

# Simple decentralised market-oriented management of OFDMA Femto-cells

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## ABSTRACT

A femto access point (FAP) is a low-cost, low-power device intended to be installed by a home owner, to turn the home into a tiny wireless communication cell, that operates underneath a standard cell, in licensed spectrum bands. Femto-cells can improve user experience in indoor locations, and increase overall system capacity with modest monetary investment. But the unplanned dynamic nature of these cells, significantly complicate resource allocation and interference control. We propose a relatively simple, decentralised market-oriented scheme for sub-channel and power allocation when femto-cell users co-exist with those of an OFDMA macro-cell. Our scheme utilises the Dutch auction (price progressively falls until a participant buys the object). Special “confirmation messages” are used to control interference, achieve channel reuse across Femto-cells, and mitigate the “hidden terminal” problem. Secure software inside each terminal may record transactions for eventual payment collection, or the auction can be interpreted as a prioritised decentralised allocation algorithm, without real money exchange.

## Categories and Subject Descriptors

C.2.3 [Computer Systems Organization]: Computer-Communication Networks—*Network Operations, Network management*

## General Terms

Algorithms, Economics, Management

## Keywords

OFDMA, femto-cells, resource allocation, power, interference, sub-channel, scheduling, auction, game theory, microeconomics, pricing, LTE, WiMAX

## 1. INTRODUCTION

Decreasing the separation between transmitters and receivers can increase overall system capacity by increasing link quality, and resource reuse. This can be accomplished in a cellular network

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through the deployment of additional infra-structure, such as distributed antenna arrays, relay nodes, or additional base stations (to reduce cell size). However, infra-structure is expensive, and may face many practical and regulatory hurdles, in particular, in connection to antenna placement.

As an alternative, a femto access point (FAP) is an inexpensive, low-power device suitable for “do-it-yourself” installation. A FAP is intended to service a very small area covering, for example, a home. The area serviced by a FAP is called a “femto-cell” (FC). Unlike Wi-Fi wireless routers, a FAP operates in licensed spectrum and is directly connected to a standard BS through a dedicated link, such as DSL, fibre or a separate radio channel [2].

Femto-cells can provide many of the benefits of traditional infra-structure, at a fraction of the cost; and this cost is at least partially absorbed by the end-user. However, the unplanned and dynamic nature of Femto-cells (they can be installed anywhere, and switched on/off at any time) creates some important challenges. These include: (i) interference mitigation and management, including macro to femto, femto to femto, and femto/handset interference; (ii) handouts (femto to macro), hand-ins (macro to femto), and handovers (femto to femto), as well as (iii) network architecture, scalability, and planning and (iv) Integration with the core network[3]. Because of the above, resource management scheme for Femto-cells must be relatively simple, and suitable for at least partially decentralised implementation.

Femto-cells operating under orthogonal frequency-division multiple access (OFDMA) are of special interest, because OFDMA is the technology of fourth-generation (4G) cellular networks. OFDMA resource management is notoriously difficult, even for a single cell without Femto-cells. It involves allocating sub-channels to users on the basis of channel-state information (CSI). CSI varies per user, per sub-carrier, and per time interval. Furthermore, a total amount of down-link power must be allocated among the various sub-carriers. Finally, adaptive modulation and coding may be needed to increase the system efficiency. The presence of Femto-cells increases the difficulty of the problem, because, among other reasons, it introduces sub-channel sharing among some Femto-cells, but only if they do not interfere one another[6].

In [10] we propose a relatively simple, decentralised market-oriented scheme for sub-channel and power allocation for the down-link of an OFDMA macro-cell. Our scheme utilises auctions to allocate sub-carriers, and pricing to allocate power. Secure software inside each terminal may record transactions for eventual payment collection as “service fees”, or the scheme can be interpreted as a prioritised decentralised low-complexity algorithm, without real money exchange. Below we extend [10] to cover Femto-cells. For interference control, and channel reuse across Femto-cells, we utilise special “confirmation messages”.

