Joint BS Selection and Subcarrier Assignment for Multicell Heterogeneous OFDM Unicasting

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Abstract—In this work resource allocation is studied for orthogonal frequency division multiplexing (OFDM) unicasting by multiple base stations (BSs). In the considered scenario, each user is restricted to being covered by one BS, i.e., BS selection, while each subcarrier is assigned to at most one user, i.e., subcarrier assignment. Thus, interference does not exist among these BSs. We aim at maximizing the weighted sum of data rates subject to limited transmission power at each BS, while the minimum rates required by users are satisfied. BS selection and subcarrier assignment are considered jointly. A heuristic method is proposed. It has linear complexity in the number of users, subcarriers and BSs. Simulations demonstrate that the performance loss of the proposed method is relatively small.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) [1] allows for allocating different rates and powers to subcarriers according to time-varying channel conditions so that system performance is enhanced. Resource allocation problems are generally classified into two groups. One aims at minimizing transmission power subject to fixed required rates. Heuristic and dual optimal solutions were suggested in [2] and [3], respectively. In the other group the sum rate is maximized while transmission power is limited [4]. However, future networks is expected to provide a mixture of delay-tolerant and delay-sensitive services, so called heterogeneous unicasting. The associated resource allocation must be heterogeneous. Obviously, the local optima for those two groups of resource allocation are not the global optimum for the heterogeneous one. Dual optimum resource allocation was provided for heterogeneous unicasting by a single base station (BS) in [5].

The number of BSs in a coverage is determined by characteristics of BSs, density of users, topology and others. It may occur that a large number of BSs is required within a small coverage, e.g., the center of metropolitan areas. In other words these BSs are near to each other. In this paper we group these BSs into one cluster. Resource allocation is considered for heterogeneous unicasting by BSs within one cluster at the control center, depicted in Figure 1. To avoid strong interference within one cluster, each subcarrier is assigned to at most one user. One user can be covered by only one BS at any specific time, i.e., BS selection, see long term evolution (LTE) [6], where resource allocation method is not yet specified. We aim at maximizing the weighted sum rate for non-real time transmission subject to power limits of BSs and the constrains on the minimum required rates. Obviously, the considered resource allocation problem is a combinatorial optimization problem and has two dimensions, i.e., BS selection and subcarrier assignment. A heuristic method is proposed to jointly perform BS selection and subcarrier assignment.

The remainder of this paper is organized as follows. Section II formulates the resource allocation problem for heterogeneous unicasting by multiple BSs. Section III gives efficient approaches for updating the objective when the subcarrier assignment changes. With these approaches, a heuristic method is proposed, while BS selection and subcarrier assignment are jointly performed. Its performance is shown by simulations in Section IV. Finally, the content is concluded.

II. PROBLEM FORMULATION

Consider OFDM unicasting by \( C \) BSs to \( K + Q \) users over \( N \) subcarriers. Transmission data is available at all BSs via fibre connection. BSs are synchronized, see [7]. This implies that signals received from different BSs are also synchronized. The delay of these received signals is smaller than the guard interval, since the BSs in one cluster are near to each other in the considered scenario, see LTE [6]. The rates required by the
first $K$ users are lower bounded, e.g., the lowest acceptable rate for keeping online movies fluent. The latter $Q$ users demands fixed data rates, e.g., phone calls. Note that rates may be variant for some real time services. It depends on the protocols of upper layers. To distinguish these two kinds of users, we call the first $K$ users rate-adaptive (RA) users and the latter $Q$ users margin-adaptive (MA) users. Perfect channel knowledge is assumed at the control center.

