PERFORMANCE EVALUATION OF SENSOR FUSION WITH SIDE INFORMATION

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ABSTRACT

The efficient evaluation of fusion algorithms becomes particularly important when different fusion schemes have to be compared with respect to an underlying performance metric. In this paper, we present explicit expressions for the global error probabilities of sensor fusion with side information for distributed detection applications. In the considered distributed detection problem, the sensors compress their observations independently and transmit local decisions to a fusion center that combines the received decisions with respect to available side information and computes the final detection result. In the special case of identical sensors, computationally efficient expressions are obtained by using the multinomial distribution. Numerical results reveal the influence of different qualities of side information on the overall detection performance.

1. INTRODUCTION

One of the primary applications of wireless sensor networks is the detection of phenomena of interest in the monitored environment, e.g., absence or presence of a target [1]. The sensors typically operate on limited energy budgets and are consequently subject to communication constraints, resulting in a finite number of bits each sensor node can transmit to the data sink before it runs out of power. In order to extend sensor network lifetime, preprocessing of measured raw data at the sensors and transmission of summary messages is recommended. In the parallel fusion topology, the sensors compress their observations independently and make preliminary decisions about the state of the observed environment. The sensors transmit the local decisions to a fusion center that combines the decisions with respect to available side information and computes the final detection result. The problem of sensor fusion is to optimally design the fusion rule according to the joint distribution of local sensor decisions and the statistics of the side information with respect to an overall performance criterion.

Sensor fusion with side information for distributed detection applications was first considered by Hashlamoun and Varshney [2]. They derived the form of the optimal fusion rule for fixed binary local sensor decision rules. In this paper, we allow for general M-ary local sensor decision rules and provide explicit expressions for the global error probabilities both for identical and non-identical sensors.

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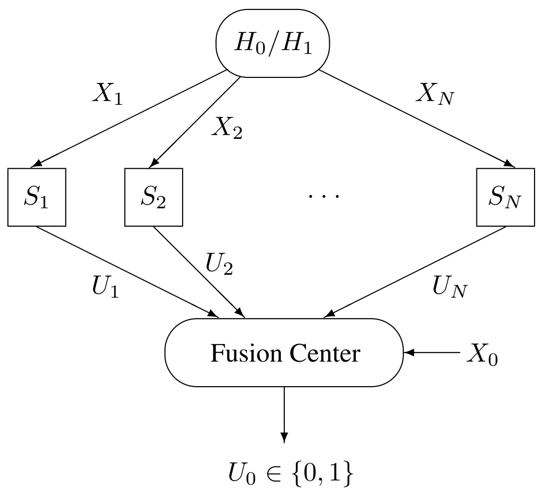

Fig. 1. Sensor fusion with side information.

2. SENSOR FUSION

The problem of sensor fusion with side information and M-ary decisions at the local sensors can be stated as follows (see Fig. 1). We consider a binary hypothesis testing problem with hypotheses H_0 and H_1 indicating the state of the monitored environment. The associated prior probabilities are $\pi_0 = P(H_0)$ and $\pi_1 = P(H_1)$. In order to detect the true state of nature, a network of N sensors $S_1, \dots, S_N$ obtains random observations

$$(X_1, \dots, X_N)' \in \mathcal{X}_1 \times \dots \times \mathcal{X}_N, \quad (1)$$

which are generated according to either H_0 or H_1. The random observations $X_1, \dots, X_N$ are assumed to be conditionally independent across sensors given the underlying hypothesis, i.e., the joint conditional probability density function of all the observations factorizes as

$$f(x_1, \dots, x_N|H_k) = \prod_{j=1}^{N} f_j(x_j|H_k), \quad k = 0, 1. \quad (2)$$

2.1. Local sensor decision rules

The sensors compress their respective observations X_j independently by forming local decisions

$$U_j = \delta_j(X_j), \quad j = 1, \ldots, N. \tag{3}$$

In the general case of M-ary quantization at the local sensors, the local sensor decision rules δ_j are mappings

$$\delta_j \colon \mathcal{X}_j \to \{1, \ldots, M\}, \quad j = 1, \ldots, N. \tag{4}$$