Below, we first introduce the OFDMA down-link system model, and the main resource allocation issues. Then, we make a detour to briefly discuss auctions in general (not specific to our proposal), with special focus on the Dutch format. Subsequently, we summarise the market-oriented scheme proposed in [10] for the basic OFDMA forward link. Then we introduce the modifications necessary to accommodate Femto-cells. We then end with some closing comments.

## 2. OFDMA SYSTEM, AND RESOURCE ALLOCATION

### 2.1 Basic OFDMA forward link

We assume a common physical model from the literature, utilised, for instance, in [11]. The total bandwidth  $B$  is divided into  $N$  sub-carriers. The signal is time slotted, and the resource manager allocates all resourced once at every slot among  $K$  data-downloading terminals. To reduce overhead, several sub-carriers may be grouped as a sub-channel that is allocated in whole.

The base station (BS) transmitter must obey a total transmit power constraint. Each transceiver has a single antenna. The frequency response is flat within a given sub-carrier. Let  $h_{k,n}$  denote the *quotient* resulting from dividing the channel gain corresponding to terminal  $k$  and sub-carrier  $n$ , by the pertinent receiver noise value.

### 2.2 OFDMA resource allocation: basic case

The critical OFDMA resource management issue is how to exploit CSI and traffic characteristics in order to efficiently allocate sub-carriers and power to improve the network's performance as much as practically possible, while considering appropriate constraints. Several adaptive resource allocation strategies are possible[12]:

- (i) Dynamic sub-carrier assignment according to channel-state information (CSI) — a sub-carrier experiencing deep fading for one terminal may be the best for another — and quality-of-service (QoS) considerations (but finding the optimal sub-channel allocation is NP-hard);
- (ii) adaptive power allocation (usually involving some form of multi-user “water filling”); and
- (iii) adaptive modulation and coding, so that higher transmission rates are sent over the “better” sub-channels.

### 2.3 OFDMA Femto-cells issues

In the down-link of a given macro-cell, a sub-channel can be concurrently used by several Femto-cells (FC) if they are sufficiently far from each other. In principle, a macro-cell terminal could also concurrently utilise that same sub-channel, if (a) this terminal is far from the concerned FC, and (b) if the BS does not cause too much interference to the terminals served by those FC. However, below we assume that a given sub-channel is never simultaneously utilised by the macro-cell and a femto-cell.

Figure 1 illustrates some of the challenges associated with OFDMA resource allocation in the presence of FC. Terminals M1 to M5 are attached to the macro-cell, whereas F1 to F3 are served by a respective FC. M1 can be interfered with by both femto-access points (FAP) 1 and 2, but not by FAP 3. M2 can be interfered only by FAP 2, while M3 can be interfered only by FAP 3, respectively. No one can interfere either M4 or M5. FAP 1 can interfere with F2, but FAP 2 cannot interfere with F1. A SC utilised by FC1 or FC2 could also be employed at FC3.

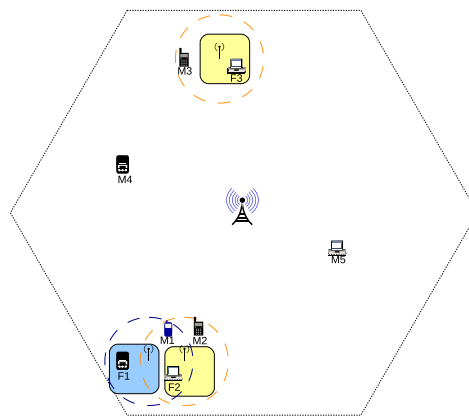


Figure 1: Femto-cell terminals F1, F2 and F3 coexist with several macro-cell terminals (M1 to M5).

