# Channel Capacity Related Power Allocation for Distributed Sensor Networks with Application in Object Classification

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Abstract—This publication analyzes the power allocation problem for a distributed wireless sensor network which is based on ultra-wide bandwidth communication technology. The network is used to classify target objects. In the considered scenarios, the absence, the presence, or the type of an object is observed by the sensors independently. Due to noisy communication channels, the interfered observations are fused into a reliable global decision in order to increase the overall classification probability. An approach based on information theory that aims at maximization of the mutual information is employed. It enables the analytical allocation of the given total power to the sensor nodes so as to optimize the overall classification probability. Furthermore, we demonstrate the feasibility of object classification by using the introduced power allocation method in ultra-wide bandwidth signaling and energy-efficient systems.

## I. INTRODUCTION

In this essay we analyze the power allocation problem for a distributed wireless sensor network with sensor nodes (SN) based entirely on ultra-wide bandwidth (UWB) technology. The network is used to perform object classification, where the kind of an object is observed by the sensors independently. UWB signals can be used for data communication between the SNs as well as for radar applications. The approach of misemploying the communication sensors as radar sensors, such that the data transmission is misused as a radar beam in order to classify a target object, helps in realizing an energy-efficient radar system with compact and cheap SNs. A further advantage of such radar systems is the fulfillment of major requirements of wireless sensor networks. This exploitation presupposes that the integration of sensing functionality into usual UWB sensors is implementable easily without the usage of any additional hardware units. Since the compact and low complexity UWB sensors are limited in power and communication capabilities, the classification performance of a single sensor is restricted compared to that of a common complex radar system. To obtain an appropriate overall system performance we consider the case of distributed classification, where the local observations of the sensors are fused into a reliable global decision. Due to noisy communication channels and differences in distances between the object and the sensors, both, the observations and their transmissions are unequally interfered. One simple way to suppress noise interference is to increase the power of each



Fig. 1. System model of the distributed wireless sensor network.

SN. But if the total power of the entire network is limited, then power allocation procedures are needed in order to increase the overall classification probability. In general, for a Bayesian-hypotheses-test criterion the mathematical function of the overall classification probability cannot be analytically evaluated [1]. This limits the usability of this criterion for analytical optimization of the power allocation. Bounds, such as the Bhattacharyya bound [2], are also difficult to use for optimizing multidimensional problems. Therefore, we employ an information theoretic approach [3] which is based on mutual information maximization. This approach yields a simple however suboptimal analytical solution for the power allocation problem. Hereby, we show the feasibility of object classification in UWB signaling and energy-efficient systems.

The origin of research on distributed detection has been the attempt to fuse signals of different radar devices [4]. Currently, distributed detection is usually discussed in the context of wireless sensor networks, where the sensor unit of the nodes might be based on radar technology [5]–[7]. In a recent publication [8], the power allocation problem is analyzed where a sensor network is used to detect target objects. Other applications for UWB radar systems, which require or benefit from the detection and classification capabilities, are for example localization and tracking [9] or through-wall surveillance [10]. The physical layer design for an integrated UWB radar network that utilizes OFDM technology was analyzed in [11].

## II. OVERVIEW AND SYSTEM DESCRIPTION

Throughout this paper we denote the set of natural, real, and complex numbers by  $\mathbb{N}$ ,  $\mathbb{R}$ , and  $\mathbb{C}$ , respectively. Note that

the set of natural numbers does not include the element zero. Furthermore, we use the subset  $\mathbb{F}_N \subseteq \mathbb{N}$  which is defined as  $\mathbb{F}_N \coloneqq \{1, \ldots, N\}$  for any given natural number N. The mathematical operations |z| and |z| denote the absolute value of a real or complex-valued number z and the Euclidian length of a real or complex vector z, respectively.

Distributed *target object* classification can be formally modeled by a multiple hypotheses testing problem with hypotheses  $H_k \forall k \in \mathbb{F}_K$  for a specified number  $K \in \mathbb{N}, K \ge 2$ of different objects. We assume that all objects have the same size, shape, alignment, and position. They only differ in material and are classified by their complex-valued reflection coefficients  $r_k \in \mathbb{C}$ , which are ordered in a strictly increasing manner  $0 \le |r_1| < \cdots < |r_K| \le 1$ . Therefore, the reflection coefficients are the only recognition features in this work. Generally, this assumption is not realistic, but, this case describes an ideal scenario for increasing the classification probability by performing a power allocation and is not really suitable for analyzing the problems of manifoldness.