As Warren and Willett have shown, local sensor decision rules leading to jointly optimal configurations are monotone likelihood ratio partitions of the sensor observation spaces $\mathcal{X}_1, \ldots, \mathcal{X}_N$, provided that the observations are conditionally independent across sensors [3]. Hence, it is sufficient to consider sensor decision rules δ_j that can be parameterized by a vector of real quantization thresholds $\boldsymbol{\tau}_j = (\tau_j^{(1)}, \ldots, \tau_j^{(M-1)})'$ with $\tau_j^{(k)} \leq \tau_j^{(k+1)}$, leading to the conditional quantization probabilities

$$\alpha_j^{(k)} = P(U_j = k|H_0) = P(\tau_j^{(k-1)} < L_j \leq \tau_j^{(k)}|H_0),$$

$$\beta_j^{(k)} = P(U_j = k|H_1) = P(\tau_j^{(k-1)} < L_j \leq \tau_j^{(k)}|H_1).$$

In the above equations, $\tau_j^{(0)} = -\infty$, $\tau_j^{(M)} = \infty$, and $L_j = \log(f_j(X_j|H_1)/f_j(X_j|H_0))$ is the local log-likelihood ratio of observation X_j. The stochastic vectors of quantization probabilities

$$\boldsymbol{\alpha}_j = (\alpha_j^{(1)}, \ldots, \alpha_j^{(M)})', \tag{5}$$

$$\boldsymbol{\beta}_j = (\beta_j^{(1)}, \ldots, \beta_j^{(M)})' \tag{6}$$

are computable given the observation statistics $f_j(x_j|H_k)$ and the quantization thresholds $\boldsymbol{\tau}_j$ for each $j = 1, \ldots, N$. Upon local decision making, the sensors transmit the local decisions $U_1, \ldots, U_N$ to the fusion center.

2.2. Optimal decision fusion with side information

At the fusion center, the received decisions $U_1, \ldots, U_N$ are fused with respect to the side information X_0 into the final detection result $U_0 = \delta_0(U_1, \ldots, U_N, X_0)$, where the fusion rule δ_0 is a binary-valued mapping

$$\delta_0 \colon \mathcal{M}^N \times \mathcal{X}_0 \to \{0, 1\}. \tag{7}$$

We assume that the random variable $X_0 \in \mathcal{X}_0$ describing the side information and the local decisions $U_1, \ldots, U_N$ from the sensors are conditionally independent given the underlying hypothesis. Sensor fusion performance is measured in terms of the global probability of error

$$P_e = \pi_0 P_f + \pi_1 P_m \tag{8}$$

at the fusion center, which is a weighted sum of the global probability of false alarm $P_f = P(U_0 = 1|H_0)$ and the global probability of miss $P_m = P(U_0 = 0|H_1)$. Optimal decision fusion with side information under the minimum probability of error criterion can be performed by evaluating a log-likelihood ratio test with a variable threshold according to

$$\ell_0 \underset{u_0=0}{\overset{u_0=1}{\gtrless}} \tau(\boldsymbol{u}), \tag{9}$$

where

$$\ell_0 = \log\left(\frac{f_0(x_0|H_1)}{f_0(x_0|H_0)}\right) \tag{10}$$

is the realization of the log-likelihood ratio L_0 of the side information X_0 and

$$\tau(\boldsymbol{u}) = \log\left(\frac{\pi_0}{\pi_1}\right) + \sum_{j=1}^{N} \log\left(\frac{\alpha_j^{(u_j)}}{\beta_j^{(u_j)}}\right) \tag{11}$$

is the decision threshold for the received values

$$\boldsymbol{u} = (u_1, \ldots, u_N)' \in \mathcal{M}^N \tag{12}$$

of local decisions. It is important to note that once the quantization probabilities (5) and (6) of the local sensor decisions $U_1, \ldots, U_N$ are determined, the optimal decision fusion rule with side information (9) is also determined.

