

Constant-Rate Power Allocation under Constraint on Average BER in Adaptive OFDM Systems

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Abstract—In adaptive orthogonal frequency division multiplexing (OFDM) systems, different powers and rates can be allocated to subcarriers optimally by water-filling. This strategy has been proved to improve the system performance significantly. However, a large signalling overhead is induced by water-filling to contain the group of presently employed mapping schemes. The faster the channel varies in time, the more frequently the large signalling overhead is needed. In this paper, to avoid such a problem, we propose that a constant rate and different powers are allocated to subcarriers while satisfying the constraints on the transmission rate and the average bit-error rate (BER). First, an approximate relationship between BER and signal-to-noise ratio (SNR) is employed. Then, based on the convex optimization framework, the transmission power is optimally distributed to the used subcarriers with an optimal constant rate. The subcarrier assignment can be determined by an upgraded bisection method, which can be also used for other resource allocation problems. Simulations demonstrate that the proposed resource allocation strategy has better performance than water-filling with the signalling overhead considered in fast time-varying environment.

I. INTRODUCTION

Inter-symbol interference (ISI) induced by the multipath fading channel may deteriorate wireless data applications significantly. To combat this effect, OFDM [1] has been suggested due to the high efficiency of its receiver. The wide transmission band impacted by the frequency selective fading is equally divided into subcarriers. This allows for assigning different powers and rates to subcarriers according to their different channel gain-to-noise ratio (CNR) in adaptive OFDM systems.

To achieve high energy efficiency for the real time transmission, the total transmission power is minimized, while the minimal transmission rate is demanded and the required BER is upper bounded. This is called the margin-adaptive problem. For the non-real time transmission, the rate-adaptive problem is taken into account, where the transmission rate is maximized while meeting the limit on the total transmission power and the required BER. They can be optimally solved by water-filling [1], where different rates and powers are allocated to subcarriers with the same BER achieved over each used subcarrier. Based on water-filling, the discrete rates and powers can be assigned to subcarriers in [2], [3]. A negligible

performance loss compared to water-filling can be reached by a constant-power allocation [4] as long as the optimal subcarrier assignment is determined, while the allocated rates may be different. This performance loss is theoretically bounded in [5], where a low-complexity implementation is suggested. The power is uniformly distributed over subcarriers while considering the BER constraint in [6]. The objective of the original problem is changed to minimizing the aggregate BER in [7] and to maximizing the parallel decoding capacity in [8].

However, if the resource allocation is centralized at the transmitter, the signalling overhead must be very large in order to take the group of employed mapping schemes to the receiver. Informed with the employed mapping schemes, the receiver can detect the following data symbols. This problem worsens in a fast time-varying environment, since a group of mapping schemes can be optimally effective only for a short period of time and must be updated frequently. Therefore, the energy efficiency would be degraded by the large power consumption on transmitting the large signalling overhead. In [9], [10] the subcarriers are clustered into blocks to reduce the computational complexity and the signalling overhead. By doing so, either the system performance would be impaired severely, or the overhead could not be reduced significantly, when the frequency selectivity is strong. To avoid such problems, the same power and rate is statically allocated to a fixed number of subcarriers, which have higher CNRs [11]. The number of used subcarriers is determined empirically and known to the receiver. This implies that the subcarrier assignment does not adapt to the channel conditions. The same BER and SNR are obtained over all subcarriers in [12]. However, the performance loss by these two methods may be very large due to the frequency selective channels.

Alternatively, as the main difference between this paper and the previous works, the rates allocated to the used subcarriers may vary but are equal, while the assigned powers are not the same under the constraint on the average BER. Consequently, the achieved BERs and SNRs over subcarriers at the receiver are different unlike the above mentioned works. The induced benefit is that the signalling overhead includes only one mapping scheme and the number of the used subcarriers. The wireless local area networks (WLAN) protocols can be easily extended to utilize the proposed resource allocation strategy.

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The remainder of this paper is organized as follows. In Section II assumptions are stated and the problem is formulated. In Section III, first, given the subcarrier assignment, the problem is optimally solved with the duality theory. Then, the bisection method is upgraded by inheriting the idea of the golden section method to efficiently determine the subcarrier assignment. At last, we compare the proposed resource allocation strategy with water-filling. Simulation results are shown in Section IV. Finally, the content of this paper is concluded.