At any instance of time a network of  $N \in \mathbb{N}$  independent and spatially distributed sensors, as shown in Fig. 1, obtains random observations  $X_1, \ldots, X_N \in \mathbb{R}$ . In the case of energy classification  $X_n$  models the received signal at the receiver of the  $n^{\text{th}}$  sensor. If a target object is present, then the received energy is a part of the radiated energy of the same sensor which is reflected from the object's surface and is weighted by its reflection coefficient. We refer to this communication channel, between the sensors and the target object, as the *first* communication link and denote all dedicated parameters by the superscript R. The random observations  $X_1, \ldots, X_N$ are assumed to be conditionally independent for each of the underlying hypotheses, i.e., the joint conditional probability density function of all the observations factorizes according to

$$f^{R}(X \mid H_{k}) \coloneqq \prod_{n=1}^{N} f_{n}^{R}(X_{n} \mid H_{k}), \ \forall k \in \mathbb{F}_{K}, \qquad (1)$$

where X denotes the sequence of random variables  $X_1, \ldots, X_N$ . In general, the observations are not identically distributed because the SNs have different distances  $d_n^R$ from the target object and their radiated powers  $P_n^R$  are also different. Therefore, the signal-to-noise ratio (SNR) varies between the SNs. Due to the distributed nature of the problem, the  $n^{\text{th}}$  sensor  $S_n$  performs independent measurements and processes its respective observation  $X_n$  by generating a local decision  $U_n := \theta_n(X_n) \forall n \in \mathbb{F}_N$ , which depends only on its own observation and not on the observations of other SNs. After deciding locally each sensor transmits its decision to a fusion center located at a remote location. The communication between the SN and the fusion center is determined by the corresponding distance  $d_n^C$  as well as by the transmission power  $P_n^C$  of the same SN. We refer to this communication channel, between the SNs and the fusion center, as the second communication link and denote all dedicated parameters by the superscript C. Furthermore, we assume that both communication channels are non-fading channels and that all data transmissions are affected only by additive white Gaussian noise (AWGN). We disregard time delays within all transmissions and assume synchronized data communication. We use two distinct pulse-shift patterns for each SN in order to distinguish its first and second communication link from the communication links of other SNs as described in [12]. Each pattern has to be suitably chosen in order to suppress inter-user interference at each receiver. Hence, the N received signals at the fusion center are uncorrelated and are assumed to be conditionally independent for each of the underlying hypotheses. These received random signals correspond to the local decisions  $U_1, \ldots, U_N$  and are mapped to  $\tilde{\mathbf{X}}_1, \ldots, \tilde{\mathbf{X}}_N \in \mathbb{R}^K$ . Their joint conditional probability density function factorizes according to

$$f^{C}(\tilde{\mathbf{X}} \mid H_{k}) \coloneqq \prod_{n=1}^{N} f_{n}^{C}(\tilde{\mathbf{X}}_{n} \mid H_{k}), \ \forall k \in \mathbb{F}_{K},$$
(2)

where  $\tilde{\mathbf{X}}$  denotes the sequence of random vectors  $\tilde{\mathbf{X}}_1, \ldots, \tilde{\mathbf{X}}_N$ . In general, these observations are – similar to the observations  $X_1, \ldots, X_N$  – not identically distributed because of variation in distances  $d_n^C$  as well as that of the radiated powers  $P_n^C$ . Unlike the local decision rules the global decision rule  $U_0 \coloneqq \theta_0(\tilde{\mathbf{X}}_1, \ldots, \tilde{\mathbf{X}}_N)$  depends on all observations in order to increase the overall classification probability.

All described assumptions are necessary in order to obtain a framework suited for analyzing the power allocation problem without studying problems of different classification methods in specific systems and their settings.