## 3. AUCTIONS FOR TELECOMMUNICATION RESOURCE ALLOCATION

### 3.1 Motivation

Since time immemorial, auctions have been employed as a practical mechanism for the transfer of ownership of articles of value, for such reasons as: (i) speed of allocation, (ii) discovery of the true “value” of the offered object, and (iii) transaction “transparency” (fraud prevention)[14]. For telecommunication resource allocation, auctions provide a form of “prioritised allocation” in that the resource is allocated to the terminal that most values it. A terminal’s valuation of the resource could either (a) represent the “true” monetary “willingness to pay” of a (selfish) human user, or (b) be a network-specified quantity (“priority index”) computed/adjusted by software inside the terminal using local information. A terminal’s priority may be “adaptive”, depending on such factors as its “importance”, packet type, location, channel state, distance travelled, battery status, etc.

Furthermore, real auctions need not rely on “altruistic” or “courteous” behaviour by “selfish” users. In this scenario, secure software inside each terminal may record transactions for eventual payment collection and system parameter tuning.

### 3.2 The Dutch auction format

The Dutch auction utilises a “price clock” which displays a progressively falling price. Each participant watches the clock while waiting for the price to reach a desired level. At some point, a participant indicates its willingness to pay the current price (the first participant to do so is the one that most values the object) [14]. Figure 2 shows a real-life Dutch flower auction in progress, in Aalsmeer, the Netherlands [1].

For telecommunication purposes, the Dutch auction retains the relative simplicity and allocation speed of other simple auction formats, and add several fundamental advantages: (i) A built-in bid-processing protocol that automatically and simply prioritise the highest bid(s); (ii) the possibility of a distributive (auctioneer-free) implementation (start times, initial price, and rate of decrease can all be pre-specified, so that a terminal can determine from its own clock the current status of the auction); (iii) Possible confirmation of transmitter-receiver pairs at auction time, with smooth continuation if the pair is infeasible; (iv) exceptional signalling economy (only one bid signal, the winner’s, is strictly necessary in a sin-



Figure 2: A real-life Dutch flower auction

gle resource scenario). The Dutch auction is discussed further in [7], where it is proposed for medium-access allocation in an infrastructureless (“ad hoc”) synchronised wireless network. Reference [5] extends [7] to consider network-layer issues, while [8] particularises [7] to a location/tracking application. A different clock-based (“ascending price” and combinatorial) auction is discussed by [4] in a dynamic-spectrum access scenario.

### 3.3 The optimal Dutch bid

A bidder’s valuation is how much the the auctioned object is “worth” to the bidder; that is, the largest monetary amount that the bidder would pay for the object, in a direct purchase. If each bidders knows that there are  $K$  bidders, and that all valuations are (statistically) uniformly distributed over a common interval, then the optimal Dutch bid for a bidder whose valuation is  $V_i$  is given by [13]:

$$\left(1 - \frac{1}{K}\right)V_i \quad (1)$$

## 4. THE DUTCH AUCTION FOR THE TYPICAL OFDMA FORWARD-LINK

In this section we summarise how our proposal works without femto-cells. The next section addresses FC.

### 4.1 Basic idea

The OFDMA resource manager makes a resource assignment once at every slot. Just before assignment, the manager sets up simultaneous Dutch auctions, one per sub-channel (as in figure 2, where several auctions are held in parallel).

The manager broadcast to the terminals the necessary information, so that each terminal can, from its own internal clock, determine the current price for each auction, and its bid. This information includes the duration of a “tick”, the starting price, and the price “step” per tick for each “price clock”. These parameters may, in principle, vary per clock and from a time slot to another.

Just before start of the auction, each terminal has an estimate of the anticipated channel state over each sub-carrier during the upcoming time slot. The terminal can, thus, compute for each sub-channel the “value” of using it during the upcoming time slot. Each terminal then waits for the price for its “best” sub-channel to be low enough. When this happens for the first time, the concerned terminal sends an appropriate signal to indicate its willingness to buy the given sub-channel at the current price. If there is no reason to decline this request, the BS broadcast an allocation confirma-

tion, along with any relevant information. After this confirmation, each terminal may re-calculate its valuations of the remaining sub-channels. The parallel auctions continue, with each “clock” decreasing its price at each tick, until another terminal determines that the price of one of the sub-channels is “low enough”. The process continues in the obvious way.