Our objective is to maximize the weighted sum rate for the $K$ RA users, $k = 1, \ldots, K$. As fairness control, their data rates are weighted by the positive vector $(w_1, \ldots, w_K)^T$. The data rate for each user $k = 1, \ldots, K + Q$ is lower bounded by $R_k$. This optimization problem is stated as

$$\begin{align*}
\text{maximize} & \quad \sum_{k=1}^{K} \sum_{c=1}^{C} \sum_{n \in S_{k,c}} r_{k,n,c} \\
\text{subject to} & \quad r_{k,n,c} = \log(1 + p_{k,n,c}g_{k,n,c}), \quad \forall k, n, c \\
& \quad \sum_{c=1}^{C} \sum_{n \in S_{k,c}} r_{k,n,c} \geq R_k, \quad k = 1, \ldots, K + Q \\
& \quad \sum_{k=1}^{K+Q} \sum_{n \in S_{k,c}} p_{k,n,c} \leq P_c, \quad c = 1, \ldots, C \\
& \quad S_{k,c} \cap S_{v,l} = \emptyset, \quad \forall (k,c) \neq (l,v) \\
& \quad \sum_{n \in S_{k,c}} r_{k,n,v} = 0, \quad \forall k, \forall c \neq v
\end{align*}$$

where $p_{k,n,c}$ and $r_{k,n,c}$ are the non-negative power and rate allocated to subcarrier $c$ for user $k$ by BS $c$, respectively. They are related by the first equality constraint, where $g_{k,n,c}$ denotes the channel gain-to-noise ratio (CNR). Note that interference from BSs outside the concerned cluster is treated as noise. The subcarriers assigned to BS $c$ for user $k$ are contained in a set $S_{k,c}$ with the cardinality $s_{k,c}$. The transmission power of each BS $c$ is limited to $P_c$. One subcarrier is assigned to only one pair of BS and user, explained by the second last constraint. One user is covered by only one BS, expressed by the last constraint. In other words, only one of $S_{k,1}, \ldots, S_{k,C}$ is not empty for user $k$. Sets $K$ and $Q$ contain RA and MA users, respectively. Users covered by BS $c$ are in a set $U_c$. The equality $U_c \cap U_v = \emptyset$ always holds for any $c \neq v$.

III. JOINT USER AND SUBCARRIER ASSIGNMENT

The above combinatorial problem has two dimensions, see Figure 2. On one hand, users must be covered by different BSs, i.e., BS selection. On the other hand, different subcarriers are separately assigned to BSs, i.e., subcarrier assignment. Obviously, these two dimensions affect each other. To achieve a good balance between BS selection and subcarrier assignment, they must be jointly performed.

In general, heuristic methods, e.g., [8], [9], are devised for resource allocation in the following way. Changing the subcarrier assignment and evaluating the induced variation of the objective are iteratively executed till some stopping criterion is met. During this iteration, water-filling is used to update the objective, while the resulting complexity is high.

In the following we present an efficient approach of updating the objective after changing the subcarrier assignment.

A. Genetic Water-Filling (GWF)

Changing the subcarrier assignment can be divided into excluding and including a subcarrier, see dashed lines in Figure 2. Given $S_{k,c} \neq \emptyset$ for MA user $k \in Q \cap U_c$ covered by BS $c$, the single-user MA problem is extracted from (1) as

$$\begin{align*}
\text{minimize} & \quad \sum_{n \in S_{k,c}} p_{k,n,c} \\
\text{subject to} & \quad \sum_{n \in S_{k,c}} r_{k,n,c} = R_k.
\end{align*}$$

The equality is met due to the complementary slackness condition [10]. If all subcarriers in $S_{k,c}$ are employed, the optimum transmission power for user $k$ at BS $c$ is

$$P_{k,c} = \sum_{n \in S_{k,c}} p_{k,n,c} = s_{k,c} \mu_k - \sum_{n \in S_{k,c}} \frac{1}{g_{k,n,c}}$$

from [1], where the water level $\mu_k$ is obtained as

$$\mu_k = 2^{\frac{n_c}{g_{k,m,c}}} \left( 1 + \prod_{n \in S_{k,c}} \frac{1}{g_{k,n,c}} \right)^{-1}.$$

