Cyclic Interference Neutralization on the $2 \times 2 \times 2$ Full-Duplex Two-Way Relay-Interference Channel

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***Abstract*—A two-way relay-interference channel describes a system of four communicating transceivers with two interjacent parallel relays arranged in a bidirectional $2 \times 2 \times 2$ relay-interference network. Two pairs of transceivers are each communicating bidirectionally with the aid of both relays. All transceivers and relays are assumed to operate in full-duplex mode.**

Since *Interference Neutralization* is known as a promising method to achieve the cut-set upper bounds on the data rates of the unidirectional relay-interference channel, we investigate a *Cyclic Interference Neutralization* scheme on the corresponding bidirectional relay-interference channel w.r.t. a conceptual channel model based on a polynomial ring. We show that, if the channel matrix satisfies a certain set of symmetry conditions, a total number of 4 degrees of freedom is asymptotically achievable.

I. Introduction

The impact of multi-user interference is a long-standing and very challenging problem in wireless communication systems. In recent years, cooperative communication schemes have significantly influenced the designs of future communication concepts. The continuing progress in that area also provided the novel theoretical concepts of *Interference Alignment* (IA) [1], [2] and *Interference Neutralization* (IN) [3]–[5].

Instead of using conventional approaches to mitigate multi-user interference, e.g., orthogonalization or treating interference as noise, IA confines all the undesired interference signals into exactly one half of the signal space at each user, while the dedicated signals are received interference-free in the other half. Even though half of the given signal space is consumed by interference, this strategy remarkably outperforms conventional communication strategies when considering a large quantity of interfering users.

If signals are moreover forwarded in cooperative multi-hop networks with multiple interjacent relays, e. g., the *relay-interference channel*, IA enables IN. The basic idea of IN to cancel multiple instances of the same interference which is forwarded by different relays such that it is "erased over the air". Such an approach is capable to provide an effectively interference-free channel.

This work has been supported by the *Deutsche Forschungsgemeinschaft* (DFG) within the project *Power Adjustment and Constructive Interference Alignment for Wireless Networks* (PACIA - Ma 1184/15-2) of the DFG program *Communication in Interference Limited Networks* (COIN) and furthermore by the UMIC Research Centre, RWTH Aachen University.

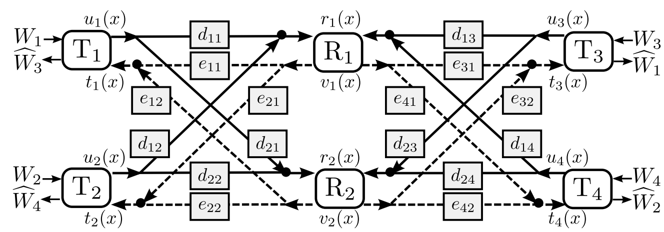

Fig. 1. The $2 \times 2 \times 2$ *full-duplex two-way relay-interference channel* in terms of the cyclic polynomial channel model: Transeivers T_i transmit signals $u_i(x)$ to relays R_j over the *uplink* channel matrix $\boldsymbol{D} = (d_{ji})_{1 \le j \le 2, 1 \le i \le 4}$ and the relays receive $r_j(x)$. Relays R_j forward signals $v_j(x)$, as functions of $r_j(x)$, over the *downlink* channel matrix $\boldsymbol{E} = (e_{ij})_{1 \le i \le 4, 1 \le j \le 2}$ to transeivers T_i who receive the corresponding $t_i(x)$.

In the recent works [6], [7], the IN scheme is also generalized for a unidirectional $K \times K \times K$ interference channel and it is termed as *Interference Diagonalization*.

Furthermore, a *two-way* channel describes a prevalently occurring communication scenario where user-pairs exchange messages bidirectionally. An exemplary *two-way relaying* scheme with a single relay is considered in [8]. The particular configuration of a $2 \times 2 \times 2$ *two-way relay-interference channel* [9], [10] considers two pairs of mutually communicating users as depicted in Fig. 1. This setup is also a generalization of the unidirectional $2 \times 2 \times 2$ relay-interference channel in [3], [5].

Both IA and IN can be motivated by the conceptual *linear deterministic channel model* [11], as in [3], [12]. The closely related *cyclic polynomial channel model* is introduced in [13], [14]. In [13], it is shown that *Cyclic IA schemes* achieve $4/3$ *degrees of freedom* (DoF) for the cyclic polynomial X- channel and $K/2$ DoF for the cyclic polynomial K- user interference channel. Note that a comparable polynomial model of a finite field X- channel is also considered in [15]. Furthermore, it is shown in our related work [14] that Cyclic IN can achieve 2 DoF on the (unidirectional) polynomial relay-interference channel. These DoF results correspond to the upper bounds given in [16], [1], [5] for Gaussian channels.

