Power Allocation for Social Benefit Through Price-taking Behaviour on a CDMA Reverse Link Shared by Energy-constrained and Energy-sufficient Data Terminals

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***Abstract*—We allocate power to maximise "social benefit" in the uplink of a CDMA cell populated by data terminals, each with its own data rate, channel gain, willingness to pay (wtp), and link-layer configuration, and with energy supplies that are limited for some, and inexhaustible for others. For both types, appropriate performance indices are specified. The social optimum can be achieved distributively through *price-taking* behaviour, if prices are based on a terminal's *fraction* of the total power received. For a given price, a terminal can choose its optimal power fraction without knowing the choices made by others because this fraction directly determines its signal-to-interference ratio (SIR), and hence its performance. By contrast, other schemes produce "games" in which terminals' optimal choices depend on each other. A "decoupled" solution has important technological and "marketing" advantages. The socially-optimal price is common to all terminals of a given energy class, and an energy-constrained terminal pays in proportion to *the square* of its power fraction.**

I. Introduction

For reverse link CDMA power allocation, many useful decentralised algorithms has been reported [1], [2], [3], [4], [5], [6]. However, previous schemes rely on per-Watt pricing, which leads to a "game" in which terminals' optimal choices are intertwined. This creates both *technological* and *marketing* problems. In a game, the typical solution concept is the Nash equilibrium (NE), which is in general inefficient[7]. And even if a unique NE exists, it is often unclear: (a) *how* will the players reach the NE, and (b) after how many "iterations". The facts that terminals frequently enter and exist the network at arbitrary times, and that they are "anonymous" to each other, further complicate matters.

Below we provide a "de-coupled" solution: for a given price per *fraction* of total power received, a terminal can individually make its optimal choice because this fraction directly determines its signal-to-interference ratio (SIR), and hence its performance. A decoupled solution is superior for both technological and marketing reasons.

Additional distinguishing features of our model are: (i) the simultaneous consideration of both limited and unlimited energy budgets as in real networks, (ii) the performance metrics utilised (one for each type of budget situation), and (iii) and the generality of our physical model (each terminal may have its own data rate, channel gain, willingness to pay (wtp), and even link-layer configuration). In [8], we allocate power to maximise network revenue.

Below, we first discuss the physical model, and the terminals' technical-economic rationale. Then, we address feasibility issues, and identify a terminal's power fraction as key. We then analyse how each terminal reacts under power-fraction pricing. Finally, we describe the "socially optimal" allocation.

II. Generalities

A. Physical Model

N terminals *upload* data *to* a CDMA base station (BS). z or x may be used as generic function arguments. E_i is the energy budget, $p_i := h_i P_i$ is the received power, and $\hat{P}_i$ the power constraint. R_i is the data rate. The chip rate is assumed equal to the bandwidth, W. $G_i := W/R_i$ is the spreading gain. M_i-bit packets carrying $L_i < M_i$ information bits are used. The signal-to-interference ratio (SIR) is defined as $\sigma_i = G_i \kappa_i$ with $\kappa_i = p_i/Y_i$ the carrier-to-interference ratio (CIR), and $Y_i = \sum_{j \neq i} p_j + p_0$, with p_0 the Gaussian noise. F_i is the packet-success rate function (PSRF) giving the probability of correct reception of a data packet as a function of the SIR. For some technical reasons, $f_i(x) := F_i(x) - F_i(0)$ replaces F_i. Its graph is assumed to have the S-shape shown in fig 1. Our analysis does *not* rely on any specific PSRF.

B. Terminals' general objective

An e-terminal focuses on the total number of information bits transferred with its total energy budget. A t-terminal focuses on information bits transferred over a reference time period.

The terminal's utility function has the form $v_i B_i + y_i$ where (i) v_i is the "willingness to pay" (wtp) (the monetary value to the terminal of one information bit successfully transferred), (ii) B_i is the (average) number of information bits the terminal has successfully transferred within a reference length of time, say τ, and (iii) y_i is the amount of money the terminal has left after any charges and rewards are computed. B_i generally depends on a vector of resources $\mathbf{z}$. When the terminal must pay $c_i(\mathbf{z})$, it chooses $\mathbf{z}$ to maximise $v_i B_i(\mathbf{z}) + [D_i - c_i(\mathbf{z})]$. D_i is the terminal's monetary budget, which limits its total expenditure and, if "large", needs not be considered, in which case the terminal maximises benefit minus cost:

$$v_i B_i(\mathbf{z}) - c_i(\mathbf{z}) \equiv S_i(\mathbf{z}) - c_i(\mathbf{z}) \tag{1}$$