## A. Local classification rules

The local decision and classification rules  $\theta_n$  are mappings of the kind  $\theta_n : \mathbb{R} \to \mathbb{F}_K, \forall n \in \mathbb{F}_N$ . In this work hard-decision rules are used for performing local classification given by

$$\theta_n(X_n = x_n) = k, \text{ if } \tau_{n,k} < x_n \le \tau_{n,k+1}, \ k \in \mathbb{F}_K \quad (3)$$

for all  $n \in \mathbb{F}_N$ , where the thresholds  $\tau_{n,k} \in \mathbb{R}$  are suitably chosen. The thresholds must be calculated separately for every SN in order to perform optimal classification. They depend on the prior probabilities of the hypotheses. Their values can be calculated by a suboptimal approach, which is described in Section III-A. In this way, every SN has a local probability of correct decision given by

$$\Pr(U_n = k \mid H_k) = \Pr(\tau_{n,k} < X_n \le \tau_{n,k+1} \mid H_k) \quad (4)$$

and a local probability of false decision given by

$$\Pr(U_n \neq k \mid H_k) = 1 - \Pr(U_n = k \mid H_k) \tag{5}$$

for all  $k \in \mathbb{F}_K$  and for all  $n \in \mathbb{F}_N$ .

## B. Fusion of local decisions and global classification rule

The local decisions  $U_1, \ldots, U_N$  at the SNs are conditionally independent due to uncorrelated and independent noisy communication channels. By applying the Bayesian-hypothesestest criterion the optimal fusion rule at the fusion center is



Fig. 2. System model of the  $n^{\text{th}}$  sensor node with circulator and antenna.

given by

$$U_0 = \theta_0(\tilde{\mathbf{X}} = \tilde{\mathbf{x}}) = \operatorname*{argmax}_{k \in \mathbb{F}_K} \left( \pi_k f^C(\tilde{\mathbf{x}} \mid H_k) \right), \qquad (6)$$

where  $\pi_k := \Pr(H_k)$  with  $\sum_{k=1}^{K} \pi_k = 1$  denotes the prior probability of hypothesis  $H_k$ . We use this formula to classify the target object. However, in order to optimize the allocation of the total power to the SNs we have to consider the overall classification probability. Therefore, we consider K pairwise disjoint regions  $\mathcal{R}_1, \ldots, \mathcal{R}_K$  with

$$\mathcal{R}_k \coloneqq \left\{ \tilde{\mathbf{x}} \in \mathbb{R}^{K \times N} \mid \pi_k f^C(\tilde{\mathbf{x}} \mid H_k) \ge \pi_l f^C(\tilde{\mathbf{x}} \mid H_l) \right\}$$
(7)

for all  $k, l \in \mathbb{F}_K$  with  $l \neq k$ . According to [1] the expected value of correct classification is given by

$$P_c \coloneqq \sum_{k=1}^{K} \Pr(\tilde{\mathbf{x}} \in \mathcal{R}_k, H_k), \tag{8}$$

which in general cannot be analytically evaluated. Therefore, the previous formula cannot be used to optimize the allocation of the total power analytically. Consequently, we choose a different approach for the optimization which is described in Section III-B.

## C. Ultra-wide bandwidth sensor nodes

In Fig. 2 the system model of the considered impulse-radio UWB (IR-UWB) sensor nodes with pulse position modulation (PPM) is shown. The transmitter generates two streams of data symbols  $s_n^C(t)$  and  $s_n^R(t)$ .

The symbol stream  $s_n^C$  is used to transmit the local decisions  $u_n(i) \in \mathbb{F}_K$  at the time index *i* to the fusion center, which are generated by the algorithm defined in (3). We describe the data symbols by Dirac delta functions  $\delta(t - [u_n(i) - 1]\Delta)$ , which are shifted pulses on the time axis. Their alignment is determined by the modulation index  $\Delta$ . We assume that the product  $K\Delta$  is much smaller than the symbol duration. Thus, K different data symbols can be transmitted to the fusion center. The transmission power  $P_n^C$  of this stream is variable in order to adjust transmission power and to enable distributed power allocation.

The symbol stream  $s_n^R$  establishes the radiation to the target object and uses always the same data symbol. Its transmission power  $P_n^R$  is also variable. In order to increase the available power range at every SN, time-division multiple-access (TDMA) method is used to separate both streams into

different time slots and to periodically share the same power amplifier.

In order to eliminate collisions due to multiple access, each user stream is assigned to a distinctive time-shift pattern after passing through the blocks  $h_n^C(t)$  and  $h_n^R(t)$ . Their transfer functions are based on time-hopping sequences [12].

After superposition of both streams a monocyclic pulse shape filter w(t) limits the bandwidth of the signal. This filter has to fulfill the Nyquist intersymbol interference (ISI) criterion in order to avoid intersymbol interferences.

