The Impact of Different Services on the Outage Probability in UMTS

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Abstract—**In this paper, we deal with the uplink outage probability for a UMTS cellular network at a reference base station equipped with a smart antenna. Users are assumed to be distributed according to a planar Poisson point process. Their respective transmission rates are modeled by independent random variables. This leads to a compound Poisson process for the total data rate to be served in a cell by code division multiplexing. Outage occurs if the total rate exceeds the effective chip rate (3.84 MChip/s divided by the minimum required bit energy to noise power ratio). Circulant matrices, a normal approximation, and Monte Carlo integration are employed to determine the outage probability as a performance parameter of the system.**

Keywords— **Code division multiple access (CDMA), universal mobile telecommunications system (UMTS), compound Poisson process, normal approximation, circulant matrices.**

I. INTRODUCTION

CODE division multiple access (CDMA) is interference limited. Third generation UMTS networks in Europe will mainly use this technology such that model based investigations of the total interference at base stations is of uttermost importance for design and deployment purposes.

In this paper, we focus on the uplink (mobile to base station). The uplink is generally accepted to demand for a careful planning since the available resources of mobile stations are a limiting factor in mobile networks. We are particularly interested in the point where the sum of all signal powers exceeds a certain threshold such that the required bit error rate cannot be maintained simultaneously for all connected stations.

A number of investigations in the literature deals with the problem of determining the total interference for spatially Poisson distributed traffic. In [1] the total interference is calculated numerically, under assumptions of uniform traffic pattern, regular triangular grid of base stations, log-normal shadowing, soft hand-over and power control. However, mobiles are not taken to be discrete entities, but are approximated by a continuous distribution instead. Earlier work on interference characterization for Poisson traffic appeared in [2], [3], however, without assuming a cellular network structure. Approximations using the first two moments have been carried out in [4] and [5] under Poisson traffic assumptions, log-normal shadowing and power control. Other planar stochastic traffic models and adaptive antennas are included in [6]. The distribution function of a mobile's power received at a reference base station is obtained in [7], taking account of distance, shadow attenuation and power control.

None of the above approaches, however, takes different transmission rates for particular users into consideration. This is particularly assumed in the present paper. Moreover, the effect of smart antennas is included such that interference may be reduced by separating different users by distinct antenna beams. In order to keep the model analytically tractable a simple circular cell shape is assumed and only intra-cell interference is considered. Other cell interference is included in the general background noise. A reasonable upper bound for the sum of all correctly decodable data rates at a base station is the effective chip rate defined as the used chip rate divided by the least allowable E_b/N_0 ratio. Assuming spatially Poisson distributed users, each marked with a random data rate requirement, we investigate the outage probability, i.e., the probability that there is a beam of fixed width containing a set of users with a total transmission rate higher than the effective chip rate of the system.

II. THE MODEL

Future universal mobile telecommunication systems (UMTS) in Europe will use a constant chip rate of $\omega = 3.84$ MChip/s. Mobiles of different transmission rate requirements d_i share the same channel in the uplink by using nearly orthogonal code sequences. After despreading and filtering the bit energy-to-noise power ratio for each user is obtained as

$$\frac{E_b}{N_0} = s_i \cdot \frac{C}{I},$$

where $s_i = \omega/d_i$ denotes the spreading factor. Let $e_{\min}$ be defined as the minimum required E_b/N_0. For a sufficient transmission quality each user should achieve

$$e_{\min} = 5\text{dB} = 3.16.$$

For the type of services provided by 3G mobile networks widely varying transmission rates will be typical. The data rate required by user i is modeled by i.i.d. random variables D_i. The corresponding spreading gain is obtained as $S_i = \omega/D_i$. Each user applies power control and transmits with the minimum power necessary to achieve the required $E_b/N_0 = S_i(C/I)$. In this situation, it is quite natural to assume that outage happens latest if the total data rate of all stations connected to a base station exceeds the *effective chip rate* $\omega' = \omega/e_{\min}$.

