solution for the local sensor decision rules and the fusion rule is very difficult to obtain and does not scale well with the number of sensors. Global optimization of distributed detection systems was first investigated by Reibman and Nolte [5]. They consider simultaneous optimization of the binary local detectors and the fusion rule under the constraints of identical local sensor decision rules and restrictions on the employed fusion rule. Numerical algorithms that find person-by-person optimal local sensor decision rules are presented in [6] and [7]. In [8], the authors use an iterated combination of a genetic algorithm for optimizing the fusion rule and a gradient-based algorithm to optimize the decision thresholds of the local detectors. All of the above authors assume independent and identically distributed observations at the local sensors and the joint optimization is done only for sensor networks with a very low number of sensors, e.g., in [8] the number of sensors varies between 2 and 8.

In this paper, we consider the design of high-quality solutions for distributed detection that are robust, scalable, and do not rely on oversimplifying assumptions. The main idea of our approach is to use an upper bound on the probability of detection error as an objective function to jointly optimize the distributed detection system. Using a probability inequality introduced by Hoeffding [9], we obtain a performance bound that is a smooth function of the system parameters and can be minimized by applying standard gradient-based algorithms for numerical optimization.

The remainder of this paper is organized as follows. In Section II, the problem of distributed detection in the parallel fusion network with noisy channels is stated. In Section III, we present an upper bound on the global probability of error which enables joint optimization of the distributed detection system. Numerical results are presented in Section IV and we conclude in Section V.

II. Distributed Detection

The problem of distributed detection in the parallel fusion network with noisy channels can be stated as follows (see Fig. 1) . We consider a binary hypothesis testing problem with hypotheses H_0 and H_1 indicating the state of the observed environment. The associated prior probabilities are $\pi_0 = P(H_0)$ and $\pi_1 = P(H_1)$. In order to detect the true state of nature, a network of N sensors $S_1, \ldots, S_N$ collects an array of random observations

$$(X_1, \ldots, X_N)' \in \mathcal{X}_1 \times \cdots \times \mathcal{X}_N \tag{1}$$

which is generated according to either H_0 or H_1. The random observations $X_1, \ldots, X_N$ are assumed to be conditionally independent across sensors given the underlying hypothesis, i.e., the joint conditional probability density function of all

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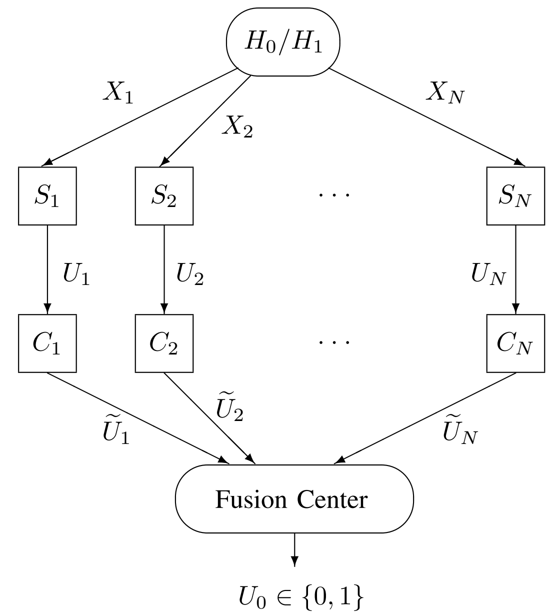

Fig. 1. Parallel fusion network with noisy channels.

the observations factorizes according to

$$f(x_1, \ldots, x_N | H_k) = \prod_{j=1}^{N} f_j(x_j | H_k), \quad k = 0, 1. \tag{2}$$

Because of the distributed nature of the problem, the sensors process their respective observations X_j independently by forming local decisions

$$U_j = \gamma_j(X_j), \quad j = 1, \ldots, N. \tag{3}$$

A. Local sensor decision rules

In the general case of M-ary quantization, i.e., $\log_2(M)$-bit sensor output, the local sensor decision rules γ_j are mappings

$$\gamma_j \colon \mathcal{X}_j \to \{1, \ldots, M\}, \quad j = 1, \ldots, N. \tag{4}$$