Table 1 displays a simple example in which there are 2 terminals, T1 and T2, and 5 sub-channels (SC). The column corresponding to a terminal has its initial bids for each SC. For simplicity, we will assume that after a sub-channel has been won, *no* terminal changes its valuation of the remaining SC’s.

Suppose that both price clocks start at 10,0, and the price drops 0,1 each “tick” of the clock. The clocks start moving to lower prices each tick, as 10,0 , 9,9 , 9,8 , 9,7 , etc. After 4 ticks, the price of SC2 has reached 9,6 which is attractive enough for T2. Thus, T2 sends the buying signal. The BS broadcast the confirmation of the allocation (in principle, each terminal could at this point re-calculate its bid for the remaining sub-channels, but we have assumed none does so). After 6 additional ticks, the price of SC 2 has reached 9,1, and accordingly T1 sends the buying signal for SC1. After 20 total ticks the price of SC4 has reached 8,0 and T1 buys it. The process continues similarly, with T2 getting SC5 (after 25 total ticks), and eventually also SC3.

Table 1: Bids by terminals 1 and 2, for the various sub-channels.

SC	T1	T2	W
1	9,1	6,5	1
2	5,1	9,6	2
3	3,5	4,0	2
4	8,0	2,0	1
5	3,5	7,5	2

## 4.2 Bid calculations and updates

### 4.2.1 Initial sub-carrier valuation for data terminals

For a terminal that has a “long” queue of delay-tolerant (data) information to transfer and that has not yet won any SC, it is reasonable to assume a valuation of the form  $\beta_i R_{i,n}(p) - c_i p$  where (i)  $\beta_i$  is the monetary value of one information bit successfully transferred multiplied by the SC bandwidth, (ii)  $R_{i,n}(p) = \log_2(1 + ph_{i,n})$  represents the number of information bits/Hertz the terminal can successfully transfer during the upcoming time slot over SC  $n$  if the amount of power  $p$  is used (with ideal link layer configuration), and  $c_i(p)$  is the associated cost. For simplicity, we assume that the power cost is linear:  $c_i(p) = c_i p$ .

The terminal can find the optimal amount of power for SC  $n$  as the solution  $x^*$  to the single-variable equation  $R'_{i,n}(x) = c_i/\beta_i$ . Of course, the amount ordered by the terminal cannot exceed the total amount of power that remains at the transmitter (considering any amount allocated to previous winners), say  $P$ . Thus, with  $p_{i,n}^* := \max(P, x^*)$ , the terminal valuation of this SC equals  $\beta_i R_{i,n}(p_{i,n}^*) - c_i p_{i,n}^*$ , and from this, under the assumptions of sub-section 3.3, the bid follows directly. Since the price clock will never reach negative numbers, a negative bid simply means that the terminal has no interest in this SC.

If the starting price, “tick” duration, and price “step” per tick are common to all price clocks, then all indicate the same price at given moment. Thus, although all SC are simultaneously auctioned, terminal  $i$  can focus on its best SC, until it is won by someone. At this

point, it can calculate its bid for its next best of the remaining SC, and focus on that until someone gets it, and so on.

### 4.2.2 Sub-carrier valuation updates

The terminal can update its valuation of the remaining carriers, after anyone has won. If someone else wins a SC, certain amount of power is allocated to that SC. Thus, a terminal may have to recalculate its valuation of SC  $n$ , if  $p_{i,n}^*$  exceeds the amount of power now available (see previous sub-section).