When this superposition is transmitted, a part of the radiated signal  $s_n^R$  will be reflected from the target surface back to the antenna. The received signal will pass through the matched-filter w(-t) and will be decoded from its timehopping sequence by  $h_n^R(-t)$ . The additive noise signal  $b_n^R(t)$ will pass as well through both filters at the receiver. We denote the corresponding noise power by  $P_{\text{noise}}$ . If all receiver components are linear, then we can describe the received power by

$$\tilde{P}_{n|k}^{R} \coloneqq P_{n}^{R} \frac{\alpha_{n}^{R} |r_{k}|^{2}}{g^{2}(2d_{n}^{R})}, \ \forall k \in \mathbb{F}_{K}, \forall n \in \mathbb{F}_{N},$$
(9)

where the transmitted power is weighted by the product of the factors  $\alpha_n^R > 0$ ,  $|r_k|^2$ , and  $g^{-2}(2d_n^R)$ . The factor  $\alpha_n^R$  includes the radar cross section, the influence of the antenna, the impacts of the filters, and all additional attenuation of the transmitted power. Due to the reflection coefficient  $r_k$  of the target object the received power depends on the underlying hypothesis. The path loss function g depends on the assumed multipath propagation channel and is usually an increasing function of the distance between transmitter and receiver. Here, the factor of two in the distance results from that back and forth transmission between the transceiver and the object. The ratio of  $\tilde{P}_{n|k}^R$  and  $P_{\text{noise}}$  is the observed conditional SNR at the receiver and is given by

$$\gamma_{n|k}^{R} \coloneqq \frac{P_{n}^{R}}{P_{\text{noise}}} \cdot \frac{\alpha_{n}^{R} |r_{k}|^{2}}{g^{2}(2d_{n}^{R})}, \ \forall k \in \mathbb{F}_{K}, \forall n \in \mathbb{F}_{N}.$$
(10)

Due to the Gaussian distribution of the noise each sample is also a Gaussian random variable, which is conditionally distributed according to

$$f_n^R(X_n = x_n \mid H_k) \coloneqq \frac{1}{\sqrt{2\pi P_{\text{noise}}}} \exp\left(-\frac{\left(x_n - \sqrt{\tilde{P}_{n\mid k}^R}\right)^2}{2P_{\text{noise}}}\right)$$
(11)

for all  $n \in \mathbb{F}_N$  and for all  $k \in \mathbb{F}_K$ . The local decision probabilities  $\Pr(U_n = l \mid H_k)$ , see (4) and (5), can be computed by solving the integral

$$\tilde{\pi}_{n,l|k} := \Pr(U_n = l \mid H_k) = \int_{\tau_{n,l}}^{\tau_{n,l+1}} f_n^R(x_n \mid H_k) \,\mathrm{d}x_n$$

$$= \frac{1}{2} \left[ \operatorname{erf}\left(\frac{\sqrt{\tilde{P}_{n|k}^R} - \tau_{n,l}}{\sqrt{2P_{\operatorname{noise}}}}\right) + \operatorname{erf}\left(\frac{\tau_{n,l+1} - \sqrt{\tilde{P}_{n|k}^R}}{\sqrt{2P_{\operatorname{noise}}}}\right) \right]$$
(12)

for all  $k, l \in \mathbb{F}_K$  and for all  $n \in \mathbb{F}_N$ . Here, the mapping  $\operatorname{erf}(z)$  denotes the error function of z.

$$\begin{array}{c} & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & & \\ & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & \\$$

Fig. 3. System model of the fusion center.

## D. Fusion center

After radiation of the stream  $s_n^C$  by the SN  $S_n$ , the signal is attenuated depending on the distance and it reaches the antenna at the fusion center as depicted in Fig. 3. The received signal is matched-filtered and decoded from its time-hopping sequence. Then a data splitter  $\mathbf{v}(t)$  is used to split the received signal into a K-dimensional vector space. This is necessary in order to retain the Euclidian distances between all transmitted symbols and achieve a higher classification probability. This filter is mathematically implemented as  $\sum_{k=1}^{K} \mathbf{e}_k \, \delta(t - (k-1)\Delta)$ , where  $\mathbf{e}_k$  is the standard basis vector of the K-dimensional space that points in the  $k^{\text{th}}$  direction. Therefore, the received signals  $\tilde{\mathbf{X}}_1, \ldots, \tilde{\mathbf{X}}_N \in \mathbb{R}^K$  are K-dimensional vectors. This new approach extends the method given by [12].