3. PERFORMANCE EVALUATION

When using the decision fusion rule according to (9), the global probability of false alarm P_f is determined by the conditional probability

$$P_f = P(\delta_0(\boldsymbol{U}, X_0) = 1|H_0) \tag{13}$$

$$= P(L_0 > \tau(\boldsymbol{U})|H_0), \tag{14}$$

where $\boldsymbol{U} = (U_1, \ldots, U_N)'$ is the discrete random vector of local decisions. Applying the theorem of total probability and after some calculation, we obtain

$$P_f = \sum_{\boldsymbol{u} \in \mathcal{M}^N} \left[1 - F_{L_0}(\tau(\boldsymbol{u})|H_0)\right] \prod_{j=1}^{N} \alpha_j^{(u_j)}, \tag{15}$$

where $F_{L_0}(\cdot|H_k)$ is the conditional cumulative distribution function of the log-likelihood ratio L_0 of the side information X_0 under hypothesis H_k. Analogously, we obtain for the global probability of miss

$$P_m = P(\delta_0(\boldsymbol{U}, X_0) = 0|H_1) \tag{16}$$

$$= P(L_0 \leq \tau(\boldsymbol{U})|H_1) \tag{17}$$

$$= \sum_{\boldsymbol{u} \in \mathcal{M}^N} F_{L_0}(\tau(\boldsymbol{u})|H_1) \prod_{j=1}^{N} \beta_j^{(u_j)}. \tag{18}$$

In order to exactly evaluate the sensor fusion performance in terms of the global probability of error P_e, one has to sum up over all $|\mathcal{M}^N| = M^N$ possible realizations $\boldsymbol{u} = (u_1, \ldots, u_N)' \in \mathcal{M}^N$ of local decisions to obtain the global probability of false alarm (15) and the global probability of miss (18). However, in the special case of identical sensors, the computational complexity can be reduced significantly by using the multinomial distribution.

4. IDENTICAL SENSORS AND MULTINOMIAL DISTRIBUTION

In the special case of identical sensors, i.e., identical stochastic vectors $\boldsymbol{\alpha}_1 = \ldots = \boldsymbol{\alpha}_N$ and $\boldsymbol{\beta}_1 = \ldots = \boldsymbol{\beta}_N$, the conditional distributions of local sensor decisions can be interpreted in terms of the multinomial distribution. The multinomial distribution allows for computationally feasible expressions for the global error probabilities.

4.1. Multinomial distribution

The discrete random vector $\boldsymbol{V} = (V_1, \ldots, V_M)' \in \mathbb{N}_0^M$ is multinomially distributed with parameters $N \in \mathbb{N}$ and $p_1, \ldots, p_M \geq 0$, $\sum_{k=1}^M p_k = 1$, if it has the probability mass function (see, e.g., [4])

$$p_{\boldsymbol{V}}(\boldsymbol{v}) = P(V_1 = v_1, \ldots, V_M = v_M) \tag{19}$$

$$= \begin{cases} \dfrac{N!}{\prod_{k=1}^M v_k!} \prod_{k=1}^M p_k^{v_k}, & \text{if } \sum_{k=1}^M v_k = N \\ 0 & \text{, otherwise} \end{cases}. \tag{20}$$

The notation is $\boldsymbol{V} \sim \mathcal{M}(N; p_1, \ldots, p_M)$. The support $\mathcal{T}$ of the multinomial distribution is given by

$$\mathcal{T} = \{(v_1, \ldots, v_M)' \in \mathbb{N}_0^M | \sum_{k=1}^M v_k = N\} \tag{21}$$

and has the cardinality

$$|\mathcal{T}| = \binom{N+M-1}{M-1}. \tag{22}$$

This is usually much smaller than M^N, the cardinality of the support of the discrete random vector of local decisions $\boldsymbol{U} = (U_1, \ldots, U_N)'$. The asymptotic behavior of (22) can be approximated by using Stirling's formula according to

$$|\mathcal{T}| \simeq \frac{(N+M-1)^{M-1}}{(M-1)!}. \tag{23}$$

Obviously, this value is much smaller than M^N. E.g., for $N = 100$ and $M = 4$, one has $|\mathcal{T}| = 176851 \ll 4^{100}$.