Warren and Willett have shown that the sensor decision rules leading to jointly optimal configurations under the Bayes criterion are monotone likelihood ratio partitions of the sensor observation spaces $\mathcal{X}_1, \ldots, \mathcal{X}_N$, provided that the observations are conditionally independent across sensors [10]. Hence, in the optimal design of distributed detection systems under the assumption of conditional independence, it is necessary only to consider sensor decision rules γ_j that can be parameterized by a set of real quantization thresholds $\tau_{j_1}, \ldots, \tau_{j_{M-1}}$, where $\tau_{j_0} = -\infty$, $\tau_{j_M} = \infty$, and $\tau_{j_k} \leq \tau_{j_{k+1}}$. In this way, each local sensor S_j is characterized by the conditional probabilities

$$\alpha_{j_k} = P(U_j = k | H_0) = P(\tau_{j_{k-1}} < L_j \leq \tau_{j_k} | H_0), \tag{5}$$
$$\beta_{j_k} = P(U_j = k | H_1) = P(\tau_{j_{k-1}} < L_j \leq \tau_{j_k} | H_1), \tag{6}$$

where $L_j = \log(f_j(X_j|H_1)/f_j(X_j|H_0))$ is the local log-likelihood ratio of observation X_j. The probability vectors $\boldsymbol{\alpha}_j = (\alpha_{j_1}, \ldots, \alpha_{j_M})'$ and $\boldsymbol{\beta}_j = (\beta_{j_1}, \ldots, \beta_{j_M})'$ are computable given the local observation statistics $f_j(x_j|H_k)$, $k = 0, 1$, and the quantization thresholds $\tau_{j_1}, \ldots, \tau_{j_{M-1}}$ for each $j = 1, \ldots, N$.

B. Transmission of local decisions

Upon local decision-making, the sensor nodes transmit a vector of local decisions

$$\boldsymbol{U} = (U_1, \ldots, U_N)' \in \{1, \ldots, M\}^N \tag{7}$$

to the fusion center in order to perform decision combining. The communication channels between the wireless sensors and the fusion center may be subject to noise and interference. We model the communication link C_j between sensor S_j and the fusion center by a discrete noisy channel with transition matrix $\boldsymbol{T}_j$. The channel transition matrix $\boldsymbol{T}_j = (T_{kl}^{(j)})_{1 \leq k, l \leq M}$ is an $M \times M$ matrix with the klth entry defined as

$$T_{kl}^{(j)} = P(\widetilde{U}_j = k | U_j = l), \quad k, l \in \{1, \ldots, M\}, \tag{8}$$

where $\sum_{k=1}^{M} T_{kl}^{(j)} = 1$ for any $l \in \{1, \ldots, M\}$. Because of the noisy channels, the fusion center receives a vector of potentially distorted decisions

$$\widetilde{\boldsymbol{U}} = (\widetilde{U}_1, \ldots, \widetilde{U}_N)' \in \{1, \ldots, M\}^N. \tag{9}$$

The distribution of the distorted decisions $\widetilde{U}_j$ is determined by the conditional probabilities

$$\widetilde{\alpha}_{j_k} = P(\widetilde{U}_j = k | H_0) = \sum_{l=1}^{M} T_{kl}^{(j)} \alpha_{j_l}, \tag{10}$$

$$\widetilde{\beta}_{j_k} = P(\widetilde{U}_j = k | H_1) = \sum_{l=1}^{M} T_{kl}^{(j)} \beta_{j_l}. \tag{11}$$

Assuming knowledge of the channel transition matrices $\boldsymbol{T}_j$, we obtain the probability vectors $\widetilde{\boldsymbol{\alpha}}_j = (\widetilde{\alpha}_{j_1}, \ldots, \widetilde{\alpha}_{j_M})'$ and $\widetilde{\boldsymbol{\beta}}_j = (\widetilde{\beta}_{j_1}, \ldots, \widetilde{\beta}_{j_M})'$ characterizing the distribution of the received local decisions $\widetilde{U}_1, \ldots, \widetilde{U}_N$ under both hypotheses.

C. Optimal channel-aware fusion rule

At the fusion center, the received decisions are combined to yield the final decision $U_0 = \gamma_0(\widetilde{U}_1, \ldots, \widetilde{U}_N)$, where the fusion rule γ_0 is a binary-valued mapping

$$\gamma_0 \colon \{1, \ldots, M\}^N \to \{0, 1\}. \tag{12}$$

The sensor network detection performance is measured in terms of the global probability of error

$$P_e = \pi_0 P_f + \pi_1 P_m, \tag{13}$$

which can be written as a weighted sum of the global probability of false alarm $P_f = P(U_0 = 1 | H_0)$ and the corresponding global probability of miss $P_m = P(U_0 = 0 | H_1)$.