In case of additive zero-mean noise and due to the assumptions of w(t) each vector sample of the received signal has the expected value of

$$\mathbf{m}_{n|l} \coloneqq \mathrm{E}\big(\tilde{\mathbf{X}}_n \mid U_n = l\big) = \sqrt{P_n^C \frac{\alpha_n^C}{g^2(d_n^C)}} \cdot \mathbf{e}_l \qquad (13)$$

for all  $l \in \mathbb{F}_K$  and for all  $n \in \mathbb{F}_N$ , which depends on the transmitted symbol  $U_n = l$ . Thus, the received power from the  $n^{\text{th}}$  SN is given by

$$\tilde{P}_n^C \coloneqq P_n^C \frac{\alpha_n^C}{g^2(d_n^C)}, \ \forall n \in \mathbb{F}_N,$$
(14)

where we assume that the path loss function is the same as for the first communication link. The power  $\tilde{P}_n^C$  is independent of the underlying hypothesis because the data stream  $s_n^C$  has the same power for all kinds of transmitted data symbols.

The additive noise signal  $b_n^C(t)$  will also pass through all the filters. We assume that the noise spectral density at the fusion center is the same as at the SNs. Due to similarity in architecture of the fusion center and the SNs the noise power in each dimension of each stream is equal to  $P_{\text{noise}}$ . Because of the whiteness of noise the interferences are uncorrelated in each dimension of each stream. Therefore, the noise covariance matrix is given by the product  $P_{\text{noise}} \cdot \mathbf{I}_K$ . Here,  $\mathbf{I}_K$ denotes the identity matrix of size K.

Similar to (10) we define the observed SNR for each data stream at the fusion center and denote it by

$$\gamma_n^C \coloneqq \frac{P_n^C}{P_{\text{noise}}} \cdot \frac{\alpha_n^C}{g^2(d_n^C)}, \quad \forall n \in \mathbb{F}_N.$$
(15)

Due to the Gaussian distribution of noise each vector sample is a Gaussian random vector, which is conditionally distributed according to

$$f_n^C(\tilde{\mathbf{X}}_n \mid H_k) \coloneqq \sum_{l=1}^K \frac{\tilde{\pi}_{n,l|k}}{(2\pi P_{\text{noise}})^{K/2}} \exp\left(-\frac{(\tilde{\mathbf{x}}_n - \mathbf{m}_{n|l})^{\mathsf{T}}(\tilde{\mathbf{x}}_n - \mathbf{m}_{n|l})}{2P_{\text{noise}}}\right)$$
(16)

for all  $k \in \mathbb{F}_K$  and for all  $n \in \mathbb{F}_N$ , where the operator  $\mathbf{z}^T$  denotes the transpose of any vector  $\mathbf{z}$ .

Because of the convex superposition of multivariate Gaussian distributions it is difficult to use (16) with the properties of (2) to optimize the distributed power allocation. Bounds such as the Bhattacharyya bound [2] are also difficult to use due to the multidimensional nature of (2) and (16). Therefore, we propose an applicable technique which is motivated by concepts of information theory and is described in the next section.

## III. SUBOPTIMAL ALLOCATION OF THE TOTAL POWER

In this section, we motivate and present an approach to suboptimally allocate transmission power to the radar and to the communication tasks. The objective is to maximize the overall classification probability given a limited total transmission power  $P_{\text{tot}}$ , which can be arbitrarily allocated to the radar task as well as to the communication task. A direct solution to this problem does not exist, since no analytical expression for the overall classification probability (8) is available. Instead, we independently maximize the mutual information of both communication channels to increase the information flow and in order to determine the power allocation. The motivation for this approach is the separation of the power allocation problem from the object classification procedure. Because in this case the data communication does not affect the classification of the target object.

Note that this theoretical concept is not realistic. However, we apply this concept as a heuristical method in this work.

#### A. Threshold calculation

For the optimization of the thresholds in Section II-A in order to increase the overall classification probability the analytic evaluation of (8) is needed. Due to the fact that the explicit form for the overall classification probability is unknown and due to the separation of the data communication from the classification task we propose the following simple approach to calculate the thresholds.

