# Distributed Sensing of a Slowly Time-Varying Sparse Spectrum Using Matrix Completion

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***Abstract*—In this paper, we consider the problem of sensing a frequency spectrum in a distributed manner using as few measurements as possible while still guaranteeing a low detection error. To achieve this goal we use the newly developed technique of matrix completion which enables to recover a low rank matrix from a small subset of its entries. We model the sensed bandwidth at different cognitive radios as a spectrum matrix. It has been shown that in many cases the spectrum used by a primary user is underutilized. Therefore the spectrum matrix often has a low rank structure. By taking few measurements at several cognitive radios and reconstructing the matrix at a fusion center, we can dramatically reduce the required number of samples to reconstruct the utilization of the bandwidth. This is a key enabler for efficient and reliable spectrum reuse.**

## I. Introduction

Bandwidth scarcity has become one of the most important problems in modern communication systems. One of the most promising approaches to tackle this problem is to reuse the underutilized bandwidth of a primary system. This approach is called dynamic spectrum access (DSA) and is performed by intelligent transceivers called cognitive radios (CRs). The main technology enabling DSA is spectrum sensing, the goal of which is to sense the spectrum occupancy in a given bandwidth and measure the marginal power in individual frequencies. By knowing the spectrum occupancy, CRs can reuse the spectrum which is left unoccupied by a primary user. It can also enable several transmitters to use the same resource inside a communication cell if it can be determined, that their signals cannot interfere. Spectrum sensing of a potentially large bandwidth is very costly when sampling at the Nyquist rate and requires expensive hardware components. In order to counterbalance this drawback, it might be interesting to sample at a rate lower than the Nyquist rate.

This goal can be achieved by using compressed sensing (CS) [1], [2], which can be employed because the frequency spectrum can often be assumed to be only sparsely occupied. There exists a rich literature on applying CS to spectrum sensing. In [3], the author presents a complete setup to perform distributed spectrum sensing using CS. The scenario consists of several CRs which sense the channel and take a number of time domain measurements much smaller than the actual dimension of the discrete frequency signal. Using CS reconstruction, the CRs recover the spectrum at their location. In order to increase the reliability of spectrum sensing and to combat the effect of fading, the author proposes a distributed approach to spectrum sensing. Using the algorithm [4], the compressed spectrum measurements can be shared among neighbors and based on several measurements, each node can reconstruct a reliable image of the frequency spectrum. Another very interesting approach is the one presented in [5], where the authors not only try to estimate the frequency spectrum, but also want to determine the position of the primary users. The authors use a deterministic path-loss model to link the received power and the distance between transmitter and receiver and use a virtual grid which represents the possible positions of the primary users, to render the optimization problem tractable.

The main idea of the present work is to use the newly developed technique of matrix completion (MC) [6], [7], to further reduce the number of measurements required to reconstruct the utilization of the bandwidth. Indeed, MC enables to reconstruct a matrix $\mathbf{M} \in \mathbf{R}^{n_1 \times n_2}$ from much fewer samples than $n_1 \cdot n_2$ if $\mathbf{M}$ has a low rank and is incoherent. In the next section we explain the basics of MC. The link between spectrum sensing and MC is clear. If we model the sensed spectrum at different CRs as a spectrum matrix, this matrix has a low rank because the spectrum is sparsely used. We can therefore reconstruct the occupancy of the bandwidth while letting each CR take much fewer measurements than necessary for traditional spectrum sensing employing Nyquist-rate sampling.

In [8], the authors introduce an approach to use MC in spectrum sensing to reconstruct a frequency spectrum. They present a framework for modeling the problem as well as simulation results showing that this approach works well.

The contribution of the present paper is manifold and is different from [8] in many aspects. First we use the algorithm OptSpace of [7] to perform MC and we explicitly consider noisy measurements in our framework. Second we give analytical bounds on the number of required samples to achieve a certain reconstruction error. Third, and most importantly, we present a new approach to the problem, utilizing past spectrum observations to further diminish the number of required samples and propose a sliding window algorithm to

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optimize the use of this knowledge. Finally we verify our idea by proceeding to numerical evaluations and analyze the system performance for different window sizes and noise conditions.