The optimal fusion rule under the minimum probability of error criterion can be performed by evaluating a log-likelihood ratio test of the form

$$\sum_{j=1}^{N} \mathcal{L}_j \underset{U_0 = 0}{\overset{U_0 = 1}{\gtrless}} \log\left(\frac{\pi_0}{\pi_1}\right) = \vartheta, \tag{14}$$

where $\mathcal{L}_j = \log(P(\widetilde{U}_j|H_1)/P(\widetilde{U}_j|H_0))$ is the log-likelihood ratio of receiving decision $\widetilde{U}_j$, and where ϑ is the fusion threshold. The log-likelihood ratio $\mathcal{L}_j$ is a discrete random variable that takes one out of M possible values

$$\ell_{j_k} = \log\Big(\frac{\widetilde{\beta}_{j_k}}{\widetilde{\alpha}_{j_k}}\Big), \quad k = 1, \ldots, M, \tag{15}$$

and has conditional probability mass functions given by

$$P(\mathcal{L}_j = \ell_{j_k}|H_0) = \widetilde{\alpha}_{j_k}, \tag{16}$$

$$P(\mathcal{L}_j = \ell_{j_k}|H_1) = \widetilde{\beta}_{j_k}. \tag{17}$$

For $j = 1, \ldots, N$, we denote by

$$\ell_{j_{\min}} = \min_k\{\ell_{j_1}, \ldots, \ell_{j_M}\}, \tag{18}$$

$$\ell_{j_{\max}} = \max_k\{\ell_{j_1}, \ldots, \ell_{j_M}\}, \tag{19}$$

the minimum and maximum value of the log-likelihood ratio $\mathcal{L}_j$, respectively.

D. Global Error Probabilities

When using the optimal fusion rule according to (14), the global probability of false alarm P_f and the global probability of miss P_m are determined by the conditional tail probabilities

$$P_f = P\Big(\sum_{j=1}^{N} \mathcal{L}_j \geq \vartheta|H_0\Big) \tag{20}$$

and

$$P_m = P\Big(\sum_{j=1}^{N} \mathcal{L}_j < \vartheta|H_1\Big). \tag{21}$$

Since closed-form expressions for these tail probabilities are not available, we consider upper bounds on the tail probabilities (20) and (21) to use them for sensor network optimization.

III. Detection Performance Bound based on Hoeffding's Inequality

In this section, we present an upper bound on the global probability of error (13), which is considered as objective function to be minimized. We apply Hoeffding's inequality [9] for bounding the tail probabilities of the sums of bounded and independent random variables to the global probability of false alarm (20) and the global probability of miss (21).

A. Probability of false alarm

In this subsection, we derive an upper bound on the global probability of false alarm (20). Introducing the conditional zero-mean random variables

$$\widehat{\mathcal{L}}_j = \mathcal{L}_j - \mathbb{E}[\mathcal{L}_j|H_0] = \mathcal{L}_j - \sum_{k=1}^{M} \widetilde{\alpha}_{j_k}\ell_{j_k}, \quad j = 1, ..., N,$$

where $\mathbb{E}[\cdot|H_0]$ denotes conditional expectation given hypothesis H_0, and the new threshold

$$\vartheta_0 = \vartheta - \sum_{j=1}^{N}\sum_{k=1}^{M} \widetilde{\alpha}_{j_k}\ell_{j_k},$$

we obtain

$$P_f = P\Big(\sum_{j=1}^{N} \widehat{\mathcal{L}}_j \geq \vartheta_0|H_0\Big). \tag{22}$$

Considering equation (22), we apply a probability inequality for the sum of zero-mean bounded random variables formulated by Hoeffding [9]. We obtain the bound

$$P_f \leq \exp\Big(- 2\vartheta_0^2 / \sum_{j=1}^{N} (\ell_{j_{\max}} - \ell_{j_{\min}})^2\Big). \tag{23}$$