II. PROBLEM FORMULATION

Consider an OFDM system consisting of N subcarriers and a pair of transmitter and receiver. It is assumed that the frequency selective fading channel is known by the transmitter. Each group of employed mapping schemes is valid only in the time period of L OFDM symbols due to the time-varying channel. It is included in the overhead and sent from the transmitter to the receiver, while it is not available at the receiver due to the computational limit or other constraints. Subcarriers with power allocated are contained in the subcarrier assignment \mathcal{A} with cardinality a .

The transmission rate, lower bounded by R , is equally distributed over subcarriers in \mathcal{A} , as r bits per subcarrier per OFDM symbol. Different transmission BERs ε_n may be allocated to these subcarriers, where n is the subcarrier index. The average BER is upper bounded by ε . We aim at minimizing the transmission power while satisfying the data rate and BER constraint. The optimization problem can be expressed as

$$\begin{aligned} & \text{minimize} && \sum_{n \in \mathcal{A}} p_n \\ & \text{subject to} && ar \geq R, \\ & && \frac{1}{a} \sum_{n \in \mathcal{A}} \varepsilon_n \leq \varepsilon. \end{aligned} \quad (1)$$

Note that the same rate r , allocated to subcarriers in \mathcal{A} , may vary and is not restricted to be integer for theoretical analysis. Let $\{p_n\}$ denote the set of allocated powers. Certainly, the allocated powers and rates must be non-negative.

The uncoded modulation scheme of quadrature amplitude modulation (QAM) is employed. In [13], the transmission BER over subcarrier n can be bounded by

$$\varepsilon_n \leq 2 \exp\left(\frac{-1.5p_n g_n}{2r_n - 1}\right),$$

where g_n indicates the CNR of subcarrier n and r_n is the allocated rate to subcarrier n . A tighter bound with 1 dB improvement is

$$\varepsilon_n \leq 0.2 \exp\left(\frac{-1.5p_n g_n}{2r_n - 1}\right). \quad (2)$$

After rearranging (2), the power-rate function under the BER constraint can be approximated as

$$r_n = \log_2\left(1 + \frac{-1.5}{\ln(5\varepsilon_n)} p_n g_n\right), \quad (3)$$

which alters and may still remain convex after employing certain channel coding schemes.

III. POWER ALLOCATION WITH BER CONTROL

The above problem belongs to the class of convex optimization problems. The duality theory can be used to solve it.

A. Water-Filling

Before focusing on problem (1), water-filling is introduced as a benchmark. In water-filling, different rates and the same transmission BER are allocated to subcarriers. It is formulated as

$$\begin{aligned} & \text{minimize} && \sum_{n \in \mathcal{A}} p_n \\ & \text{subject to} && \sum_{n \in \mathcal{A}} r_n \geq R, \\ & && r_n = \log_2\left(1 + \frac{-1.5}{\ln(5\varepsilon)} p_n g_n\right). \end{aligned} \quad (4)$$

with (3) as an equality constraint considered. Given \mathcal{A} , the optimal solution to (4) is

$$\begin{aligned} r_n &= \log_2\left(\frac{-1.5}{\ln(5\varepsilon)} \lambda g_n\right), \\ p_n &= \lambda - \frac{\ln(5\varepsilon)}{-1.5g_n}, \end{aligned} \quad (5)$$

which must be non-negative. The water level λ is

$$\lambda = \frac{\ln(5\varepsilon)}{-1.5} 2^{\frac{R}{a}} \left(\prod_{n \in \mathcal{A}} \frac{1}{g_n}\right)^{\frac{1}{a}}. \quad (6)$$

The optimal subcarrier assignment can be determined by the approach in [1] or the following procedure. Set \mathcal{A} is initialized to be $\{1, \dots, N\}$. The power allocated to subcarriers in \mathcal{A} can be obtained with (5) and (6). The subcarriers with non-positive power allocated are removed from \mathcal{A} . Power calculation and removing subcarriers are iteratively performed until all subcarriers in \mathcal{A} are allocated with positive power. Then, the minimal consumed power by water-filling is derived as $P^{(WF)}$. Both procedures need to compute (6) iteratively. However, it may happen that the product operation in (6) violates the precision limit of computers, when the number of subcarriers is very large. We can simply change (6) to

$$\lambda = \frac{\ln(5\varepsilon)}{-1.5} 2^{\frac{R}{a}} \left(\prod_{n \in \mathcal{A}} \frac{1}{g_n}\right)^{\frac{1}{a}}$$

to avoid this problem, but the average computing time would increase significantly as the number of subcarriers increases.