If terminal  $i$  has won SC  $m$ , then to determine its valuation for SC  $n$  it should find the power levels  $x, y$  that maximise  $\beta_i(R_{i,m}(x) + R_{i,n}(y)) - c_i(x+y)$ . Of course,  $x+y$  must be less than the available (not already allocated) power.

Notice that the terminal must solve this bi-variate problem, only after it has won a SC, and — when clocks parameters are common as discussed in section 4.2.1 — only for its best of the remaining SC. Once this SC is won, then the terminal should repeated this calculation for the best of those still remaining one, and so on.

More formally, if the terminal has won  $\mu - 1$  sub-carriers, it should solve the optimal power-allocation problem for  $\mu$  sub-carriers with a per-Watt price, which leads to “water-filling” with costly power. This problem is sufficiently interesting to merit an independent discussion[9].

## 5. CONSIDERATION OF FEMTO-CELLS

In principle, the scheme discussed in section 4 continues to apply. Femto-cells are intended to serve at most a few *simultaneous* terminals [2]. For simplicity, we assume below that each FC has at most one active terminal. The extension to consider several active terminals per FC is relatively simple.

### 5.1 Basic idea

As before, the manager sets up simultaneous Dutch (descending price) auctions, one per sub-channel, and broadcasts the necessary information (e.g., starting price of each “clock”, the “tick” duration, the price “step”, etc.) to enable each terminal — including those in FC — to determine its bid, and from its own internal clock — infer the current price for each sub-channel. An FAP also receives this information through its dedicated BS-FAP channel.

The per-Watt price only applies to macro-cell terminals, since a FC terminal does not use BS power. Since there is at most one active terminal per FC, the FAP can allocate all its down-link power to the SC won by its single terminal, applying standard procedures (e.g., single-user “water-filling”).

When a macro-cell sends a buying message (including desired power level), the BS directly receives it, and if appropriate immediately broadcast a confirmation that includes the amount of remaining power, as in section 4.

When a FC terminal sends a buy signal, its FAP sends an approval message if it knows the SC to be available, both over the air (for interference control, as explained below), and through the dedicated link to the BS. Immediately after, the buying FC terminal sends a short confirmation message, also for interference control.

Optionally, the BS could then broadcast a notification letting the macro-terminals know that that SC is no longer available. This is not strictly necessary because if a macro-terminal later tries to purchase this SC, the BS would not send the necessary approval.

### 5.2 Interference control through confirmation messages

We consider the forward link (down-link) of a single-cell system, and assume that a given sub-channel cannot be simultaneously used by a macro-cell and a FC terminal, because the BS would create too

much interference at the FC. Because BS is always made aware of all SC purchases, it will never interfere with any terminal in its cell. Likewise, no FAP will interfere with a macro-cell terminal, because when a macro-cell terminal wins a SC, all terminals and FAP’s are notified by the BS.

However, several FC can use the same SC, if they are sufficiently separated. The confirmation messages sent by the FC terminals and FAP’s are the key to interference prevention between FC (and the avoidance of the so-called “hidden terminal problem”).

Under the common assumption of symmetric channels, if a FC terminal can “hear” a FAP, this FAP can also hear this terminal. Thus, this FAP would hear the confirmation message sent by any “foreign” terminal, and correctly infer that it must NOT assign this SC to its own terminal. Likewise, if a FC terminal hears an approval message sent by a “foreign” FAP, this terminal will correctly infer that that channel is no longer available, because the “foreign” FAP that will use it is sufficiently close to interfere with this terminal.

For instance, suppose that in fig. 1, F2 has sent a buy signal, FAP-2 has approved, and F2 has sent the short confirmation message. Although F1 cannot hear F2, FAP-1 does hear F2’s confirmation. Thus, if later F1 attempts to buy this SC, FAP-1 will not approve this purchase, and F1 will never send a confirmation.