4.2. Global error probabilities for identical sensors

For sensors $S_1, \ldots, S_N$ with identical stochastic vectors of quantization probabilities $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, optimal decision fusion with side information can be performed by evaluating the log-likelihood ratio test

$$\ell_0 \underset{u_0=0}{\overset{u_0=1}{\gtrless}} \theta(\boldsymbol{v}), \tag{24}$$

where

$$\theta(\boldsymbol{v}) = \log\left(\frac{\pi_0}{\pi_1}\right) + \sum_{k=1}^M v_k \cdot \log\left(\frac{\alpha^{(k)}}{\beta^{(k)}}\right) \tag{25}$$

is the decision threshold and

$$v_k \in \{0, \ldots, N\} \tag{26}$$

denotes the number of sensors deciding for local decision $k \in \{1, \ldots, M\}$. Per definition, the values $v_1, \ldots, v_M$ are realizations of random variables $V_1, \ldots, V_M$ where the vector $\boldsymbol{V} = (V_1, \ldots, V_M)'$ follows a multinomial distribution with parameters N and $\boldsymbol{\alpha}$ or $\boldsymbol{\beta}$, depending on the underlying hypothesis, i.e.

$$H_0 \colon \boldsymbol{V} \sim \mathcal{M}(N; \alpha^{(1)}, \ldots, \alpha^{(M)}), \tag{27}$$

$$H_1 \colon \boldsymbol{V} \sim \mathcal{M}(N; \beta^{(1)}, \ldots, \beta^{(M)}). \tag{28}$$

For identical sensors, the global probability of false alarm P_f of the optimal fusion rule (24) is given by

$$P_f = P(L_0 > \theta(\boldsymbol{V}) | H_0) \tag{29}$$

$$= \sum_{\boldsymbol{v} \in \mathcal{T}} \left[1 - F_{L_0}(\theta(\boldsymbol{v}) | H_0)\right] p_{\boldsymbol{V}}(\boldsymbol{v} | H_0). \tag{30}$$

The global probability of miss P_m is given by

$$P_m = P(L_0 \leq \theta(\boldsymbol{V}) | H_1) \tag{31}$$

$$= \sum_{\boldsymbol{v} \in \mathcal{T}} F_{L_0}(\theta(\boldsymbol{v}) | H_1) p_{\boldsymbol{V}}(\boldsymbol{v} | H_1). \tag{32}$$

Accordingly, for computing the global error probabilities P_f and P_m of sensor fusion with side information and identical M-ary sensors, one only needs to consider the $|\mathcal{T}| \simeq (N+M-1)^{M-1}/(M-1)!$ possible outcomes of the multinomial distribution.

5. NUMERICAL RESULTS

We provide numerical results for the probability of error of sensor fusion with side information obtained by exact calculation. Sensor networks consisting of $N = 2, \ldots, 50$ identical binary and quaternary sensors are assumed, i.e., $M = 2$ and $M = 4$. The hypotheses H_0 and H_1 are equally likely to occur, i.e., $\pi_0 = \pi_1 = 0.5$. As an illustrative example, we assume that the sensor observations X_j and the side information X_0 follow a normal distribution, i.e., we assume that the random variables X_0 and X_j are conditionally distributed according to

$$H_0 \colon X_0 \sim \mathcal{N}(0, \sigma_0^2),\ X_j \sim \mathcal{N}(0, \sigma^2), \tag{33}$$

$$H_1 \colon X_0 \sim \mathcal{N}(\mu_0, \sigma_0^2),\ X_j \sim \mathcal{N}(\mu, \sigma^2). \tag{34}$$

We evaluate the sensor fusion performance for different combinations of values for the sensor observation and the side information signal-to-noise ratio (SNR). The determination of the local sensor decision rules is done by maximizing the Chernoff information between $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, an approach presented in [5]. The results for binary sensors are depicted in Fig. 2 and Fig. 3. For a sensor observation SNR of -5 dB, a significant reduction of the probability of error P_e can be obtained by using high quality side information. If the sensor observation SNR is 0 dB, this effect only occurs when the number of sensors is low. The results for quaternary sensors are shown in Fig. 4 and Fig. 5 and resemble very much the binary case.