III. Feasibility of Power Ratios

A. SIR/CIR feasibility

Let

$$\pi_i := \frac{\kappa_i}{1+\kappa_i} \equiv \frac{\sigma_i}{G_i + \sigma_i} \tag{2}$$

A system of N equations like

$$\frac{p_i}{\sum_{\substack{j=1 \\ j \neq i}}^{N} p_j + p_0} = \frac{\sigma_i}{G_i} := \kappa_i \tag{3}$$

has a unique solution given by :

$$p_i = p_0 \pi_i / (1 - \bar{\pi}) \tag{4}$$

provided that [9],

$$\bar{\pi} := \sum_{j=1}^{N} \pi_j < 1 \tag{5}$$

Notice that $\kappa_i = \kappa(\pi_i)$, with

$$\kappa(z) := \frac{z}{1-z} \equiv \frac{1}{z^{-1} - 1} \tag{6}$$

κ is strictly increasing: $\kappa'(z) = (1-z)^{-2} > 0$.

If the cell is "interference limited", (i.e., $p_0 \approx 0$) then the feasibility condition is

$$\sum_{j=1}^{N} \pi_j = 1 \tag{7}$$

and any power vector proportional to the vector of π_i's is a solution.

B. Power limits and the "ruling terminal"

With $\hat{P}_i$ denoting the power limit and $\hat{p}_i := h_i \hat{P}_i$, from eq. (4) one obtains

$$\frac{\pi_i}{1-\bar{\pi}} p_0 \leq \hat{p}_i \rightarrow \bar{\pi} \leq 1 - \frac{\pi_i}{\hat{p}_i / p_0} \tag{8}$$

If $\pi_k / \hat{p}_k \geq \pi_i / \hat{p}_i \quad \forall i \in \{1, \ldots, N\}$, we call terminal k the "ruling terminal", and notice that then the *only* necessary feasibility condition is

$$\bar{\pi} \leq 1 - \frac{\pi_k}{\hat{p}_k / p_0} \tag{9}$$

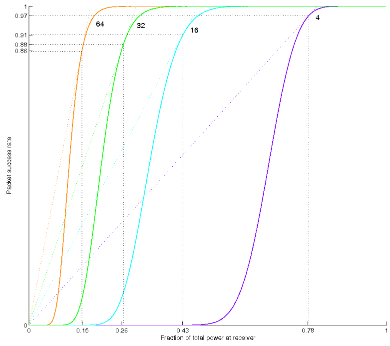


Figure 1. The composite function $f(G\kappa(z))$, with f the PSRF plays a key role, especially its tangenu (tangent to (0,0)) and genu (tangency point). $f(x) = [1 - \exp(-x/2)/2]^{80} - 2^{-80}$ is displayed.

IV. Dividing "a Pie" among the Terminals

A. A terminal's fractional "power slice"

Notice that, with

$$\bar{p} := \sum_{i=1}^{N} p_i \tag{10}$$

and with $Y_i := \bar{p} - p_i + p_0$ (the interference experienced by terminal i), and $\kappa_i = p_i / Y_i$, π_i can be written as:

$$\frac{p_i / Y_i}{p_i / Y_i + 1} \equiv \frac{p_i}{p_i + Y_i} \equiv \frac{p_i}{\bar{p} + p_0} \tag{11}$$

Thus, π_i is terminal i's fraction of the total power at the receiver, $\Pi = \sum_{i=1}^{N} p_i + p_0$. Thus, the network faces a "pie division" problem (the role of power constraints is clarified below). The case of the interference-limited cell is even simpler. The "pie size" is necessarily 1 (eq. (7)), and the "slices" immediately indicate the received power levels.

B. Power-fraction pricing

For a given value of π_i, the terminal can obtain directly the corresponding CIR (equation (6)), and hence its performance. Thus, the network can set a price c_i at which terminal i can "buy" π_i and, for the given c_i, i can choose optimally π_i, *irrespective* of the choices made by others.