In order to analyze the performance of such systems, we consider a circular cell of radius r corresponding to a reference base station located at the center. Furthermore, we assume that users are scattered according to a two-dimensional Poisson point process with intensity λ (see Fig. 1). Then the number of users in the circle segment between the rays at angle 0 and t, $0 \le t < 2\pi$ forms an homogeneous one-dimensional Poisson process N_t with intensity $\lambda' = \lambda r^2/2$. Hence, the sum of all data rates of users in the circle segment from 0 to t, $0 \le t < 2\pi$ is given

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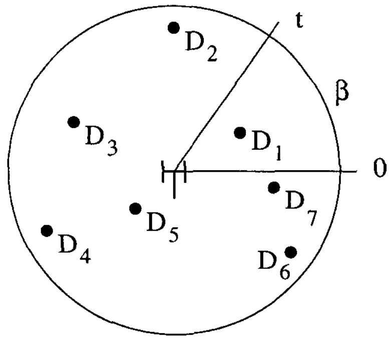

Fig. 1. A circular cell with the base station in the middle. Black dots indicate a spatial random process of users with random transmission rates $D_1, \ldots, D_7$.

by

$$X_t = \sum_{i=1}^{N_t} D_i. \quad (1)$$

X_t is a so called compound Poisson process, it has independent increments, and hence is Markovian (see, e.g., [8]).

Example. Assume an omnidirectional antenna and two different classes of users with transmission rates $d_1 = 30$ KBit/s and $d_2 = 60$ KBit/s. Users are distributed according to independent spatial Poisson processes of intensity $\lambda_i, i = 1, 2$. Random variables N_i count the number of each type in the cell. N_i are Poissonian with parameter $\lambda_i' = \lambda_i \pi r^2$, and $X_{2\pi}$ from (1) may be written as

$$X_{2\pi} = N_1 d_1 + N_2 d_2.$$

The corresponding outage probability is given by

$$P(N_1 d_1 + N_2 d_2 > \omega') = 1 - \sum_{n_1, n_2 \geq 0,\ n_1 d_1 + n_2 d_2 < \omega'} e^{-(\lambda_1' + \lambda_2')} \frac{\lambda_1'^{n_1} \lambda_2'^{n_2}}{n_1!\, n_2!},$$

which can be determined numerically. ∎

From now on, we assume that a smart antenna is deployed at the BTS, forming a receive beam of angle β, and filtering out all interference from stations outside the beam. Under the above assumptions outage happens in a fixed beam of width β with base line at angle t, $t \leq 2\pi - \beta$ if

$$I_\beta(t) = X_{t+\beta} - X_t > \omega' = \omega / e_{\min}.$$

If $t + \beta > 2\pi$ extend $I_\beta(t)$ cyclically as

$$I_\beta(t) = I_{t+\beta-2\pi}(0) + I_{2\pi-t}(t).$$

The probability that no outage occurs in the whole cell is hence given by

$$P(I_\beta(t) \leq \omega / e_{\min} \text{ for all } 0 \leq t < 2\pi). \quad (2)$$

In the following we aim at determining probability (2). For this purpose we consider two distinct approaches.

A. Normal Approximation

Let the interval $[0, 2\pi)$ be divided into n subintervals of equal length, and let

$$I_k = X_{2\pi k/n} - X_{2\pi(k-1)/n},\ k = 1, \ldots, n,$$

denote the increment of (1) in the k-th interval with $X_0 = 0$. $I_1, \ldots, I_n$ are i.i.d. random variables, whose distribution depends on the distribution of D_i, λ', and n. A discrete approximation to (2) is given by the following

$$P(I_1 + \cdots + I_k \leq \omega',\ I_2 + \cdots + I_{k+1} \leq \omega', \cdots \cdots, I_n + \cdots + I_{k-1} \leq \omega'), \quad (3)$$

with $c = \omega / e_{\min}$. (3) is built by adding k cyclically shifted terms, respectively. Let $\boldsymbol{C} = \boldsymbol{C}(n, k)$ denote the circulant matrix whose first row is given by k 1's followed by $n - k$ 0's (for circulant matrices cp. [9]), and define

$$\boldsymbol{I} = (I_1, \ldots, I_n)' \text{ and } \boldsymbol{X} = \boldsymbol{C}(n, k)\boldsymbol{I}.$$