The rest of the present work is organized as follows. Section II gives the basics of MC. In section III, we present our mathematical framework and in section IV, we develop bounds on the required number of measurements and error for the simple MC reconstruction and describe a new algorithm making use of past spectrum observations. Finally, we verify our method experimentally in section V and conclude this paper in section VI.

## II. Basics of Matrix Completion

In this section we focus on the work of [7]. We want to reconstruct a matrix $\mathbf{M} \in \mathbf{R}^{n_1 \times n_2}$. The measurements we take are noisy and thus we only have access to $\mathbf{P} = \mathbf{M} + \mathbf{N}$, where $\mathbf{N}$ is a noise matrix. In [7], the authors demonstrate that it is possible to recover $\mathbf{M}$ from fewer measurements than $n_1 \cdot n_2$ using their algorithm called OptSpace, given that $\mathbf{M}$ and the sampling process fulfill several requirements detailed below. Note that we present OptSpace in more detail in section IV-A.

### A. Uniform Sampling

We measure $m$ entries of $\mathbf{P}$ uniformly distributed at random.

### B. Low Rank

The matrix $\mathbf{M}$ has to have a low rank denoted by $r$.

### C. Incoherence

Let $\mathbf{M} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathrm{T}}$ be a scaled singular value decomposition of $\mathbf{M}$, where $\mathbf{U}^{\mathrm{T}}\mathbf{U} = n_1\mathbf{I}_{n_1}$, $\mathbf{V}^{\mathrm{T}}\mathbf{V} = n_2\mathbf{I}_{n_2}$ and $\mathbf{\Sigma}$ is a diagonal matrix with $\Sigma_{ii} = \sigma_i$ being the $i$-th singular value of $\mathbf{M}$, scaled by $1/\sqrt{n_1 n_2}$. Let $\sigma_1 \geq \cdots \geq \sigma_r$. The matrix $\mathbf{M}$ is $(\mu_0, \mu_1)$-incoherent if the following two conditions hold.

A1 $\forall i{=}1, ..., n_1, j{=}1, ..., n_2, \sum_{k=1}^{r} U_{ik}^2 \leq \mu_0 r, \sum_{k=1}^{r} V_{jk}^2 \leq \mu_0 r.$

A2 $\forall i{=}1, ..., n_1, j{=}1, ..., n_2, |\sum_{k=1}^{r} U_{ik}(\frac{\sigma_k}{\sigma_1})V_{jk}| \leq \mu_1 r^{1/2}.$

### D. Minimum Required Number of Samples and error bound

We define $\kappa = \sigma_1/\sigma_r$ and $\alpha = n_1/n_2 \geq 1$. There exist numerical constants $C$ and $C'$ such that if

$$m \geq C n_2 \sqrt{\alpha}\kappa^2 \max\{\, \mu_0 r \sqrt{\alpha} \log\, n_2,\ \mu_0^2 r^2 \alpha \kappa^4,\ \mu_1^2 r^2 \alpha \kappa^4\}, \tag{1}$$

then with a probability of at least $1 - 1/n_2^3$, OptSpace recovers $\mathbf{M}$ with an error

$$\frac{1}{\sqrt{n_1 n_2}} \|\hat{\mathbf{M}} - \mathbf{M}\|_{\mathrm{F}} \leq C' \kappa^2 \frac{n_2\sqrt{r\alpha}}{m} \|\mathbf{N}^E\|_2, \tag{2}$$

provided that the right-hand side is smaller than $\sigma_r$. $\|.\|_{\mathrm{F}}$ denotes the Frobenius norm, $\|.\|_2$ denotes the maximum singular value, $\hat{\mathbf{M}}$ is the reconstruction of $\mathbf{M}$ and $\mathbf{N}^E$ equals $\mathbf{N}$ at the positions where $\mathbf{P}$ has been sampled and is zero otherwise.