After excluding $m$ from $S_{k,c}$ or including $m$ in $S_{k,c}$, the new water levels and the resulting power variations are

$$\mu_k^{(ex)}(m) = \mu_k(\mu_k g_{k,m,c})^{1/g_{k,m,c}}$$

$$\Delta P_k^{(ex)}(m) = s_{k,c}(\mu_k(m) - \mu_k) - (\mu_k(m) - \frac{1}{g_{k,m,c}})$$

$$\mu_k^{(in)}(m) = \mu_k(\mu_k g_{k,m,c})^{1/g_{k,m,c}}$$

$$\Delta P_k^{(in)}(m) = s_{k,c}(\mu_k(m) - \mu_k) + (\mu_k^{(in)} - \frac{1}{g_{k,m,c}}),$$

respectively. In the equations above, (ex) indicates the operation of excluding a subcarrier, while (in) indicates the one of including a subcarrier. The power variations on the primal $s_{k,c}$ subcarriers are the same as $(\mu_k^{(ex)}(m) - \mu_k)$ or $(\mu_k^{(in)}(m) - \mu_k)$. The last terms in the power variations $(\mu_k^{(ex)}(m) - 1/g_{k,m,c})$ and $(\mu_k^{(in)} - 1/g_{k,m,c})$ are the power allocated to subcarrier $m$ with the new water level. Only one exponential operation is necessary for each case, while the others are simple, like additions, subtractions and multiplications.
The idea above is extended for RA users. The relaxed multiuser RA problem covered by BS $c$ is separated from (1) by omitting the constraints on the minimum required rates. For all $c = 1, \ldots, C$, it is expressed as

\[
\begin{align*}
\text{maximize} & \quad M_c = \sum_{k \in K \cap U_c} \sum_{n \in S_{k,c}} r_{k,n,c} \\
\text{subject to} & \quad \sum_{k \in K \cap U_c} \sum_{n \in S_{k,c}} p_{k,n,c} = P^{(RA)}_c
\end{align*}
\]

given $S_{k,c}$ and $U_c$, where $P^{(RA)}_c = P_c = \sum_{k \in Q \cap U_c} P_{k,c}$ is the transmission power for RA users at BS $c$. It can be solved as

\[
M_c = \sum_{k \in K \cap U_c} \sum_{n \in S_{k,c}} \log_2(v_c w_k g_{k,n,c})
\]

see [1], where $v_c w_k$ is the water level, determined by

\[
v_c = \frac{P^{(RA)}_c}{\sum_{k \in K \cap U_c} \sum_{n \in S_{k,c}} 1/g_{k,n,c}}.
\]

All BSs achieve the weighted sum rate $\sum_{c=1}^C M_c$. Due to the last constraint, $M_1, \ldots, M_C$ are separable given $U_c$ and $S_{k,c}$. After removing subcarrier $m$ from $S_{k,c} \neq \emptyset$ for RA users $k \in U_c \cap K$, $v_c$ becomes

\[
v^{(\text{ex})}_c(m, k) = \frac{v_c \sum_{l \in K \cap U_c} w_l s_{l,c} - 1/g_{k,m,c}}{\sum_{l \in K \cap U_c} w_l s_{l,c} - w_k}.
\]

The resulting decrement of the weighted sum rate is

\[
\Delta M^{(\text{ex})}_c(m, k) = \log_2 \left( \frac{v^{(\text{ex})}_c(m, k)}{v_c} \right) \sum_{l \in K \cap U_c} w_l s_{l,c} - w_k \log_2(v^{(\text{ex})}_c(m, k) w_k g_{k,m,c}).
\]

After including $m$ in $S_{k,c} \neq \emptyset$ for RA user $k \in U_c \cap K$, the water levels are related to

\[
v^{(\text{in})}_c(m, k) = \frac{v_c \sum_{l \in K \cap U_c} w_l s_{l,c} + 1/g_{k,m,c}}{\sum_{l \in K \cap U_c} w_l s_{l,c} + w_k}.
\]

The variation of the weighted sum rate is derived as

\[
\Delta M^{(\text{in})}_c(m, k) = \log_2 \left( \frac{v^{(\text{in})}_c(m, k)}{v_c} \right) \sum_{l \in K \cap U_c} w_l s_{l,c} + w_k \log_2(v^{(\text{in})}_c(m, k) w_k g_{k,m,c}).
\]