Contributions. In the present paper, we apply and further generalize the concept of Cyclic IN [14] to the full-duplex two-way $2 \times 2 \times 2$ relay-interference channel in the cyclic polynomial channel model. Our proposed scheme achieves $\frac{4n-2}{n}$ DoF for n dimensions, if the channel matrices satisfy certain symmetries.

This scheme includes network-coded signal alignment at the relays and cancellation of back-propagated self-interference. Our result corresponds to the cut-set upper bound of 4 DoF for $n \to \infty$ and supports recent results by Lee et al. in [10].

Organization. The system model is introduced in Section II. The basic concept of the two-way Cyclic IN scheme is proposed in Section III-A. In Section III-B, we investigate an *asymptotic* Cyclic IN scheme that achieves the upper bound of four DoF. We briefly discuss a feasibility problem concerning the corresponding Aligned IN scheme in Section IV and we conclude in Section V.

Notation. The operator $\operatorname{diag}(a_1, \ldots, a_n)$ specifies a diagonal matrix with the entries $a_1, \ldots, a_n$ on the main diagonal and zero entries elsewhere. A univariate polynomial of degree $n-1$ in the indeterminate x and with coefficients $p^{[i]}$ is denoted by $p(x) = \sum_{i=0}^{n-1} p^{[i]} x^i$.

II. System Model

The conceptual communication model of the cyclic polynomial channel refers to the works [13], [14] and the notation is adopted here.

A $2 \times 2 \times 2$ *two-way relay-interference channel* as given in [9] comprises of four full-duplex transceivers $\mathrm{T}_1, \mathrm{T}_2, \mathrm{T}_3, \mathrm{T}_4$, and two full-duplex relays $\mathrm{R}_1, \mathrm{R}_2$ as depicted in Fig. 1. The *set of transceiver-indices* is $\mathcal{K} = \{1,2,3,4\}$ and the *set of relay-indices* is $\mathcal{R} = \{1,2\}$. Each transceiver T_i transmits a message W_i, $i \in \mathcal{K}$. The user-pair $(\mathrm{T}_1, \mathrm{T}_3)$ desires to exchange messages W_1 and W_3 over the given channel and the user-pair $(\mathrm{T}_2, \mathrm{T}_4)$ desires to exchange messages W_2 and W_4, respectively. There are no direct links between the transceivers and no direct links between the two relays.

The *set of dedicated transmitter-indices* for a receiver T_i is denoted by $\mathcal{D}_i$, i.e., the singleton sets $\mathcal{D}_1 = \{3\}$, $\mathcal{D}_2 = \{4\}$, $\mathcal{D}_3 = \{1\}$ and $\mathcal{D}_4 = \{2\}$. We combine the indices of the two communicating user-pairs in the sets $\mathcal{G}_{13} = \{1,3\}$ and $\mathcal{G}_{24} = \{2,4\}$. The *set of interfering transmitter-indices* at a receiver R_i is denoted by $\mathcal{I}_i$, i.e., $\mathcal{I}_1 = \mathcal{I}_3 = \{2,4\}$ and $\mathcal{I}_2 = \mathcal{I}_4 = \{1,3\}$.

A transmitted signal from T_i, $i \in \mathcal{K}$, is described by a polynomial $u_i(x)$ which is limited to a number of n dimensions. A single dimension $k \in \{0, \ldots, n-1\}$ and its assigned coefficient $u^{[k]}$ is addressed by an offset x^k as follows:

$$u_i(x) = \sum_{k=0}^{n-1} u^{[k]} x^k. \tag{1}$$

Let the indices denoted in squared brackets be reduced by the modulus n for notational convenience. A message W_i, $i \in \mathcal{K}$, is partitioned into an n-dimensional vector $\boldsymbol{w}_i$ of n *submessages* $W_i^{[k]}$ with $k \in \{0, \ldots, n-1\}$:

$$\boldsymbol{w}_i = (W_i^{[0]}, W_i^{[1]}, \ldots, W_i^{[n-1]}). \tag{2}$$

The transceiver Tx_i maps the submessages $\boldsymbol{w}_i$ to the transmitted signal by $u_i(x) = \boldsymbol{w}_i \boldsymbol{x}^\mathsf{T}$. We define the vector addressing the n offsets by $\boldsymbol{x} = (x^0, x^1, \ldots, x^{n-1})$.

The influence of the wireless channel on the signal $u_i(x)$ is represented by a parameterized cyclic right-shift of the coefficients over n dimensions. For polynomials, it is common to describe such a cyclic right-shift by k positions by a multiplication with x^k and then taking the modulus $x^n - 1$.