## III. Analytical Model and Optimization Problem

We consider a cognitive radio (CR) network with $n_2$ CRs, sensing a bandwidth of size $n_1$. A primary system owns this bandwidth and transmits a signal at time slot $t$, resulting in a frequency spectrum $\mathbf{f}(t) \in \mathbf{C}^{n_1}$, where $\mathbf{f}(t)$ is $k$-sparse, i.e. it has no more than $k$ nonzero components. The sparsity of $\mathbf{f}(t)$ represents the fact that the bandwidth is scarcely used. We observe $N_t$ time slots, i.e. $t = 1, \ldots, N_t$.

Each CR $i$ ($i = 1, \ldots, n_2$) senses a signal $\hat{\mathbf{f}}_i(t)$ given by

$$\hat{\mathbf{f}}_i(t) = \mathbf{H}_i(t)\mathbf{f}(t) + \mathbf{n}_i(t), \tag{3}$$

where $\mathbf{H}_i(t) \in \mathbb{C}^{n_1 \times n_1}$ is a diagonal matrix, $[\mathbf{H}_i(t)]_{jj}$ ($j = 1, \ldots, n_1$) is the channel gain encountered by $[\mathbf{f}(t)]_j$ at CR $i$ and $\mathbf{n}_i(t)$ is a noise vector, the entries of which are Gaussian independently and identically-distributed with mean zero and variance $\sigma^2$. Note that in the present paper we assume that the CRs have no knowledge of $\mathbf{H}_i(t)$. We define the matrix $\hat{\mathbf{F}}(t) \in \mathbb{C}^{n_1 \times n_2}$ as

$$\hat{\mathbf{F}}(t) = [\hat{\mathbf{f}}_1(t), \ldots, \hat{\mathbf{f}}_{n_2}(t)], \tag{4}$$

which can be expressed as a sum of matrices as follows

$$\hat{\mathbf{F}}(t) = \mathbf{F}(t) + \mathbf{N}(t) \tag{5}$$

where $\mathbf{F}(t) \in \mathbb{C}^{n_1 \times n_2}$ is given by

$$\mathbf{F}(t) = [\mathbf{H}_1(t)\mathbf{f}(t), \ldots, \mathbf{H}_{n_2}(t)\mathbf{f}(t)], \tag{6}$$

and $\mathbf{N}(t) \in \mathbb{C}^{n_1 \times n_2}$ is given by

$$\mathbf{N}(t) = [\mathbf{n}_1(t), \ldots, \mathbf{n}_{n_2}(t)]. \tag{7}$$

Each CR $i$ undersamples the signal $\hat{\mathbf{f}}_i(t)$ in time, i.e. it takes only $m_i(t)$ time samples of $\mathbf{\Psi}\hat{\mathbf{f}}_i(t)$, where $\mathbf{\Psi} \in \mathbf{C}^{n_1 \times n_1}$ is the inverse discrete Fourier transform (IDFT) matrix. The total number of measurements $m(t)$ made by the CRs at time slot $t$ is given by

$$m(t) = \sum_{i=1}^{n_2} m_i(t). \tag{8}$$

We define $\Omega(t)$ as a set of cardinality $m(t)$ containing the indices of the sampled entries of $\mathbf{\Psi}\hat{\mathbf{F}}(t)$. Further we introduce the projection operator $\mathcal{P}_{\Omega(t)}$ defined as follows. Given a matrix $\mathbf{X}$,

$$[\mathcal{P}_{\Omega(t)}(\mathbf{X})]_{ij} = \begin{cases} [\mathbf{X}]_{ij} & \text{if } (i,j) \in \Omega(t), \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

The result of the sampling by the CRs is given by the matrix

$$\hat{\mathbf{T}}(t) = \mathcal{P}_{\Omega(t)}(\mathbf{\Psi}\hat{\mathbf{F}}(t)), \tag{10}$$

of size $n_1 \times n_2$. $\hat{\mathbf{T}}(t)$ contains only $m(t)$ non-zero entries.

All measurements are collected at a fusion center. The goal of the fusion center is to recover the matrix $\mathbf{F}(t)$ which represents the spectrum at each CR. To achieve this, we need an algorithm taking $\hat{\mathbf{T}}(t)$ as input and producing a matrix $\mathbf{F}_{\mathbf{r}}(t) \in \mathbb{C}^{n_1 \times n_2}$ such that the reconstruction error

$$\frac{1}{\sqrt{n_1 n_2}} \|\mathbf{F}_{\mathbf{r}}(t) - \mathbf{F}(t)\|_{\mathrm{F}} \tag{11}$$

is as small as possible.