V. Terminal's Choice under Pricing

A. Choice by a throughput-driven terminal

The period of interest is some pre-specified time length, say a time unit. For a given π_i, the terminal's cost is $c_i \pi_i$, and (with $\kappa(z)$ given by eq. (6)) the average number of information bits transferred over a time unit is B_i:

$$\frac{L_i}{M_i} R_i f_i(G_i \kappa(\pi_i)) \tag{12}$$

The terminal should choose the value of π_i that maximises $v_i B_i$:

$$v_i \frac{L_i}{M_i} R_i f_i(G_i \kappa(\pi_i)) - c_i \pi_i \tag{13}$$

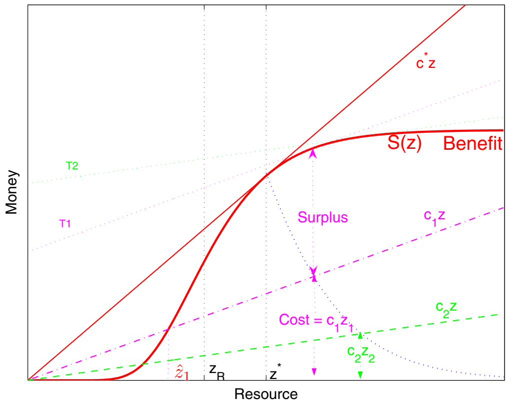


Figure 2. With a power share z, the terminal maximises$S(z) - c(z)$. The largest acceptable c is the slope of the tangenu of S.

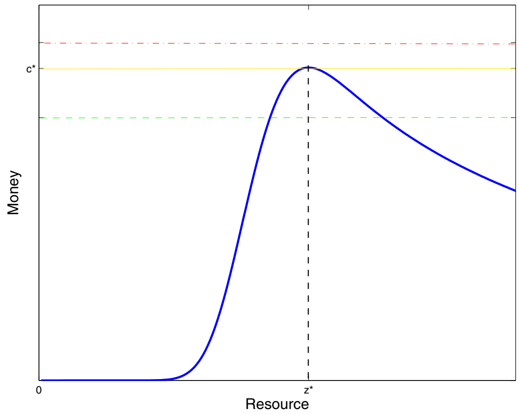


Figure 3. For $c \leq c^*$ the e-terminal chooses z^*; else $z = 0$ is optimal.

f_i is the (slightly modified) packet-success-rate function, whose graph as a function of its argument, the SIR, is an S-curve. Let

$$g_i(z; G_i) := f_i(G_i \kappa(z)) \tag{14}$$

g_i is a composite function of f_i and κ, with independent variable z and parameter G_i (denoted simply as $g_i(z)$ when no confusion is caused). It turns out that the graph of g_i (as a function of z) inherits the S-shape of f_i (see figure 1). Thus, we need to understand the maximisation of an expression of the form $S(z) - cz$, where S is some S-curve.

Figure 2 illustrates the solution procedure. First, if the line cz lies entirely above S, except at the origin, the terminal should choose $z = 0$ (decline to operate), since its cost would exceed its benefit for any positive z. Otherwise, two values of z satisfy $S'(z) = c$, and the maximiser is the largest of the two. The largest acceptable c is denoted as c^*, and as shown in fig. 2, is obtained as the slope of the tangenu (the unique tangent of S that goes through (0,0)). It can be shown that replacing S with a multiple of S has no effect on the value of z^*. c^* and z^* are related by $c^* = S'(z^*) = S(z^*)/z^*$.

B. *Choice by an energy-limited terminal*

For an energy-limited terminal, the natural period of interest is battery life. For π_i the corresponding transmission power is

$$P_i = p_i/h_i \equiv \pi_i(\bar{p} + p_0)/h_i \tag{15}$$

With energy E_i, battery life is

$$T_i = E_i/P_i \equiv \frac{E_i h_i}{\pi_i(\bar{p} + p_0)} \tag{16}$$

The terminal's benefit is $v_i B_i$, with B_i the total (average) number of (energy-earned) information bits over the period T_i:

$$B_i(\pi_i) = \frac{L_i}{M_i} R_i f_i(G_i \kappa(\pi_i)) T_i \tag{17}$$

In view of equation (16), the terminal's total cost over the period of interest is given by

$$c_i \pi_i T_i \equiv c_i \frac{E_i h_i}{\bar{p} + p_0} \tag{18}$$

Notice that π_i drops out of the total cost expression.

The terminal chooses π_i to maximise utility (total benefit minus total cost):

$$\frac{E_i h_i}{\bar{p} + p_0} \left(\frac{L_i}{M_i} v_i R_i \frac{f_i(G_i \kappa(\pi_i))}{\pi_i} - c_i \right) \tag{19}$$

c_i is a known constant. Thus, the positive utility-maximising choice maximises:

$$\frac{f_i(G_i \kappa(z))}{z} \equiv \frac{g_i(z; G_i)}{z} \tag{20}$$

("benefit per Watt"). The graph of g_i as a function of z retains the S-shape of f_i (recall eq. (14) and see fig. 1): $g_i(z)/z$ is the familiar ratio of an S-curve to its argument which is "single-peaked", as shown by fig. 3. Its maximiser is unique, and found at the genu of the S-curve (see fig. 1) . If the tangency occurs beyond the feasible range the maximiser is the highest reachable value of z [10]. The terminal should choose the value of z that maximises $g_i(z)/z$, unless, of course, that choice results in a cost greater than the terminal's benefit (in which case the terminal should decline to operate).