Alternatively, when $P^{(RA)}_c$ changes, the resulting variation is

\[
\Delta M^{(p)}_c(D^{(RA)}) = \log_2 \left( \frac{\Delta P^{(RA)}_c}{v_c} \right) \sum_{l \in K \cap U_c} w_l s_{l,c}
\]

where (p) indicates the operation of changing the transmission power for RA users by $D^{(RA)}_c$ and

\[
\frac{\Delta P^{(RA)}_c}{v_c} = \frac{\Delta P^{(RA)}_c}{\sum_{l \in K \cap U_c} w_l s_{l,c}}.
\]

In the above equations, $\sum_{l \in K \cap U_c} w_l s_{l,c}$ is always required. It can be buffered and updated to accelerate the calculation. Similarly to the MA problem, only two logarithmic operations are necessary besides those simple operations.

The equations derived above can substitute water-filling to obtain the rate variation after changing the subcarrier assignment. For example, see the dashed line 2, a subcarrier was reassigned from an MA user to RA users. First, the transmission power for this MA user would increase by (2). Due to this increment the weighted sum rate for RA users would decrease by (6) and then increase by (5) because of employing this subcarrier.

For another example, see the dashed line 3, a subcarrier was reassigned from an RA user to an MA user. First, the weighted sum rate for RA users would decrease by (4). The transmission power for this MA user would then decrease by (3). Due to this decrement the weighted sum rate would increase by (6). The above equations can be used for other branches in similar ways. These approaches utilize the previous water-filling solution to update the objective. Thus, we call them the genetic water-filling (GWF).

### B. Iterative BS Selection and Subcarrier Assignment

In general, a joint signal processing is realized by some iterative procedure to keep solutions reliable. For example, joint channel estimation and data detection is implemented by the iteration between channel estimation and data detection, e.g., [11]. Similarly, joint BS selection and subcarrier assignment for (1) can also be given by the following iterative process, where the GWF is used to accelerate it.

1) Initialization: The subcarrier assignment and the BS selection are initialized by Algorithm 1 and Algorithm 2. The BS selection is initialized by two steps in Algorithm 1. The average channel condition is denoted by

\[
\bar{g}_{k,c} = \frac{1}{N} \sum_{n=1}^N g_{k,n,c}, \quad k = 1, \ldots, K + Q, c = 1, \ldots, C.
\]

If $K \cap U_c$ is empty, rate maximization cannot be performed for RA users at BS $c$. Thus, each BS serves at least one RA user. To ensure this, one RA user is assigned to each BS in the first step. In the second step each remaining user is covered by that BS, from which it has the best average channel condition. Finally, the BS selection $U_1, \ldots, U_C$ is initialized.

Since users are separated to different BSs after Algorithm 1 as $U_c \cap U_v = \emptyset$ and $s_{k,c}, s_{k,v} = 0$ hold for any $c \neq v$, the multicell subcarrier assignment can be viewed as a single-cell.

#### Algorithm 1 Initialization for BS selection

#### Step 1:

for $c = 1, \ldots, C$ do

$k \leftarrow$ the non-assigned RA user with the greatest $\bar{g}_{k,c}$

$U_c \leftarrow U_c \cup \{k\}$

end for

#### Step 2:

for each non-assigned user $k$ do

$\hat{c} \leftarrow$ the BS, at which user $k$ has the greatest $\bar{g}_{k,c}$

$U_c \leftarrow U_c \cup \{k\}$

end for
Algorithm 2 Initialization for Subcarrier Assignment

Step 1:

\[ s_k \leftarrow 1, \quad k = 1, \ldots, K + Q \]

repeat

\[ (k, \Delta P_k) \leftarrow \arg \max_{k=K+1, \ldots, K+Q} \Delta P_k \]

\[ (\hat{k}, P^{\text{RA}}(\hat{k})) \leftarrow \arg \max_{l \in \{1, \ldots, K\}} P^{\text{RA}}(l) \]

if \( R^{\text{MA}}(\hat{k}) > R^{\text{RA}}(\hat{k}) \) then

\[ s_\hat{k} \leftarrow s_\hat{k} + 1 \]

else

\[ s_\hat{k} \leftarrow s_\hat{k} + 1 \]

end if

until \( \sum_{k=1}^{K+Q} s_k = N \)