One of the main contributions of the present paper is, that in order to recover $\mathbf{F}(t)$, we will make use of previous observations of the channel. In other words, in addition to $\hat{\mathbf{T}}(t)$, we use the knowledge of $\hat{\mathbf{T}}(1), \ldots, \hat{\mathbf{T}}(t-1)$ to recover $\mathbf{F}(t)$. We assume that the primary user slowly changes its usage of the bandwidth, i.e. $\mathbf{f}(t)$ and $\mathbf{f}(t+1)$ are very similar. To model this behavior we will assume that

- if $[\mathbf{f}(t)]_j$ is not used at time $t$, it is used at time $t+1$ with probability $p_1$.
- if $[\mathbf{f}(t)]_j$ is used by the primary user at time $t$, it stops being used at time $t+1$ with probability $p_2$.

The probabilities $p_1$ and $p_2$ model how fast the spectrum occupancy is changing and they are typically small. However, although the spectrum is changing, we consider the spectrum to be on average $k$-sparse at each time slot $t$. Therefore, it holds that

$$(n_1 - k)p_1 = kp_2. \tag{12}$$

## IV. Spectrum Sensing using Matrix Completion

In order to recover the matrix $\mathbf{F}(t)$ we want to exploit the fact that $\mathbf{F}(t)$ has a low rank. Indeed, since $\mathbf{f}(t)$ is $k$-sparse, $\mathbf{F}(t)$ has at most rank $k$ for $k \leq n_2$. The idea is to use the algorithm OptSpace of [7] to reconstruct $\mathbf{F}(t)$ based on its undersampled noisy version in time $\hat{\mathbf{T}}(t)$. We first explain how to recover $\mathbf{F}(t)$ observing only one time slot and in a second step we introduce a method utilizing past observations.

### A. Spectrum Sensing without Prior Spectrum Information

We use OptSpace to recover $\mathbf{F}(t)$ using algorithm (1).

**Algorithm 1** OptSpace for spectrum sensing

$\tilde{\mathbf{T}}(t) = \text{trim}(\hat{\mathbf{T}}(t) = \mathcal{P}_{\Omega(t)}(\mathbf{\Psi}\hat{\mathbf{F}}(t)))$.
$P_r(\tilde{\mathbf{T}}(t)) =$ rank-$k$ projection of $\tilde{\mathbf{T}}(t)$.
Decompose $P_r(\tilde{\mathbf{T}}(t))$ as $P_r(\tilde{\mathbf{T}}(t)) = \mathbf{U}_0\mathbf{\Sigma}_0\mathbf{V}_0^{\mathrm{H}}$.
$\underset{\mathbf{U},\mathbf{V}}{\text{minimize}} \ \underset{\mathbf{\Sigma}}{\min.} \ \frac{1}{2}\|\mathcal{P}_{\Omega(t)}(\hat{\mathbf{T}}(t) - \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathrm{H}})\|_{\mathrm{F}}$, with initial condition $(\mathbf{U}_0, \mathbf{V}_0)$.
**return** $\mathbf{T_r}(t) = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathrm{H}}$.

For details about the *trim* and *projection* operations, refer to [7]. In this form, the algorithm (1) returns a matrix $\mathbf{T_r}(t) \in \mathbb{C}^{n_1 \times n_2}$. To recover $\mathbf{F}(t)$ we simply apply a DFT matrix to $\mathbf{T_r}(t)$, i.e. $\mathbf{F_r}(t) = \mathbf{\Psi}^{-1}\mathbf{T_r}(t)$.