VI. Socially Optimal Allocation

A. *Power limits of data terminals*

To simplify the exposition, we assume that there is a power-constrained media terminal with a service level agreement that specifies its spreading gain G_M and SIR, σ_M, and denote the total "slice" available for data terminals as $1-d$. In principle, any combination of π_i satisfying $\bar{\pi} = \sum \pi_i \leq 1 - d$ can be assigned to the data terminals. The largest possible value of p_i occurs when $\pi_i = \bar{\pi} = 1-d$ (i is the *only* active data terminal), which yields $p_i = p_0(1-d)/d$ (eq. (4)). We assume below that for all i, $\hat{p}_i/p_0 \geq (1-d)/d$ (data terminals are very "well-situated"). Therefore, each data terminal can reach its resulting power level.

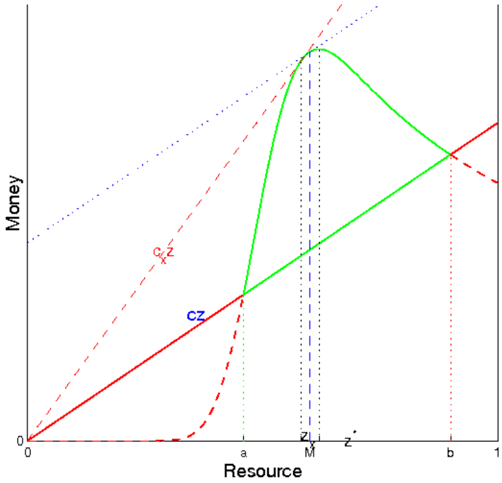


Figure 4. E-terminal under quadratic power-share pricing.

B. Objective function

With $V_i(\pi_i)$ denoting the appropriate benefit function (depends on the terminal's energy class), a reasonable criterion for a social planner is to solve

$$\text{maximise:} \quad \sum_{i=1}^{N} \mathcal{V}_i(\pi_i) \tag{21}$$

subject to,

$$\sum_{i=1}^{N} \pi_i = 1 - d \tag{22}$$

$$\pi_i \geq 0 \tag{23}$$

The necessary optimising conditions are:

$$\mathcal{V}_i'(\pi_i) - \mu_0 \leq 0 \text{ with equality for } \pi_i > 0 \tag{24}$$

C. Social optimum by linear pricing

From (24), any terminal receiving a positive share of the power must satisfy $\mathcal{V}_i'(\pi_i) = \mu_0$. From subsec. V-A, and fig. 2, a t-terminal can satisfy (24), and hence reach a "socially optimal" allocation, provided that a common price is set, which coincides with μ_0.

D. Quadratic pricing for E-terminals

However, per subsection V-B and fig. 3, in order for the e-terminal's behaviour to lead to (24),a price function of the form cz^2 is needed. With energy E_i, battery life T_i is $E_i h_i/(\pi_i(\bar{p}+p_0))$ (eq. (16)). Total cost is now $c_i\pi_i^2 T_i$, or

$$\frac{E_i h_i}{\bar{p}+p_0} c_i \pi_i \tag{25}$$

Now, π_i does *not* drop out (compare to (18)).

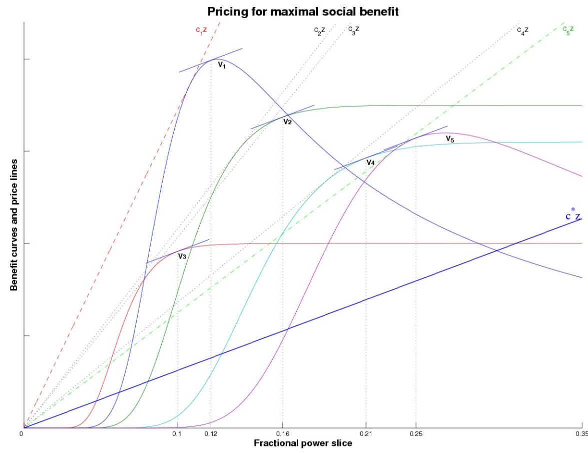
The terminal chooses π_i to maximise total benefit minus total cost:

$$\frac{E_i h_i}{\bar{p}+p_0}\left(\frac{L_i}{M_i} v_i R_i \frac{f_i(G_i\kappa(\pi_i))}{\pi_i} - c_i\pi_i\right) \tag{26}$$