We increase the probability of correct decision of each SN independently to achieve suboptimal values for the thresholds. Thus, the overall classification probability should be increased as well. According to equations (4) and (12) the local probability of correct decision, which has to be maximized, is given by

$$\sum_{k=1}^{K} \Pr(H_k) \Pr(U_n = k \mid H_k)$$
$$= \sum_{k=1}^{K} \frac{\pi_k}{2} \left[ \operatorname{erf}\left(\frac{\sqrt{\tilde{P}_{n|k}^R} - \tau_{n,k}}{\sqrt{2P_{\text{noise}}}}\right) + \operatorname{erf}\left(\frac{\tau_{n,k+1} - \sqrt{\tilde{P}_{n|k}^R}}{\sqrt{2P_{\text{noise}}}}\right) \right]. \quad (17)$$

Its solution can be found explicitly by using differential calculus. The corresponding result is identical to the one obtained by using the Bayesian-hypotheses-test criterion. It is given by

$$\tau_{n,k} = \begin{cases} \inf(\mathbb{I}_{n,k}) & \text{if} \quad \mathbb{I}_{n,k} \neq \emptyset, k \in \mathbb{F}_K, \\ \tau_{n,k+1} & \text{if} \quad \mathbb{I}_{n,k} = \emptyset, k \in \mathbb{F}_K, \\ \infty & \text{if} \quad k = K+1, \end{cases}$$
(18)

for all  $n \in \mathbb{F}_N$ , where the function  $\inf(\mathbb{I}_{n,k})$  is the infimum of the interval  $\mathbb{I}_{n,k}$  that is defined by

$$\mathbb{I}_{n,k} \coloneqq \{ x \in \mathbb{R} \mid \pi_k f_n^{\scriptscriptstyle R}(x \mid H_k) > \pi_l f_n^{\scriptscriptstyle R}(x \mid H_l), \forall l \neq k \}.$$
(19)

# B. Mutual information-based power allocation

For the maximization of the information flow we set the mutual information of both communication channels equal. This leads to the same symbol error probabilities on both sides for low SNR values. For each SN an upper bound for the mutual information of its first and second link can simply be calculated. The identity of obtained bounds

$$\frac{1}{2}\log\left[1 + \frac{P_n^R \alpha_n^R (|r_K| - |r_1|)^2}{4P_{\text{noise}} g^2 (2d_n^R)}\right] = \frac{K}{2}\log\left[1 + \frac{P_n^C \alpha_n^C (K-1)}{P_{\text{noise}} g^2 (d_n^C) K^2}\right]$$
(20)

has to be computed in order to find the relationship between the powers for all  $n \in \mathbb{F}_N$ . After calculation and usage of the approximation  $\sqrt[K]{1+x} \approx 1 + \frac{x}{K}$  for small values of x we obtain the analytical relationship

$$P_n^C = P_n^R \cdot \frac{\alpha_n^R}{\alpha_n^C} \frac{g^2(d_n^C)}{g^2(2d_n^R)} \frac{K}{K-1} \frac{(|r_K| - |r_1|)^2}{4}, \ \forall n \in \mathbb{F}_N.$$
(21)

In the next step we increase the overall mutual information by maximization of the cumulative mutual information subject to the given total power of the sensor network. Then the optimization problem is given by

$$\underset{P_{1}^{R},\dots,P_{N}^{R}}{\text{maximize}} \quad \sum_{n=1}^{N} \frac{1}{2} \log \left[ 1 + \frac{P_{n}^{R} \alpha_{n}^{R} \left( |r_{K}| - |r_{1}| \right)^{2}}{4P_{\text{noise}} g^{2} \left( 2d_{n}^{R} \right)} \right]$$
(22)

subject to  $\sum_{n=1}^{N} P_n^C + P_n^R \leq P_{\text{tot}}$ . It has to be considered that the sum of concave functions is also concave and that the arguments of the logarithms are linear functions of the powers. Furthermore, the domain of the feasible set is a closed convex set and, therefore, only one global maximum of the problem exists. This maximum can be explicitly calculated by using the method of Lagrange multipliers which is equivalent to the water-filling power allocation result [3]. The result is given by

$$P_n^R = P_{\text{noise}} \frac{g^2 (2d_n^R)}{\alpha_n^R} \frac{4}{(|r_K| - |r_1|)^2} \cdot \max\left(0, \ \frac{\lambda}{\beta_n} - 1\right)$$
(23)

for all  $n \in \mathbb{F}_N$ , where the factor  $\beta_n$  is defined by

$$\beta_n \coloneqq \frac{g^2(2d_n^R)}{\alpha_n^R} \frac{4}{(|r_K| - |r_1|)^2} + \frac{g^2(d_n^C)}{\alpha_n^C} \frac{K}{K-1}.$$
 (24)