In order to bound the error between $\mathbf{F_r}(t)$ and $\mathbf{F}(t)$ we now consider the decomposition of the matrix $\mathbf{\Psi F}(t)$ into its real and imaginary part:

$$\mathbf{\Psi F}(t) = \mathbf{T}_1(t) + i\mathbf{T}_2(t). \tag{13}$$

First we determine the number of samples required for reconstructing these two parts separately from noisy observations with OptSpace. Subsequently, an upper bound for the resulting reconstruction error is found. We define $\mathbf{T_{r1}}(t)$ and $\mathbf{T_{r2}}(t)$ as the reconstruction of $\mathbf{T}_1(t)$ and $\mathbf{T}_2(t)$ respectively. $\mathbf{T}_1(t)$ and $\mathbf{T}_2(t)$ are $(\mu_0, \mu_1)$-incoherent with rank $k$ and $(\mu_0', \mu_1')$-incoherent with rank $k'$ respectively, $\sigma_1 \geq \ldots \geq \sigma_r$ and $\sigma_1' \geq \ldots \geq \sigma_r'$ are the ordered scaled singular values of $\mathbf{T}_1(t)$ and $\mathbf{T}_2(t)$ respectively, $\kappa = \sigma_1/\sigma_r$, $\kappa' = \sigma_1'/\sigma_r'$, $\alpha = n_1/n_2 \geq 1$ and we sampled $m(t)$ complex entries of $\mathbf{\Psi}\hat{\mathbf{F}}(\mathbf{t})$. When using algorithm (1), there exist numerical constants $C_1$, $C_1'$, $C_2$ and $C_2'$ such that if

$$\begin{aligned} m(t) \geq C_1 n_2 \sqrt{\alpha}\kappa^2 \max\{ & \mu_0 k \sqrt{\alpha} \log n_2, \\ & \mu_0^2 k^2 \alpha \kappa^4, \\ & \mu_1^2 k^2 \alpha \kappa^4\}, \end{aligned} \tag{14}$$

and

$$\begin{aligned} m(t) \geq C_2 n_2 \sqrt{\alpha}\kappa'^2 \max\{ & \mu_0' k' \sqrt{\alpha} \log n_2, \\ & \mu_0'^2 k'^2 \alpha \kappa'^4, \\ & \mu_1'^2 k'^2 \alpha \kappa'^4\}, \end{aligned} \tag{15}$$

then with a probability of at least $1 - 1/n_2^3$,

$$\begin{aligned} \frac{\|\mathbf{T_{r1}}(t) - \mathbf{T}_1(t)\|_{\mathrm{F}}}{\sqrt{n_1 n_2}} &\leq C_1' \kappa^2 \frac{n_2\sqrt{k\alpha}}{m(t)} \|\mathcal{P}_{\Omega(t)}(\mathrm{Re}(\mathbf{\Psi N}(t)))\|_2 \\ &\triangleq \epsilon_1, \end{aligned} \tag{16}$$

and

$$\begin{aligned} \frac{\|\mathbf{T_{r2}}(t) - \mathbf{T}_2(t)\|_{\mathrm{F}}}{\sqrt{n_1 n_2}} &\leq C_2' \kappa'^2 \frac{n_2\sqrt{k'\alpha}}{m(t)} \|\mathcal{P}_{\Omega(t)}(\mathrm{Im}(\mathbf{\Psi N}(t)))\|_2 \\ &\triangleq \epsilon_2, \end{aligned} \tag{17}$$

provided that $\epsilon_1 < \sigma_r$ and $\epsilon_2 < \sigma_r'$. Using the triangular inequality, it follows that

$$\frac{1}{\sqrt{n_1 n_2}} \|\mathbf{T_r}(t) - \mathbf{\Psi F}(t)\|_{\mathrm{F}} \leq \epsilon_1 + \epsilon_2, \tag{18}$$

where

$$\mathbf{T_r}(t) = \mathbf{T_{r1}}(t) + i\mathbf{T_{r2}}(t). \tag{19}$$

Finally, since the IDFT matrix $\mathbf{\Psi}$ is unitary, we find

$$\frac{1}{\sqrt{n_1 n_2}} \|\mathbf{F_r}(t) - \mathbf{F}(t)\|_{\mathrm{F}} \leq \epsilon_1 + \epsilon_2. \tag{20}$$