The complementary slackness condition states that the constraint is satisfied with equality if and only if the dual variable associated with the inequality is strictly greater than zero [14]. In water-filling, this condition implies that the positive water level (the scaled dual variable) assures that the achieved rate must be equal to R for the optimal solution. As assumed before, the resource allocation can be performed only at the transmitter. If each specific mapping scheme is denoted by M bits, at least MN/R OFDM symbols, included in the signalling overhead, must be used to indicate the currently employed mapping schemes for the receiver due to the different rates allocated to subcarriers.

B. Constant-Rate Adaptation

To reduce the signalling overhead, strategy (1) is proposed. It can be solved by the duality theory [14], which is explained especially for resource allocation problems in [15]. Provided the subcarrier assignment \mathcal{A} , the rate on each subcarrier in \mathcal{A} is R/a due to the constant-rate constraint. Consequently, based on the Karush-Kuhn-Tucker conditions, the Lagrangian of problem (1) is defined as

$$\mathcal{L}(\beta, \{p_n\}) = \sum_{n \in \mathcal{A}} p_n + \beta \left(\frac{1}{a} \sum_{n \in \mathcal{A}} \varepsilon_n - \varepsilon \right), \quad (7)$$

where β is the Lagrange multiplier. The dual optimization problem can be expressed by

$$\begin{aligned} & \text{maximize} && \mathcal{D}(\beta) \\ & \text{subject to} && \beta \geq 0, \end{aligned}$$

where the dual objective $\mathcal{D}(\beta)$ is an unconstrained minimization of the Lagrangian, obtained as

$$\begin{aligned} \mathcal{D}(\beta) &= \min_{\{p_n\}} \mathcal{L}(\beta, \{p_n\}) \\ &= -\beta\varepsilon + \min_{\{p_n\}} \sum_{n \in \mathcal{A}} \left(p_n + \frac{\beta}{a} \varepsilon_n \right). \end{aligned}$$

The power variables $\{p_n\}$ are separable over subcarriers. It follows that

$$\mathcal{D}(\beta) = -\beta\varepsilon + \sum_{n \in \mathcal{A}} \min_{p_n} \left(p_n + \frac{\beta}{a} \varepsilon_n \right). \quad (8)$$

As earlier mentioned, the power-rate function (3) transforms to the power-BER function

$$\varepsilon_n = 0.2 \exp\left(\frac{-1.5g_n}{2^r - 1} p_n\right) \quad (9)$$

such that the same rate $r = R/a$ may be distributed over the subcarriers in \mathcal{A} . After taking (9) to (8), the minimum can be acquired by the derivative over p_n , shown as

$$p_n = \frac{c}{g_n} \ln\left(\frac{g_n}{5\mu}\right) \quad (10)$$

$$\varepsilon_n = \frac{\mu}{g_n}. \quad (11)$$

These expressions are simplified by using the substitutions $(2^r - 1)/1.5 = c$ and $a(2^r - 1)/(1.5\beta) = \mu$. Similar to the analysis on the optimum condition of water-filling, the complementary slackness condition also ensures that our objective can be minimized, while the average BER constraint in problem (1) holds with equality. Then, the substituted Lagrange multiplier can be obtained as

$$\mu = \frac{a\varepsilon}{\sum_{n \in \mathcal{A}} 1/g_n}, \quad (12)$$

which is positive. If the subcarrier assignment \mathcal{A} is derived, the minimum to the constant-rate power allocation is obtained as $P^{(CR)}$.

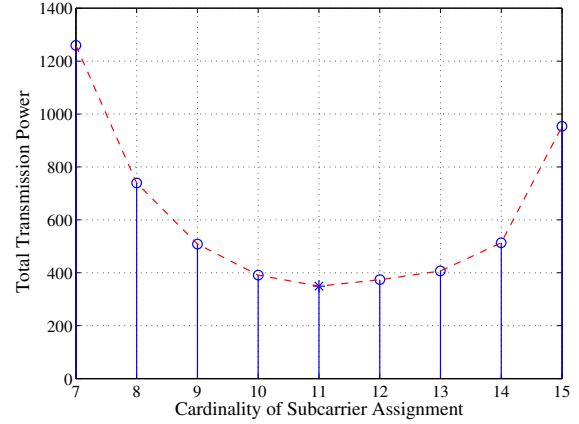


Fig. 1. Example of transmission power vs. cardinality of \mathcal{A} .