For the following equations we assume that the factors  $\beta_n$  are ordered in an increasing manner. Then the water-filling level  $\lambda$  is a value specified by the inequality

$$\beta_{\tilde{N}} < \lambda \le \frac{1}{\tilde{N}} \left[ \frac{P_{\text{tot}}}{P_{\text{noise}}} + \sum_{n=1}^{\tilde{N}} \beta_n \right],$$
(25)

where the number  $\tilde{N}$  with  $1 \leq \tilde{N} \leq N$  is a suitably chosen integer value for which the inequality

$$\sum_{n=1}^{\tilde{N}} (\beta_{\tilde{N}} - \beta_n) < \frac{P_{\text{tot}}}{P_{\text{noise}}}$$
(26)

holds. From (21) and (23) the allocated power for the second channel is determined as

$$P_n^C = P_{\text{noise}} \frac{g^2(d_n^C)}{\alpha_n^C} \frac{K}{K-1} \cdot \max\left(0, \frac{\lambda}{\beta_n} - 1\right).$$
(27)

This allocation has the following interpretation. The SN  $S_n$  with the lowest  $\beta_n$  gets the largest part of the total power because its communication channels are possibly the best due to the low distances. Therefore, the observation of the target object is less interfered by noise and consequently results in better data communication. SNs with higher distances get smaller parts of the total power and some of them do not get any power at all. The last ones participate neither in the data communication nor in the classification of the target object. Their information reliability is too poor to be considered for data fusion. More and more SNs will become active by increasing the total power. Then the overall classification probability increases because more correct information is provided by the observations.

## **IV. NUMERICAL RESULTS**

In this section we present some numerical results obtained by applying the proposed optimization method from Section III. We simulate target objects with equal prior probabilities  $\pi_k = \frac{1}{K} \ \forall k \in \mathbb{F}_K$  in sensor networks with different settings as described in Section II. In all results, we consider three different kinds of target objects with reflection coefficients chosen as  $|r_1| = 0$ ,  $|r_2| = \frac{1}{2}$ , and  $|r_3| = 1$ . Furthermore, the path loss function is modeled as line-of-sight propagation. The ratio SNR =  $10dB \log(\frac{P_{\text{tot}}}{P_{\text{noise}}})$ , instead of *received* SNRs, is depicted on the abscissa of all figures.



Fig. 4. Verification of proposed power allocation between the two communication links of a single sensor node network.

The verification of the proposed power allocation between both communication links of a single SN is shown in Fig. 4. The overall error probability of the classification increases for higher SNR values for the case where the allocated power of one link is reduced by 10% and at the same time the power of the other link is stepped up by this 10%. When we reallocate a power amount of 10% - 30% to both links in an inverse manner, then the classification probability remains almost valid. This result shows that the proposed method allocates the given total power nearly optimal to both communication links, especially for higher SNR values.



Fig. 5. Verification of proposed power allocation between two sensor nodes.

In Fig. 5 another verification of the proposed power allocation is shown, where a network of two SNs is considered. The overall error probability of the classification decreases if we decrease the allocated power of the SN, which has the smallest part of the total power, by 10% and allocate this amount of power to the other SN. This result shows that the proposed method assigns the given total power suboptimal to the SNs. The curves disperse, because of the approximation which has been used for the equation (21).



Fig. 6. Comparison of proposed power allocation to a uniform power allocation in a network of ten sensor nodes.

As shown in Fig. 6 the proposed method yields a better classification probability in comparison to a uniform power allocation where a network of ten SNs is considered. In particular, it is shown that the same overall classification probability can be achieved with much lower transmission power, especially for low SNR values, by using an efficient power allocation method. Furthermore, the symbol-error probability of the SN with the highest part of the total power is also shown. The classification accuracy is better than the best symbol-error probability for higher SNR values, which affirms the gain of data fusion and illustrates the feasibility of object classification in this kind of distributed sensor networks.

## V. CONCLUSION

The goal of the power allocation is the maximization of the classification probability in a distributed wireless sensor network, which is based on ultra-wide bandwidth communication technology. The object classification procedure is based on a novel two-stage decision process. We have shown that any given amount of total transmission power can be analytically allocated to each node of the considered networks by using the proposed suboptimal method. Numerical results illustrate the performance of the described algorithm as well as the achieved classification probability. As the performance of the optimal power allocation is still unknown, we are not able to state how close the proposed algorithm approaches the theoretical limit.

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