### B. Spectrum Sensing with Prior Spectrum Information

We now consider the case where we can use past channel observations $\hat{\mathbf{T}}(1), \ldots, \hat{\mathbf{T}}(t-1)$ to recover $\mathbf{F}(t)$. The intuition underlying our idea is as follows. The rank of the matrix $\mathbf{F}(t)$ and the matrix $[\mathbf{F}(t)\ \mathbf{F}(t+1)]$ is nearly the same since the spectrum occupancy changes slowly. From (14), we can see that, in that case, to recover $\mathbf{F}(t)$ and $\mathbf{F}(t+1)$ separately, we need more measurements than to recover $[\mathbf{F}(t)\ \mathbf{F}(t+1)]$. In other words, reusing past channel observations, we need less measurements per time slot to recover $\mathbf{F}(t)$.

We will proceed using a sliding window algorithm. We first define the matrix $\mathbf{W}(t_1, t_2)$ given by

$$\mathbf{W}(t_1, t_2) = \mathbf{\Psi}[\mathbf{F}(t_1)\ \ \mathbf{F}(t_1+1)\ \ \ldots\ \ \mathbf{F}(t_2)], \tag{21}$$

which is a window matrix defined as the concatenation of time-domain matrices from time slot $t_1$ to $t_2$. Our algorithm is given in (2). Further we define $t_w$ as the maximum number of time slots in the window matrix we want to reconstruct. At $t = 1$ the window matrix only contains $\mathbf{\Psi F}(1)$ and we reconstruct $\mathbf{F}(1)$ using the method described in the previous section. As $t$

increases, we append $\mathbf{\Psi F}(t)$ to the window matrix and recover $\mathbf{W}(1,t)$ (and consequently $\mathbf{F}(t)$). When $t$ has reached $t_w$ we begin sliding the window. In other words we remove the part of the window matrix representing the first time slot and append $\mathbf{\Psi F}(t)$, hence getting $\mathbf{W}(2,t_w+1)$. We slide the window until $t = N_t$.

**Algorithm 2** Sliding window algorithm

Window size $t_w$.
Initialization: recover $\mathbf{F}(1)$.
**for** $i = 2,\ldots,t_w$ **do**
  Recover $\mathbf{W}(1,i)$ and consequently $\mathbf{F}(i)$.
**end for**
Sliding phase:
**for** $j = 1,\ldots,N_t - t_w$ **do**
  Recover $\mathbf{W}(1+j,t_w+j)$ and consequently $\mathbf{F}(t_w+j)$.
**end for**

In the following section it will be demonstrated that this approach largely improves the performance of the system compared to the simple approach without prior spectrum information.

## V. Numerical Evaluation

To validate our concept we simulate the scenario described in Table I. The non-zero entries of $\mathbf{F}(t)$ are generated randomly from the Rayleigh distribution with standard deviation $\sigma = 0.5$. The signal to noise ratio (SNR) is defined as the signal energy of the wideband signal over the entire spectrum divided by the overall noise energy (again over the entire spectrum). Note, that due to the sparse spectrum occupation,

TABLE I
Scenario Parameters

| Parameters | Symbol | Value |
|---|---|---|
| Number of frequency subchannels | $n_1$ | 300 |
| Number of CRs | $n_2$ | 5 |
| Average number of busy subchannels | $k$ | 5 |
| Number of observed time slots | $N_t$ | 50 |
| Probability of subchannel to become occupied | $p_1$ | $1/5000$ |
| Window size | $t_w$ | $\{10,20\}$ |
| Signal to noise ratio (SNR) | | $\{0,-10,-20\}$dB |

the considered values for the SNR, i.e. 0dB, $-10$dB and $-20$dB might seem low. However, defining the SNR as the signal energy of a single subchannel divided by the noise energy of named subchannel would yield SNR values of 17.8dB, 7.8dB and $-2.2$dB respectively for the occupied channels.