Then (3) reads as

$$P(\boldsymbol{X} \leq c\boldsymbol{1}_n) = P(\boldsymbol{C}(n, k)\boldsymbol{I} \leq c\boldsymbol{1}_n), \quad (4)$$

where the inequality sign is to be understood componentwise and $\boldsymbol{1}_n$ denotes the n-vector of ones. From Campbell's theorem (see [10]) it follows that

$$\mathrm{E}(I_1) = \frac{\lambda' 2\pi}{n} E(D_1),$$
$$\mathrm{V}(I_1) = \frac{\lambda' 2\pi}{n} E(D_1^2).$$

Hence, for the expectation vector and the covariance matrix of $\boldsymbol{I}$ we get

$$\mathrm{E}(\boldsymbol{X}) = \frac{k \lambda' 2\pi}{n} \mathrm{E}(D_1) \boldsymbol{1}_n, \quad (5)$$

$$\mathrm{Cov}(\boldsymbol{X}) = \frac{\lambda' 2\pi}{n} \mathrm{E}(D_1^2) \boldsymbol{B}(n, k). \quad (6)$$

$\boldsymbol{B}(n, k) = \boldsymbol{C}(n, k)\boldsymbol{C}'(n, k)$ is a $n \times n$ circulant matrix with first row

$$\boldsymbol{b} = \begin{cases} (k, (k-1)^+, \ldots, (k - n/2)^+, (k - n/2 + 1), \\ \quad \ldots, k-1), \text{ for } n \text{ even}, \\ (k, (k-1)^+, \ldots, (k - (n-1)/2)^+, \\ \quad (k - (n-1)/2), \ldots, k-1), \text{ for } n \text{ is odd}. \end{cases}$$

$x^+ = \max\{0, x\}$ denotes the positive part. $\boldsymbol{B}(n, k)$ is regular if and only if $C(n, k)$ is. It can be shown that this is the case iff $(k, n) = 1$, i.e., k and n are relatively prime.

In Section III a normal approximation for $\boldsymbol{X}$ with expectation (5) and covariance (6) is applied to determine probability (2) for finite n and k.

B. Monte Carlo Approximation

Generalizing the approach in [6] we may write (2) as

$$
\begin{aligned}
&P(I_\beta(t) \le \omega/e_{\min} \text{ for all } 0 \le t < 2\pi) \\
&= \sum_{k \ge 0} P(N_{2\pi} = k) \int_0^{2\pi} dt_1 \dots \int_{t_{k-1}}^{2\pi} dt_k \frac{k!}{(2\pi)^k} \\
&\int_0^{\infty} dP(d_1) \dots \int_0^{\infty} dP(d_k) \\
&\prod_{1 \le i \le k} \Bigg(\mathbb{I}[t_i > \beta] \mathbb{I}[\sum_{1 \le j \le i} d_j \mathbb{I}[t_j + \beta > t_i] \le \omega'] \\
&+ \mathbb{I}[t_i \le \beta] \\
&\mathbb{I}[\sum_{1 \le j \le i} d_j + \sum_{i+1 \le j \le k} d_j \mathbb{I}[t_j + \beta > 2\pi + t_i] \le \omega'] \Bigg) \quad (7)
\end{aligned}
$$

We now briefly sketch the proof of Equation (7). Conditional on the number k of users in the whole cell, the angular coordinates in increasing order are distributed as the order statistics of k independent, on $[0, 2\pi)$ uniformly distributed random variables. We furthermore condition on the respective locations $t_1 \le \dots \le t_k$ and the independent data rates $d_1, \dots, d_k$. It clearly holds that $I_\beta(t) \le \omega'$ for all $0 \le t < 2\pi$ if and only if the sum of all data rates d_j corresponding to locations t_j in the interval $[t_i - \beta, t_i)$ does not exceed ω' for all $i = 1, \dots, k$. Because of cyclic repetition, in case of $t_i - \beta < 0$ the sum of all d_j with t_j in any of the intervals $[0, t_i)$ or $[2\pi + t_i - \beta, 2\pi)$ must not be greater than ω'. The integrand in (7) is combined of indicator functions which yields a 1 iff none of the above sums is greater than ω' and 0, otherwise. Successive integration and summation over the respective joint distribution gives probability (7).