C. Upgraded Bisection Method

If the power (10) allocated to subcarrier n is non-positive, $g_n \leq 5\mu$ must hold. However, this can rarely happen in practice, since the target average BER in (12) is generally very small compared to the CNRs. This implies that all subcarriers in \mathcal{A} may be allocated with positive power, even when the subcarrier assignment is not optimal. The procedures of finding the optimal \mathcal{A} for water-filling, as above described, cannot be used for our problem. Hence, to the best of our knowledge, an upgraded bisection method is novelly applied in the following in order to determine the subcarrier assignment for the constant-rate power allocation.

Obviously, the exhaustive search for the optimal \mathcal{A} cannot work in practice due to its complexity of $\mathcal{O}(2^N)$. Subcarriers are arranged in a descending order of CNRs with complexity of $\mathcal{O}(N \log(N))$. Without loss of generality, $g_1 \geq \dots \geq g_N$ is assumed. With \mathcal{A} initialized with $\{1, \dots, N\}$, the exhaustive search can be performed by excluding the last subcarrier in \mathcal{A} in each iteration. Then, the minimum can be selected and (10) has to be calculated $\sum_{n=1}^N n$ times so that the complexity reduces to $\mathcal{O}(N^2)$. Fig. 1 draws an example of the transmission power against the cardinality of \mathcal{A} after sorting subcarriers. In this example, the optimal a^* is 11. On the left side of a^* , the transmission power is monotonically decreasing in a . On the other side, it is monotonically increasing in a . If the relationship between the transmission power and the cardinality of \mathcal{A} is an inverse unimodal function, the exhaustive search can be performed from the right side by removing the last subcarrier in \mathcal{A} till the transmission power begins increasing. This does not reduce the complexity, but decreases the average computing time by 50%.

Alternatively, the bisection method can be used after an alteration, if the transmission power is an inverse unimodal function of a . The conventional bisection method cannot be directly used for our problem due to the inverse unimodality shown in Fig. 1. We inherit the idea of the golden section search, which cannot be used either because of the discrete relationship between the transmission power and the cardinality of \mathcal{A} . Here, the upgraded bisection method is derived as shown in Algorithm 1. In each iteration, the searching interval

Algorithm 1 Upgraded Bisection Method

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 $a^{(l)} \leftarrow 1$ 
 $a^{(r)} \leftarrow N$ 
 $a^{(m)} \leftarrow \lceil 0.5N \rceil$ 
 $c^{(m)} \leftarrow (2^{R/a^{(m)}} - 1)/1.5$ 
 $\mu^{(m)} \leftarrow a^{(m)}\varepsilon / (\sum_{n=1}^{n=a^{(m)}} 1/g_n)$ 
 $P^{(m)} \leftarrow c^{(m)} \sum_{n=1}^{n=a^{(m)}} (\ln(g_n/\mu^{(m)}/5)/g_n)$ 
 $P^{(l)} \leftarrow 0$ 
 $P^{(r)} \leftarrow 0$ 
repeat
  if  $a^{(m)} - a^{(l)} \leq a^{(r)} - a^{(m)}$  then
     $a^{(x)} \leftarrow \lceil 0.5(a^{(m)} + a^{(r)}) \rceil$ 
  else
     $a^{(x)} \leftarrow \lfloor 0.5(a^{(m)} + a^{(l)}) \rfloor$ 
  end if
   $a^{(x)} \leftarrow \lceil 0.5N \rceil$ 
   $c^{(x)} \leftarrow (2^{R/a^{(x)}} - 1)/1.5$ 
   $\mu^{(x)} \leftarrow a^{(x)}\varepsilon / (\sum_{n=1}^{n=a^{(x)}} 1/g_n)$ 
   $P^{(x)} \leftarrow c^{(x)} \sum_{n=1}^{n=a^{(x)}} (\ln(g_n/\mu^{(x)}/5)/g_n)$ 
  if  $a^{(m)} - a^{(l)} \leq a^{(r)} - a^{(m)}$  then
    if  $P^{(m)} \leq P^{(x)}$  then
       $a^{(r)} \leftarrow a^{(x)}$ 
       $P^{(r)} \leftarrow P^{(x)}$ 
    else
       $a^{(l)} \leftarrow a^{(m)}$ 
       $P^{(l)} \leftarrow P^{(m)}$ 
       $a^{(m)} \leftarrow a^{(x)}$ 
       $P^{(m)} \leftarrow P^{(x)}$ 
    end if
  else
    if  $P^{(m)} \leq P^{(x)}$  then
       $a^{(l)} \leftarrow a^{(x)}$ 
       $P^{(l)} \leftarrow P^{(x)}$ 
    else
       $a^{(r)} \leftarrow a^{(m)}$ 
       $P^{(r)} \leftarrow P^{(m)}$ 
       $a^{(m)} \leftarrow a^{(x)}$ 
       $P^{(m)} \leftarrow P^{(x)}$ 
    end if
  end if
until  $a^{(r)} - a^{(l)} \leq 2$ 
 $P^{(CR)} \leftarrow \min(P^{(l)}, P^{(m)}, P^{(r)})$ 
    