In Figure 1, we plot the reconstruction error at SNR = 0dB without past spectrum information (window size 1) and with the window sizes 10 and 20. In this case we take the same number of samples at each iteration. The plot enables us to compare the effect of the window size on the performance of the system. First we can see that using past information provides a significant decrease in the reconstruction error. Note that in both cases we take the same number of samples per

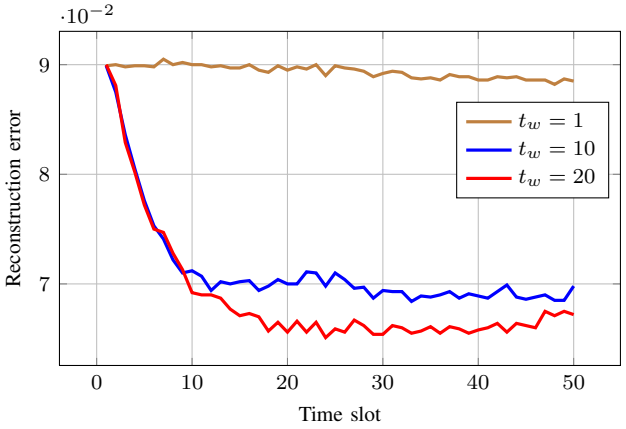


Fig. 1. Reconstruction error at each time slot, SNR = 0dB, 375 samples per time slot

time slot, but in the window case we keep the old ones. In this case our approach provides a gain of up to 25%. The second thing we can see is that a larger window size actually further decreases the reconstruction error.

In Figure 2, we plot the reconstruction error without past information against the reconstruction error with a window of size 20 for different SNRs. This figure illustrates the behavior

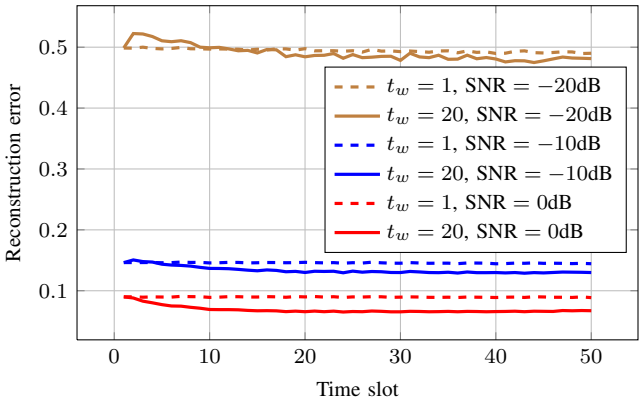


Fig. 2. Reconstruction error at each time slot, 375 samples per time slot

of the system with respect to noise intensity. First we can observe that a larger noise power induces a larger reconstruction error in both reconstruction methods. Comparing the method without past information with the one employing the sliding window, we can see that the amount by which the usage of past information decreases the reconstruction error gets bigger as the SNR goes up. Another observation that can be made, is that applying the multi time slot reconstruction with a non-decreasing number of samples per time slot yields larger errors than the single time slot reconstruction for low SNRs in the transient stage. Indeed, in the first time slots, the gain of having past information does not compensate the increase in error resulting from the higher amount of sampled noise.

In Figure 3 and 4, we illustrate a scenario at SNR = $-10$dB, where the sliding window method is applied using a decreasing number of samples per time slot. Figure 3 plots the number of samples taken and Figure 4 shows the reconstruction error at