B. Probability of miss

The construction of an upper bound on the global probability of miss (21) follows the same lines as for the probability of false alarm. We obtain the bound

$$P_m \leq \exp\Big(- 2\vartheta_1^2 / \sum_{j=1}^{N} (\ell_{j_{\max}} - \ell_{j_{\min}})^2\Big), \tag{24}$$

where

$$\vartheta_1 = \sum_{j=1}^{N}\sum_{k=1}^{M} \widetilde{\beta}_{j_k}\ell_{j_k} - \vartheta.$$

C. Detection performance bound

With the results (23), (24) and after some calculation, we obtain an upper bound on the global probability of error according to

$$\begin{aligned} P_e \leq \pi_0 \cdot \exp\Bigg(- \frac{\big(\sum_{j=1}^{N} \mathrm{D}(\widetilde{\boldsymbol{\alpha}}_j \| \widetilde{\boldsymbol{\beta}}_j) + \vartheta\big)^2}{\frac{1}{2}\sum_{j=1}^{N} (\ell_{j_{\max}} - \ell_{j_{\min}})^2}\Bigg) + \\ + \pi_1 \cdot \exp\Bigg(- \frac{\big(\sum_{j=1}^{N} \mathrm{D}(\widetilde{\boldsymbol{\beta}}_j \| \widetilde{\boldsymbol{\alpha}}_j) - \vartheta\big)^2}{\frac{1}{2}\sum_{j=1}^{N} (\ell_{j_{\max}} - \ell_{j_{\min}})^2}\Bigg), \end{aligned} \tag{25}$$

where $\mathrm{D}(\cdot\|\cdot)$ denotes the Kullback-Leibler distance. The exponents on the right hand side of the inequailty (25) are essentially squared and normalized sums of Kullback-Leibler distances of the probability vectors $\widetilde{\boldsymbol{\alpha}}_j$ and $\widetilde{\boldsymbol{\beta}}_j$.
It is important to note, that once the quantization thresholds $\tau_{j_1}, \ldots, \tau_{j_{M-1}}$ are determined for all sensors S_j, the value of the upper bound (25) can be calculated immediately. Furthermore, the bound (25) is a smooth function of the probability vectors $\widetilde{\boldsymbol{\alpha}}_j$ and $\widetilde{\boldsymbol{\beta}}_j$ for $j = 1, \ldots, N$, which facilitates the use of standard gradient-based optimization algorithms.

IV. Numerical Results

In the following, we provide detailed numerical results obtained by minimization of the performance bound (25). In particular, the dependencies of the quantization thresholds $\tau_{j_1}, \ldots, \tau_{j_{M-1}}$ on the prior probabilities π_0 and π_1, the total number of sensors N, and the local observation SNR are revealed. We consider the cases of binary and quaternary sensors, i.e., the cases of 1-bit and 2-bit quantization at the local sensors.

A. Joint distribution of sensor observations

As an illustrative example, we consider the problem of detecting the presence or absence of a known signal in Gaussian noise, i.e., we assume that the observations $X_1, \ldots, X_N$ at the local sensors are conditionally independent and identically distributed according to

$$\begin{aligned} H_0 &: X_j \sim \mathcal{N}(0, \sigma^2), \\ H_1 &: X_j \sim \mathcal{N}(\mu, \sigma^2), \end{aligned} \tag{26}$$

for $j = 1, \ldots, N$. The variance σ^2 describes the Gaussian background noise and the mean μ indicates the deterministic signal component under hypothesis H_1. At each sensor, the local observation signal-to-noise ratio (SNR) is given by

$$\mathrm{SNR} = 10 \log_{10} \left(\frac{\mu^2}{\sigma^2} \right). \tag{27}$$

The local log-likelihood ratios L_j are again Gaussian random variables with conditional marginal distributions according to

$$\begin{aligned} H_0 &: L_j \sim \mathcal{N}\Big(-\frac{\mu^2}{2\sigma^2}, \frac{\mu^2}{\sigma^2} \Big), \\ H_1 &: L_j \sim \mathcal{N}\Big(\frac{\mu^2}{2\sigma^2}, \frac{\mu^2}{\sigma^2} \Big). \end{aligned} \tag{28}$$

For the sake of illustrative simplicity, we assume ideal communication links between the sensors and the fusion center.