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$[a^{(l)}, a^{(r)}]$ is divided into two sections by the third point $a^{(m)}$ with $a^{(l)} < a^{(m)} < a^{(r)}$. A new point $a^{(x)}$ is chosen either between $a^{(l)}$ and $a^{(m)}$ or between $a^{(m)}$ and $a^{(r)}$. By taking the latter choice, if the transmission power $P^{(m)}$ with $a^{(m)}$ is smaller than the transmission power $P^{(x)}$ with $a^{(x)}$. Then, the new bracketing triplet of points is $a^{(l)} < a^{(m)} < a^{(x)}$; otherwise, the new bracketing triplet is $a^{(m)} < a^{(x)} < a^{(r)}$. This iteration procedure finishes until $a^{(x)}$ is equal to one of the triplets from

 TABLE I
 COMPLEXITY COMPARISON

Exhaustive search	Upgraded bisection method	Water-filling
$\mathcal{O}(N^2)$	$\mathcal{O}(N \log(N))$	$\mathcal{O}(N \log(N))$

the previous iteration, i.e., $a^{(r)} - a^{(m)} = a^{(m)} - a^{(l)} = 1$. The ceiling and floor functions in Algorithm 1 are used due to the number of subcarriers being integer. The optimal point may be lost by using rounding here. At last, the three total transmission powers $P^{(l)}, P^{(m)}, P^{(r)}$ with different numbers of used subcarriers are compared and the optimum is determined.

D. Complexity Analysis and Comparison

Since Algorithm 1 is an upgraded bisection method, $(a^{(r)} - a^{(l)})/2$ elements are excluded from set \mathcal{A} in every two iterations and $2 \log_2(N)$ iterations in total are performed. In each two iterations, the transmission power must be allocated to $(a^{(m)} - a^{(l)})/2$ subcarriers and $(a^{(r)} - a^{(m)})/2$ subcarriers. Hence, the complexity of Algorithm 1 is $\mathcal{O}(N \log(N))$, which is equal to the complexity of the sorting. The complexities are listed in Table I. Water-filling [1] also needs to sort the subcarriers according to their CNRs so that it has complexity of $\mathcal{O}(N \log(N))$ also. As earlier mentioned, the average computing time for water-filling may be significantly increasing in the large N due to the product operation in (6). This problem does not exist in the proposed strategy, since product is not needed in (12). For some channel realization, the transmission power is not unimodal in a . However, such cases happen very rarely shown in the simulation.

Only M bits for expressing the employed mapping scheme and $\log_2(N)$ bits for the number of used subcarriers are needed in the signalling overhead for the receiver. As previously illustrated, a group of employed mapping schemes can be highly effective only for L data OFDM symbols, where L is determined by the time-varying channel. To apply water-filling, the transmission power of at least $P^{(WF)}(L + MN/R)$ is consumed in order to transmit the $L + MN/R$ data and signalling OFDM symbols. To apply our strategy, the signalling overhead could be very small, and the transmission power of $P^{(CR)}(L + M/R)$ is sufficient. This means that the water-filling has better performance than the proposed strategy, only when the following inequality holds:

$$L > L^{(\min)} = \frac{M(NP^{(WF)} - P^{(CR)})}{R(P^{(CR)} - P^{(WF)})}. \quad (13)$$

In the fast time-varying environment, the mapping schemes must vary frequently and inequality (13) may not hold.