The BS learns about F2 intentions through the confirmation message sent by FAP-2 through the dedicated BS-FAP link. As discussed above, the BS could send an over the air notification to let all macro-terminals know that this SC is no longer available to them (leaving total BS power unchanged; after a macro-cell terminal wins a SC the available down-link power is reduced). This message is not strictly necessary because the BS could simply refuse any future attempt by a macro-cell terminal to purchase this SC. In any event, M1 and M2 could hear FAP-2’s approval message, and immediately infer that the concerned SC is no longer available (FAP-2 would interfere with either of them).

However, any FAP that did not hear F2 confirmation message, and any FC terminal that did not hear FAP-2 confirmation will continue to treat the SC as available (even if it hears the BS notification). For example F3 cannot hear the approval from FAP-2, nor can FAP-3 hear F2’s confirmation message. Thus, the concerned SC would remain available at FC3. This would not affect M3, because the BS would not use any SC won by any FC terminal.

### 5.3 SC valuation by data terminal in FC

For a terminal that has a “long” queue of delay-tolerant (data) information to transfer that has not yet won any SC, it is reasonable to assume a valuation of the form  $\beta_i R_{i,n}(p)$  where (i)  $\beta_i$  is the monetary value of one information bit successfully transferred multiplied by the SC bandwidth, (ii)  $R_{i,n}(p) = \log_2(1 + ph_{i,n})$  represents the number of information bits/Hertz the terminal can successfully transfer during the upcoming time slot over SC  $n$  if the amount of power  $p$  is used (with ideal link layer configuration). Since we have assumed only one active terminal per FC,  $p$  is set to  $P_F$ , the maximal power available to the FAP (which is “low” compared to the BS power).

As before, the terminal’s highest bid will correspond to its “best” of available SC. Thus, the terminal can focus on its best SC, until it is won by someone, at which point it can calculate its bid for its next best of the available SC, and focus on that one, and so on.

If this terminal has already won sub-channels, it proceeds as discussed in sec. 4.2.2.

## 6. SOME IMPLEMENTATION ISSUES

Evidently, the auction requires tight synchronisation among all transceivers, including the FAP, i.e., a “common clock”. This is no

problem in our case, because 4G networks are time-slotted, which requires tight synchronisation.

The parameters of the auction (initial price, the clock “tick”, and price “step” per tick) should be chosen judiciously. The statistics of the terminals’ “valuations” are among the factors to be considered in choosing the system parameters. The clock tick must be sufficiently long to allow the exchange of the necessary short messages after a purchase.

A FAP communicates with the BS over dedicated link, and this link must be sufficiently fast to permit the operations that have been discussed. Presently, typical home connection Internet speeds are of the order of several megabits per second (Mbps) in many parts of the world, and much higher than that (approaching 100 Mbps) in certain regions, such as Japan and Korea. They likely will continue to improve. Thus, the BS-FAP link may not be a problem.

Furthermore, since the valuations (and corresponding bids) are channel dependent, they can be idealised as continuous random variables. Hence the probability that 2 terminals have the exact same bid can be neglected.

## 7. DISCUSSION

We have presented a low-complexity decentralised sub-channel and power allocation scheme for the forward-link of an orthogonal frequency-division multiple-access (OFDMA) wireless access network in the presence of Femto-cells. The unplanned and dynamic nature of Femto-cells (they can be installed anywhere, and switched on/off at any time) necessitates a decentralised resource management scheme, as proposed herein. The scheme is based on a simple descending-price auction run in parallel for each sub-channel by the access point for each time slot, and can be taken literally as involving real money to be paid by users as service fees, or may also be interpreted as a (semi)-decentralised, low-complexity algorithm in which the only significant computation is performed by each terminal while calculating its bid in a “channel by channel” basis. Further discussion and detail about our scheme (*without* femto-cells) can be found in [10].

We have emphasised qualitative/conceptual aspects, and provided simple numerical and diagrammatic examples. We realise that it would be very useful and interesting to implement our proposal within a 4G simulation environment.

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