Monte Carlo methods described in Chapter 5 in [11] are used to evaluate Equation (7) numerically.

III. Numerical Results

To obtain numerical values for the outage probability (4) under different parameters, complicated high-dimensional integrals must be evaluated.

In order to compute probability (4), an extension of the algorithm in [12] has been employed. This algorithm makes extensive use of Monte Carlo integration. In Figure 2, the corresponding results are compared with direct Monte-Carlo approximation of (7). In general, the outage probabilities obtained by the second approximation method are greater, and hence underestimate the performance of the system. The difference between the two approximations decreases with increasing radius r of the cell. In the following we confine ourselves to the second method (7).

Three types of services are of interest as proposed in [13], namely

	Bit rates (kb/s)
Voice	8
Service Class B	64
LCD 144	144

The outage probability is calculated as a function of the radius r (1.2 km $\le r \le$ 2.4 km) of the cell. In Figure 3, four different

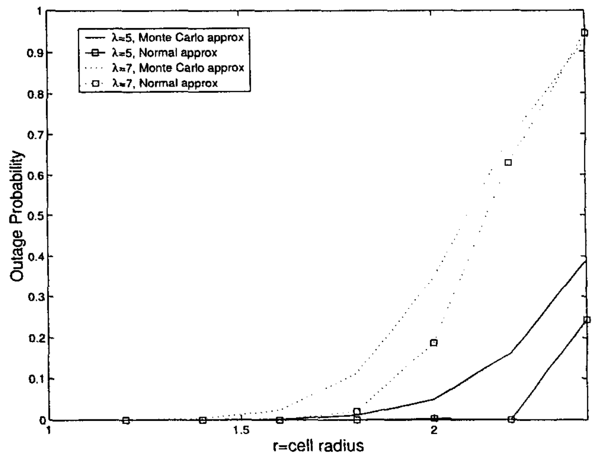

Fig. 2. Outage probability versus the radius of the cell, the two approximation methods are compared.

Poisson process with $\lambda = 5, 7, 9, 11, 13$ users/km^2, respectively, are considered. The bit rate service distribution is uniform, namely P[bit rate $= 8$] $= P$[bit rate $= 64$] $= P$[bit rate $= 144$] $= 1/3$.

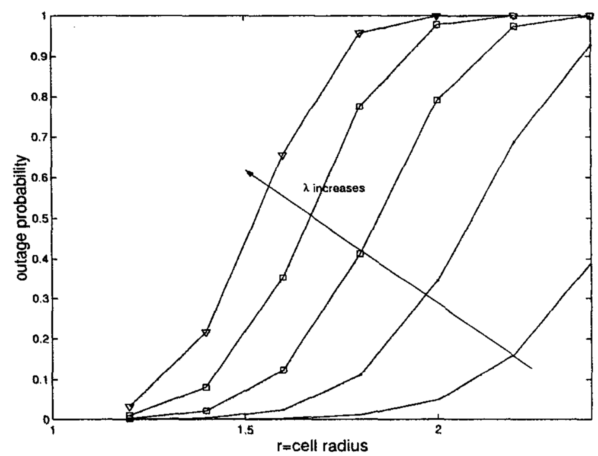

Fig. 3. Outage probability versus the radius of the cell, for intensities $\lambda = 5, 7, 9, 11, 13$.

As expected, the greater the radius (or intensity), the greater the outage probability. In Figures 4 and 5 we compare the outage probability if only two types of services are of interest. The corresponding weights are as follows.