IV. SIMULATION RESULTS

The proposed strategy is compared to water-filling in simulations. The simulation system is built with the parameters from worldwide interoperability for microwave access (WiMAX) [16]. It consists of 256 subcarriers. The OFDM symbol duration is 102.9 μ s. Channel coding is not used to extract the performance loss by the proposed strategy. The required BER is 10^{-3} . The demanded rate varies from 64 to 1024 bits per OFDM symbol, i.e., on average 0.25 to 4 bits

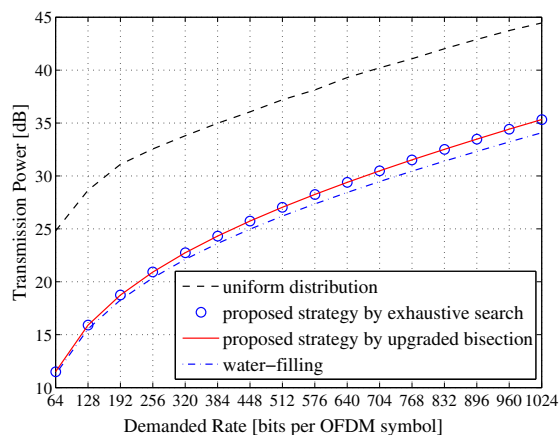


Fig. 2. Instantaneous per-symbol transmission power for each OFDM symbol against different demanded rates

per subcarrier per OFDM symbol. Each mapping scheme is expressed by 3 bits. In simulations, the frequency selective channel is modeled as consisting of 8 independently Rayleigh distributed multipaths with an exponential decaying profile. The expected channel gain on each subcarrier is normalized to one and the noise power is set to be -10 dB.

Fig. 2 draws the instantaneous per-symbol transmission power against different demanded rates. The solution provided by the upgraded bisection method is almost the same as the optimal solution. The proposed strategy reduces the power consumption significantly compared to that the demanded rate is uniformly distributed to all subcarriers. The gap between the power consumptions by the proposed strategy and water-filling is increasing in the demanded rate. It is limited to 1.25 dB in our simulation. If the signalling overhead is taken into the comparison, water-filling has better performance as inequality (13) holds. However, this condition is violated, when the demanded rate is relatively small, shown in Fig. 3.

V. CONCLUSION

In this work, our aim is to minimize the total transmission power while the demanded transmission rate and the required average BER are lower and upper bounded, respectively. In stead of water-filling, we have proposed that the constant rate and different BERs are allocated to subcarriers. This constant-rate power allocation problem has been optimally solved. To the best of our knowledge, an upgraded bisection method has been newly utilized to find the subcarrier assignment. This approach can be also used on other resource allocation problems. As long as that the subcarrier assignment is determined, the demanded rate is equally distributed over the used subcarriers. With the duality theory, different BERs, subject to the average BER constraint, are assigned to the used subcarriers such that the comparable performance to water-filling can be acquired. In the proposed strategy, large signalling overheads taking the employed mapping schemes from the transmitter to the receiver can be avoided so that the resource allocation can be performed frequently. Since the same mapping scheme is applied on the used subcarriers, the existing WLAN protocols can be easily extended to adopt our strategy and the receiver

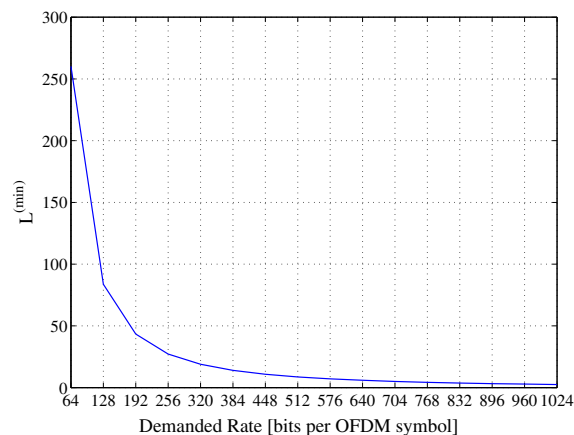


Fig. 3. Minimal number of OFDM symbols for water-filling with the better performance against different demanded rates.

is still simple. Due to the fast fading channel, a group of employed mapping schemes can be effectively used only for a few OFDM symbols. By considering this, simulations have shown that the proposed strategy has better performance than water-filling, when the rate demand is relatively low in the fast time-varying environment.

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