Step 2:

\[ N' \leftarrow \{1, \ldots, N\} \]

\[ \hat{s}_k \leftarrow \left[ s_k / \left( \sum_{k=1}^{K+Q} s_k / (K + Q) \right) \right], \quad k = 1, \ldots, K + Q \]

repeat

for each \( k \in \{1, \ldots, K + Q\} \)

\[ \mathcal{T} \leftarrow \{ \hat{s}_k \text{ subcarriers with the largest CNRs, } n \in N' \} \]

\[ S_k \leftarrow S_k \cup \mathcal{T} \]

\[ N' \leftarrow N' \setminus \mathcal{T} \]

end for

until \( N' = 0 \)

\[ S_{k,c} \leftarrow S_k, \quad k \in \mathcal{U}_c \]

Algorithm 3 Iterative BS Selection and Subcarrier Adjustment

repeat

\( (K, Q) \leftarrow \text{independent water-filling with } S_{k,c}, \mathcal{U}_c \)

for each \( n \in \{1, \ldots, N\} \)

if \( \sum_{k \in K \cap \mathcal{U}_c} \sum_{n \in S_{k,c}} p_{k,n,c} > P_n \) then

\[ \min \sum_{k \in K \cap \mathcal{U}_i} \sum_{n \in S_{k,c}} p_{k,n,c} \]

\[ \leftarrow \text{adjusting } n \text{ among users by GWF} \]

else

\[ \max \sum_{c=1}^{C} \sum_{k \in K \cap \mathcal{U}_c} w_k \sum_{n \in S_{k,c}} r_{k,n,c} \]

\[ \leftarrow \text{adjusting } k \text{ among BSs by water-filling} \]

end if

end for

for each \( k \in \{1, \ldots, K + Q\} \)

\[ \max \sum_{c=1}^{C} \sum_{k \in K \cap \mathcal{U}_c} w_k \sum_{n \in S_{k,c}} r_{k,n,c} \]

\[ \leftarrow \text{adjusting } k \text{ among BSs by water-filling} \]

end for

until no improvement

With the evaluated \( s_1, \ldots, s_{K+Q} \), subcarriers are simply assigned to users in the second step. In each iteration only \( s_k \) subcarriers with the greatest CNR are assigned to each user \( k \), where \( s_k \) is determined by the average of the evaluated cardinalities. In doing so, every user has the opportunity to select the subcarriers with relatively good qualities. Finally, the BS selection and the subcarrier assignment are initialized.

2) BS Selection and Subcarrier Adjustment: After the initialization above, water-filling is performed for RA and MA users so that the sets of RA and MA users are determined. Note that \( Q \) may include some primal RA users, for whom only the minimum required rates are reached. The set \( K \) contains the RA users, for whom the achieved rates are strictly greater than the minimum required rates. These two sets remain constant within one iteration of the outer loop in Algorithm 3.

First, subcarriers are successively adjusted among users. Each subcarrier is reassigned to different users to see whether the weighted sum rate can be improved. While adjusting one subcarrier, the assignment of others is fixed and the GWF is used to accelerate this reassignment. In this successive procedure the power limits may be violated. If such a case occurs, the sum power for reaching all minimum required rates is minimized by adjusting subcarriers among users. Otherwise, the weighted sum rate is maximized.

Users are then adjusted among BSs. Each user is reassigned to different BSs to see whether the objective can be improved. While adjusting one user, the subcarrier assignment and the assignment of other users are fixed. These two successive procedures iterate till no improvement can be made. Since one subcarrier can be assigned to one of \( K + Q \) users and one

### Resource Allocation with Subcarrier Assignment

resource allocation with \( S_k = \bigcup_{c=1}^{C} S_{k,c} \) and \( s_k = \sum_{c=1}^{C} s_{k,c} \).