Service	Curve 1	Curve 2	Curve 3
Service Class B	0.1	0.2	0.3
LCD 144	0.9	0.8	0.7

In Figure 4 and Figure 5 a traffic intensity of $\lambda = 5$ and $\lambda = 7$ respectively is set.

As expected the greater the contribution of users with a high data rate, the greater the outage probability. When comparing Figure 4 to 5 we obtain Figure 6 where the differences between corresponding outage probabilities are plotted. First there exists an identical behavior between three curves when the intensity is

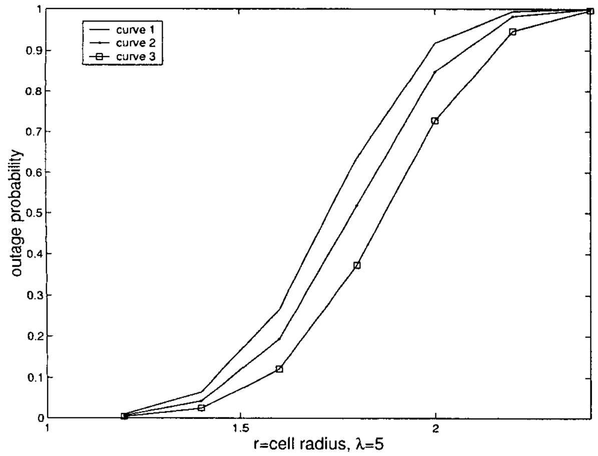

Fig. 4. Outage probability versus the radius of the cell, $(0.1, 0.9), (0.2, 0.8)$ and $(0.3, 0.7)$ are the weights of services of $(64, 144)$ kb/s, $\lambda = 5$.

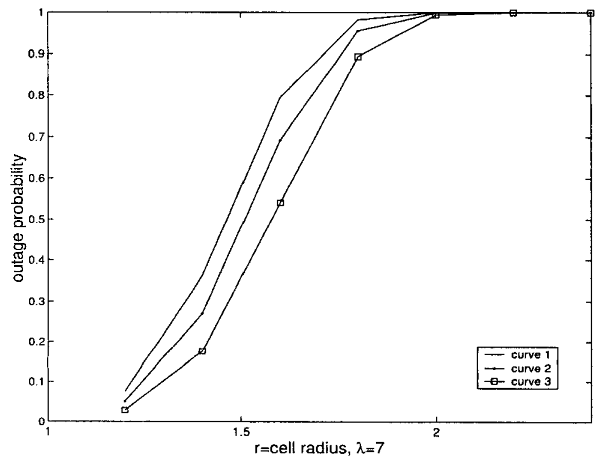

Fig. 5. Outage probability versus the radius of the cell, $(0.1, 0.9), (0.2, 0.8)$ and $(0.3, 0.7)$ are the weights of services of $(64, 144)$ kb/s, $\lambda = 7$.

fixed. The difference first increases to reach a maximum point and then this difference decreases again. The maximum point corresponds to two different radii according to the intensity λ at which the outage probability is evaluated.

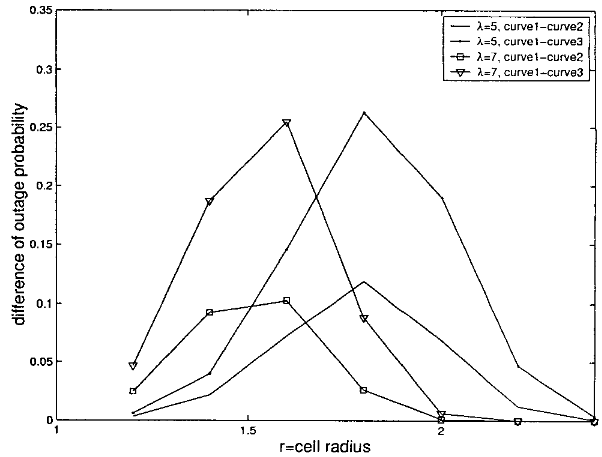

Fig. 6. Comparison of outage probabilities for different weights

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