B. Centralized detection system

For the optimal centralized detection system that has access to all the unquantized observations $X_1, \ldots, X_N$, the minimum probability of error P_e^* can be calculated explicitly. Introducing the Mahalanobis distance d_M which is given by

$$d_M = \frac{\mu}{\sigma} \sqrt{N} \tag{29}$$

for the conditional distributions according to (26), the minimum probability of error P_e^* of the optimal centralized detection system is given by

$$P_e^* = \pi_0 \cdot \left(1 - \Phi\Big(\frac{\vartheta + \frac{1}{2} d_M^2}{d_M} \Big) \right) + \pi_1 \cdot \Phi\Big(\frac{\vartheta - \frac{1}{2} d_M^2}{d_M} \Big), \tag{30}$$

where Φ is the cumulative distribution function of the standard normal distribution. The minimum probability of error P_e^* will be the benchmark for the performance of the distributed detection systems. It should be noted however, that the optimal centralized detection system achieving P_e^* is characterized by full access to the unquantized observations $X_1, \ldots, X_N$.

C. Binary and quaternary sensors

In the case of binary sensors, i.e., $M = 2$, there is only one local quantization threshold τ_j at every sensor. Accordingly, the minimization of the bound (25) is done over the vectors of quantization thresholds $(\tau_1, \ldots, \tau_N)' \in \mathbb{R}^N$. In the case of quaternary sensors, i.e., $M = 4$, three quantization thresholds $\tau_{j_1}, \tau_{j_2}, \tau_{j_3}$ have to be determined for each sensor. In both cases, we assume that the joint distribution of sensor observations is given by (26) and that quantization of the local log-likelihood ratio L_j is performed. After the optimization

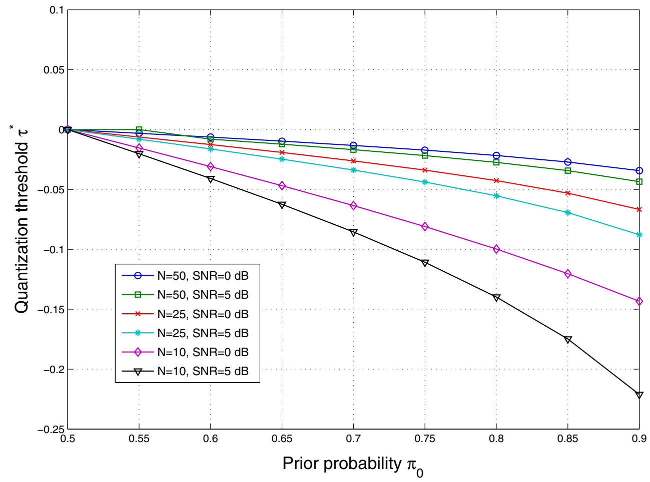

Fig. 2. Optimal quantization threshold τ^* for $N = 10, 25$ and 50 binary sensors for 0 and 5 dB.

procedure, we employ a technique presented in [11] in order to numerically evaluate the actual value of the probability of error P_e with high accuracy.

The first result is that both for binary and quaternary sensors the numerically optimal decision thresholds τ_j^* and $\tau_{j_1}^*, \tau_{j_2}^*, \tau_{j_3}^*$ are the same for every sensor, i.e.

$$\tau_1^* = \tau_2^* = \ldots = \tau_N^* = \tau^* \tag{31}$$

for the binary case and

$$\begin{aligned} \tau_{1_1}^* &= \tau_{2_1}^* = \ldots = \tau_{N_1}^* = \tau_1^*, \\ \tau_{1_2}^* &= \tau_{2_2}^* = \ldots = \tau_{N_2}^* = \tau_2^*, \\ \tau_{1_3}^* &= \tau_{2_3}^* = \ldots = \tau_{N_3}^* = \tau_3^*. \end{aligned} \tag{32}$$

for the quaternary case. This is consistent with the analytical results for the Gaussian detection problem obtained by Irving and Tsitsiklis in [12].

Fig. 2 illustrates the results for binary sensors. The optimal decision threshold τ^* is plotted against the prior probability π_0 for $N = 10$, 25 and 50 sensors for a local observation SNR of 0 and 5 dB. The larger the prior probability π_0, the smaller the optimal threshold τ^*. On the other hand, the higher the number of sensors and the lower the SNR, the more τ^* approaches 0 and the smaller is the influence of the prior probability π_0. This suggests the conclusion that with a large number of sensors operating in low SNR environments, setting the quantization thresholds uniformly to 0 might yield near-optimal performance.