The idea from [12] is used to evaluate \( s_k, k = 1, \ldots, K + Q \).

In the first step of Algorithm 2, we assume that all subcarriers of each user have the same CNR as the average of their CNRs from the BS that the user is assigned to, i.e.,

\[ \text{if } k \in \mathcal{U}_c, \text{ then } g_k = g_{k,c} \]

In each iteration we increase only one of \( s_1, \ldots, s_{K+Q} \) by 1, which may result in the greatest increment to the weighted sum rate. For MA users the power decrement by increasing cardinalities by one is

\[ \Delta P_k = \frac{s_k}{g_k} \left( 2^{\frac{r_k}{g_k}} - 1 \right) - \frac{s_k + 1}{g_k} \left( 2^{\frac{r_k}{g_k}} - 1 \right) \]

for all \( k = 1, \ldots, K + Q \). Then, the resulting increment to the weighted sum rate is denoted by \( R^{\text{MA}}(k) \) and equals to

\[
\sum_{k=1}^{K} w_k s_k \log_2 \left( \frac{\sum_{c=1}^{C} P_c - \sum_{i=1}^{K+Q} P_i + \sum_{k=1}^{K} \left( s_k/g_k \right)}{\sum_{i=1}^{K} s_i w_i} \right) + w_k \left( s_k+1 \right) \log_2 \left( \frac{\sum_{c=1}^{C} P_c - \sum_{i=1}^{K+Q} P_i + \sum_{k=1}^{K} \left( s_k/g_k + 1/g_k \right)}{\sum_{i=1}^{K} s_i w_i + w_k} \right)
\]
user can be served by one of \(C\) BSs, the proposed method has linear complexity \(O((K+Q)NC)\).

### IV. Simulations

In this section, the offered method is evaluated by simulations. In the polar coordinate, BSs are located equally on the circle with the radius \(D = 1\). Each user’s location is denoted by \((\theta, \gamma)\), while \(\theta\) and \(\gamma\) are uniformly distributed within \([0, 360^\circ]\) and within \([0, 1.5D]\), respectively. The transmission fading between each pair of BS and user is subject to the free space path loss with \(f = 2.5\) GHz. The transmission power is limited to \(P_c = 20\) dBW. Each required rate is uniformly distributed within \([10, 20]\) bits per OFDM symbol. Each weight is uniformly distributed within \([1, 10]\). Their sum is normalized to 1. The channel is modeled as consisting of \(N/8\) independently Rayleigh fading multiple paths with an exponentially decaying profile.

As explained before, when the BS selection is settled, the subcarrier assignment shrinks to the single-cell case. For comparison the optimal BS selection is determined by exhaustive \((K+Q)^C\) searches, since each user may be served by one of \(C\) BSs. For each search a dual optimum of subcarrier assignment is obtained as suggested in [5].

The proposed method is compared to the dual optimum for different numbers of subcarriers in Fig. 3. Here \(I\) indicates the number of iterations in Algorithm 3. In our simulations improvement becomes insignificant as \(I\) is larger than 4. This implies Algorithm 3 converges fast. The gap between the dual optimum and the proposed heuristic solution is relatively small, since even distributing only one user to an inappropriate BS leads to a big difference between the resulting subcarrier assignment and the optimal one.

The performance of the proposed method is given in Fig. 4 against different numbers of users. The dual optimum is ignored due to the computational limit by exhaustive search. The case for initialization may be viewed as the sequential (non-iterative) BS selection and subcarrier assignment. Compared to it, our method rapidly converges to much better performance.

### V. Conclusion

In this paper, we have investigated resource allocation for heterogeneous unicasting by multiple BSs which are near to each other. We have aimed at maximizing the weighted sum rate subject to the power limits while satisfying the minimum required rates. Since each subcarrier or each user is allowed for at most one BS, the system performance is very sensitive to BS selection. To enhance performance BS selection and subcarrier assignment have been jointly considered. An iterative method has been proposed. Simulations have demonstrated that the performance loss of the proposed method is relatively small for the concerned scenario.

### REFERENCES