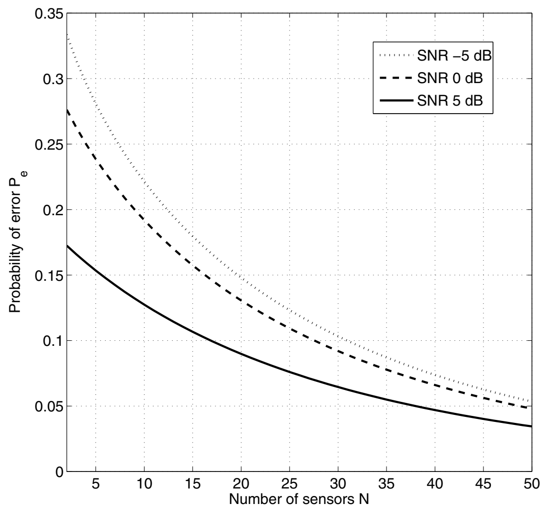

Fig. 2. Binary sensor fusion performance at a sensor observation SNR of -5 dB for different values of the side information SNR.

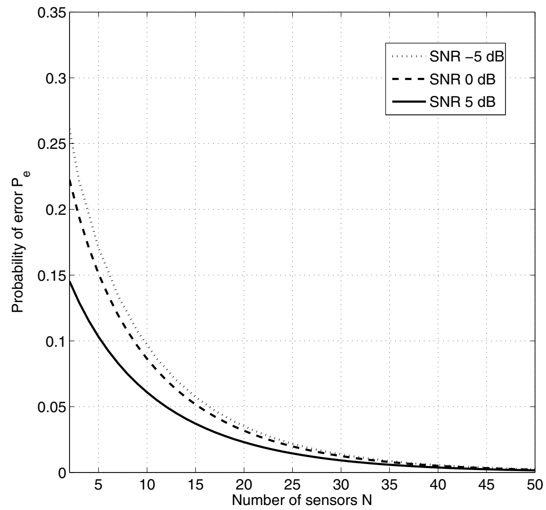

Fig. 3. Binary sensor fusion performance at a sensor observation SNR of 0 dB for different values of the side information SNR.

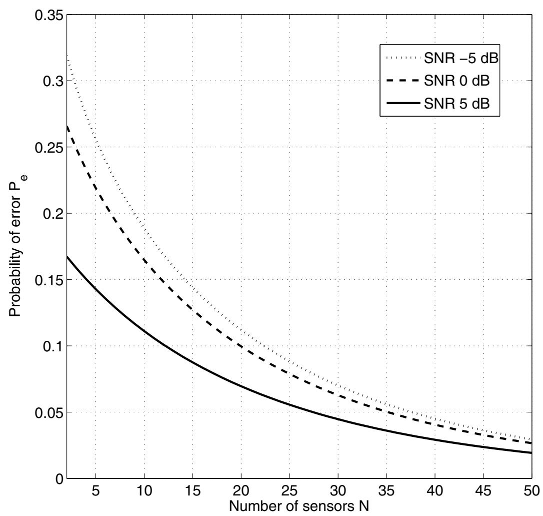

Fig. 4. Quaternary sensor fusion performance at a sensor observation SNR of -5 dB for different values of the side information SNR.

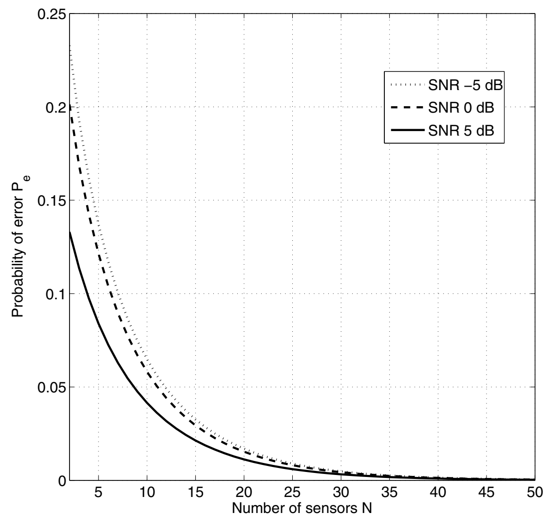

Fig. 5. Quaternary sensor fusion performance at a sensor observation SNR of 0 dB for different values of the side information SNR.

6. CONCLUSIONS

In this paper, we have presented explicit expressions for sensor fusion performance with side information where the number of quantization levels at the sensors is arbitrary. For the case of identical sensors, computationally efficient expressions have been obtained by using the multinomial distribution. Numerical results illustrate the influence of side information on the fusion performance.

7. REFERENCES

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