Fig. 3 illustrates the results for quaternary sensors. Interestingly, the distance between the lowest optimal threshold τ_1^* and the highest optimal threshold τ_3^* is very small, so that only τ_1^* is plotted. The small distance between τ_1^* and τ_3^* means that in almost all cases the sensors only transmit the lowest and the highest value of the possible local decisions. As for binary sensors, the optimal threshold τ_1^* decreases with

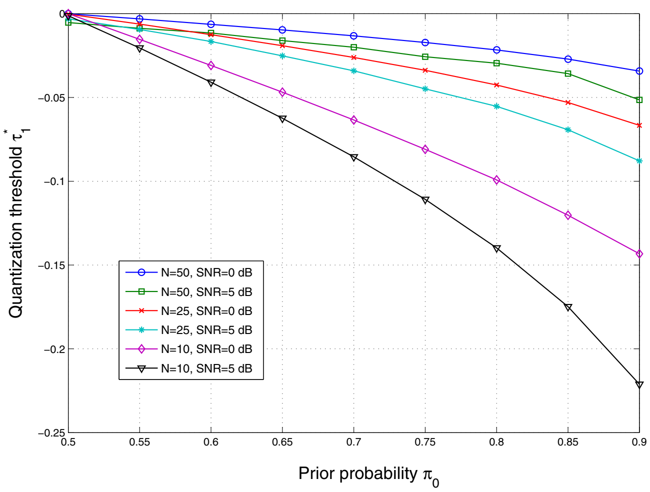

Fig. 3. Optimal quantization threshold τ_1^* for $N = 10, 25$ and 50 quaternary sensors for 0 and 5 dB.

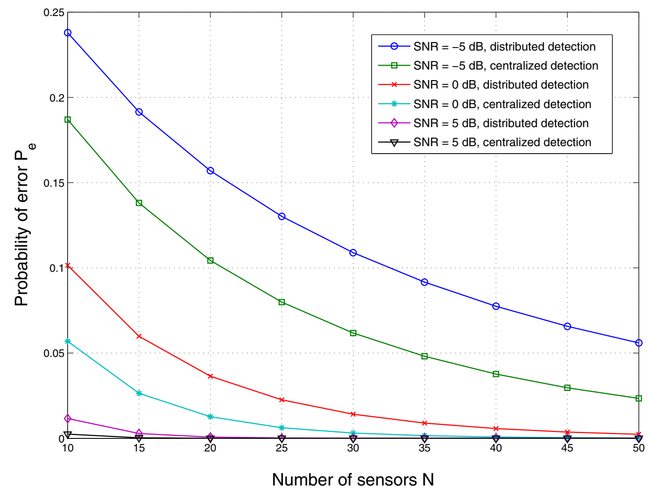

Fig. 4. Global probability of error P_e for $N = 10, 15, \ldots, 50$ binary and quaternary sensors in comparison to the minimum probability of error P_e^* of the optimal centralized detection system for -5, 0 and 5 dB.

increasing π_0. Furthermore, τ_1^* approaches 0 for an increasing number of sensors and decreasing observation SNR.

The probability of error P_e of the optimized distributed detection systems with binary and quaternary sensors evaluated by a method presented in [11] in comparison to the probability of error P_e^* of the optimal centralized system is depicted in Fig. 4. Surprisingly, using quaternary sensors does not lead to any performance gain compared to the use of binary sensors. This corresponds to the before mentioned observation that the three optimal quantization thresholds τ_1^*, τ_2^* and τ_3^* practically coincide. These results strongly indicate that binary quantization at the sensors is sufficient for minimizing the performance bound (25), at least for the problem of detecting a known signal in the presence of Gaussian noise. However, there is a performance degradation compared to the centralized system as can be seen from Fig. 4.

V. Conclusions

In this paper, we presented an approach to the joint optimization of sensor networks for distributed detection applications that is based on an upper bound of the probability of detection error. The performance bound is derived using Hoeffding's inequality and is a smooth function of the system parameters. The bound can be minimized by applying standard gradient-based algorithms for numerical optimization. By considering the problem of detecting a known signal in the presence of Gaussian noise, the numerical results show dependencies of the obtained solutions on the prior probabilities, the total number of sensors, and the local observation SNR. Furthermore, it is indicated under which circumstances high-quality solutions might be obtained by only considering binary quantization at the local sensors.

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