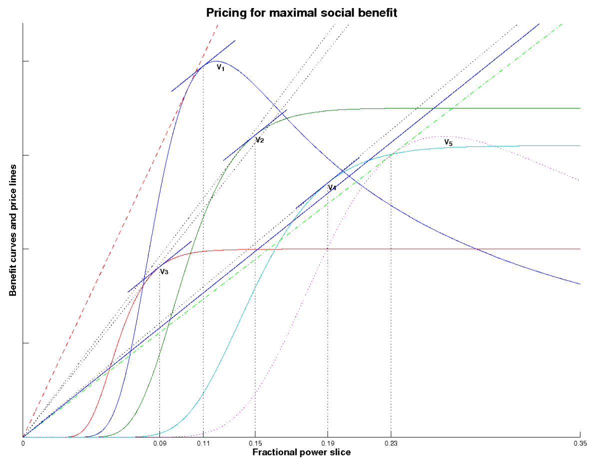
As in subsec. V-B, $f_i(G_i\kappa(z))/z := g_i(z; G_i)/z$ is the familiar ratio of an S-curve to its argument, which is "bell-shaped", as in fig. 3 (see also[10]). Thus, we must maximise $\mathcal{B}(z) - cz$ $\mathcal{B}$ has a bell-shaped graph.

In fig. 4, for $c > c_x$, the line cz lies entirely over the curve except at the origin, thus, $\mathcal{B}(z) - cz < 0$ for any $z > 0$, therefore $z = 0$ maximises $\mathcal{B}(z) - cz$. At the other extreme, if $c \approx 0$, the maximiser is $\approx z^*$, the same value chosen under linear pricing in subsec. V-B. For $0 \ll c \leq c_x$, there is an interval (a, b) on which $\mathcal{B}(z) > cz$. The maximiser satisfies $\mathcal{B}'(M) = c$ (at such a point a tangent to the curve is parallel to the cost line). $M < z^*$ because $\mathcal{B}'(z) \leq 0$ for $z \geq z^*$. It can be shown that $\mathcal{B}'$ is monotonic over (z_x, z^*), thus M is unique. If $\hat{z}$ is the largest value that can be reached, and $a < \hat{z} \leq M$, then $\hat{z}$ is optimal. However, $\hat{z} < a \rightarrow \mathcal{B}(\hat{z}) < 0$, therefore 0 is optimal, for $\hat{z} < a$.

E. Numerical illustration



(a) Total slice = 0.84



(b) Total slice = 0,54

Figure 5. Bell and S curves are benefit graphs. A terminal's optimal allocation is identified by a short tangent, parallel to the solid blue line representing the socially optimal price, for the given resource constraint. All can be served when total resource is 0,84 (subfig 5(a)), but terminal 5 is left out when the resource drops to 0,54.

One can solve the system of (nonlinear) equations (24) by algebraic or numerical means. But fig. 5 provides greater insight (recall figs. 2 and 4). The planner sweeps a price line, from vertical to horizontal, until it finds the optimal slope. Any price greater than c_1 (a line to the left of $c_1 z$ (red, dash)) is "too high". When the price falls to c_1, terminal 1 chooses to operate, and as the price continues to drop, more terminals become active and/or those already active increase their purchase. The planner stops when the sum of "slices" equals $1 - d$.

VII. Discussion

In a CDMA uplink, each can achieve SIR σ_i only if $\sum \pi_i \leq 1 - d$ where $\pi_i = \sigma_i / (\sigma_i + G_i)$ (G_i is the spreading gain and $0 < d < 1$). Given a price on π_i, i's share of the total power at the receiver, i can make its *optimal* choice *independently* from others because its SIR is $G_i \pi_i / (1 - \pi_i)$. This is a *major* advantage discussed above.

In [8], the network sets revenue maximising prices. As shown by fig. 5, under the social-planner scheme the first terminals to become active are those with the "steepest" tangenu, which is precisely the criterion used by the revenue-maximising network. However, the network chooses an individual price per terminal such that each operates at the genu ("knee"), where each maximises "benefit per Watt". The planner chooses a common price, and each active terminal ends up paying less that it would under the network's price. But at the lower planner's price, each active terminal consumes more, which may reduce the total number of terminals that can receive service.

Acknowledgement

We gratefully acknowledge partial financial support from the European Commission through the PULSERS II project, as well as from the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through the UMIC project.

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