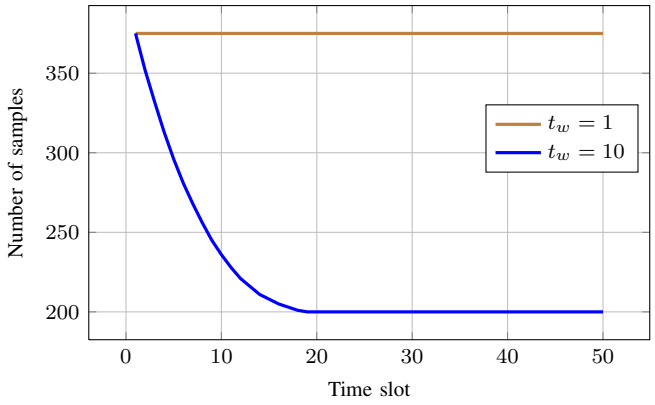


Fig. 3. Number of samples taken at each time slot

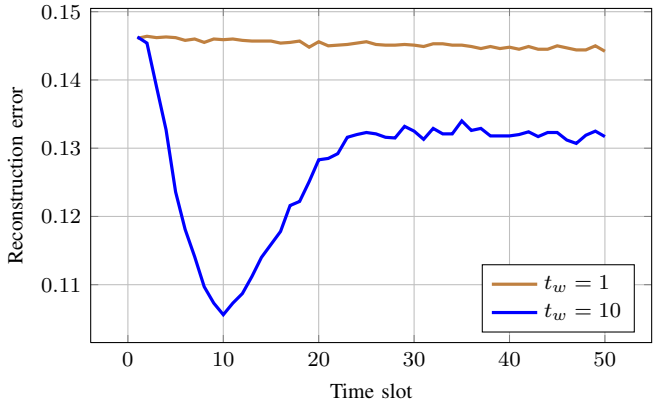


Fig. 4. Reconstruction error at each time slot, decreasing number of samples, SNR = $-10$dB

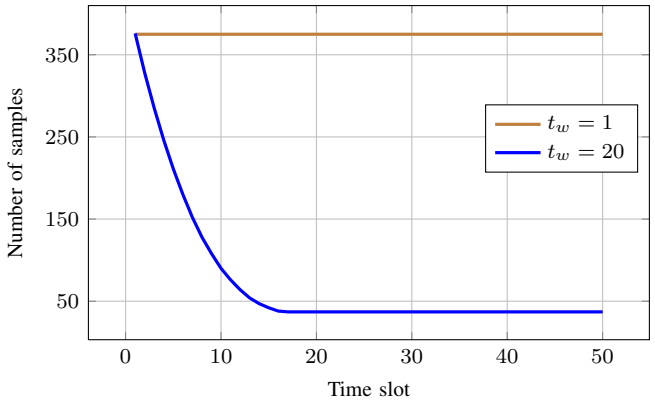


Fig. 5. Number of samples taken at each time slot

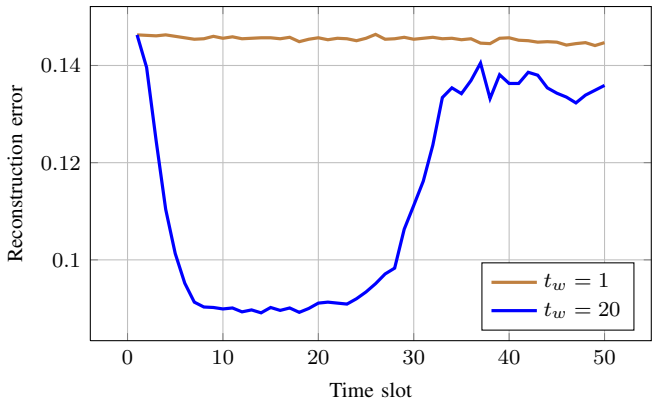


Fig. 6. Reconstruction error at each time slot, decreasing number of samples, SNR = $-10$dB

each time slot. In this case we consider a window of size 10. These two curves demonstrate the relevance of our approach. First we accumulate samples and the reconstruction error gets smaller. Then, when the window starts sliding, the number of total samples we consider is reduced until we reach time slot 20. From there on we have a constant total number of samples and the reconstruction error is constant. The results speak for themselves. In order to achieve a smaller reconstruction error, the sliding window algorithm needs 200 measurements per time slot, while the simple MC algorithm without prior spectrum knowledge requires 375.

In Figure 5 and 6 we analyze a similar scenario. This time however, the window is of size 20. Increasing the window size allows us to reduce the number of required samples even more. The sliding window algorithm achieves a smaller reconstruction error with 37 measurements per time slot than the simple MC algorithm without prior spectrum knowledge does with 375.

## VI. Conclusion

In this paper we have presented a new approach to spectrum sensing using matrix completion and exploiting past spectrum information to reduce the number of required samples while still guaranteeing a small error. We have given analytical bounds on the necessary number of samples and error and we have demonstrated the validity of our approach through numerical evaluation. A future research direction is the implementation of such a concept on a software defined radio platform to verify real world performance.

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