To model individual cyclic shifts for each transmitter-receiver link, the uplink channel from the four transceivers to the two relays is described by the *uplink channel matrix* $\boldsymbol{D} = (d_{ji})_{1 \le j \le 2, 1 \le i \le 4}$ and the downlink is described by the *downlink channel matrix* $\boldsymbol{E} = (e_{ij})_{1 \le i \le 4, 1 \le j \le 2}$ with $d_{ji}, e_{ji} \in \mathcal{D} := \{x^k \mid k \in \mathbb{N}\}$, respectively. These coefficients are assumed to be static over the whole communication period. The channel matrices are fully and globally known. We denote the offsets by δ_{ji}, $\eta_{ji} \in \mathbb{N}$, i.e., $d_{ji} = x^{\delta_{ji}}$ and $e_{ij} = x^{\eta_{ij}}$.

In both two-hop and two-way relay communication systems, the channel access is described by two different access phases: The *multiple-access phase* or *first hop* describes the communication from transceivers to relays and the *broadcast phase* or *second hop* describes the communication from relays to transceivers, accordingly. For multiple-relays we will term these steps by the *uplink-phase* (UL-phase) and the *downlink-phase* (DL-phase).

1) UL-phase: The sources T_i, $i \in \mathcal{K}$, map the message W_i to a polynomial $u_i(x)$. The polynomials $u_i(x)$ are transmitted to the relays R_j, $j \in \mathcal{R}$, over the uplink matrix $\boldsymbol{D}$ so that the relays R_j receive a superposition of interfering polynomials:

$$r_j(x) = \sum_{i=1}^{4} d_{ji} u_i(x) \bmod (x^n - 1). \tag{3}$$

2) DL-phase: The relays R_j use a causal relaying function on their received polynomials $r_j(x)$ which are mapped to the polynomials $v_j(x)$. Then, the relays R_j forward $v_j(x)$ to the destinations T_i, $i \in \mathcal{K}$, over the downlink matrix $\boldsymbol{E}$. The destinations T_i receive the following superposition:

$$t_i(x) = \sum_{j=1}^{2} e_{ij} v_j(x) \bmod (x^n - 1). \tag{4}$$

The superposition of those polynomials that are not dedicated for a destination T_i causes undesired interference. Only if all dedicated signals can be received interference-free, the four destinations can decode their dedicated messages to obtain $\widehat{W}_1, \widehat{W}_2, \widehat{W}_3$ and $\widehat{W}_4$, respectively.

For notational convenience, the transmission vector of the UL-phase is denoted by $\boldsymbol{u} = (u_1(x), u_2(x), u_3(x), u_4(x))$ and the received vector is denoted by $\boldsymbol{r} = (r_1(x), r_2(x))$. In the DL-phase, we utilize the vectors $\boldsymbol{v} = (v_1(x), v_2(x))$ and $\boldsymbol{t} = (t_1(x), t_2(x), t_3(x), t_4(x))$. Then, the transfer functions of the given channel are compactly expressed by:

$$\boldsymbol{r}^\mathsf{T} = \boldsymbol{D}\boldsymbol{u}^\mathsf{T} \bmod (x^n - 1), \tag{5}$$

$$\boldsymbol{t}^\mathsf{T} = \boldsymbol{E}\boldsymbol{v}^\mathsf{T} \bmod (x^n - 1), \tag{6}$$

where the modulo operation is taken component-wise.

To evaluate the achieved data rate, the metric of the *degrees of freedom* (DoF) is defined as the maximal number M of messages conveyed interference-free over the channel in n dimensions [13], [14]:

$$\mathrm{DoF} = \frac{M}{n}. \tag{7}$$

III. Cyclic Two-Way Interference Neutralization

For an interference-free transmission between four users with n dimensional signals, a total number of $M = 4n$ independent submessages must be decodable, i. e., n messages per user. Then a total number of exactly 4 DoF would be achieved. Such a result corresponds to the cut-set upper bound on the DoF of the $2 \times 2 \times 2$ two-way relay-interference channel. In the following subsections, we propose an Cyclic IN scheme that achieves the given bound in the asymptotical limit for $n \to \infty$.

A. Cyclic IN for the Two-Way $2 \times 2 \times 2$ Channel

Firstly, we introduce the conditions of a Cyclic IN scheme.

1) UL-phase: Each transceiver T_i, $i \in \mathcal{K}$, transmits $u_i(x)$. The relays R_1 and R_2 receive the following superposition of four submessages per dimension k as given by (3) for $j \in \mathcal{R}$:

$$r_j^{[k]} = \sum_{i=1}^{4} W_i^{[k-\delta_{ji}]}. \tag{8}$$

The relays R_1, R_2 forward their received messages as follows:

$$v_1(x) = \quad x^{\gamma_1} r_1(x) \bmod (x^n - 1), \tag{9}$$
$$v_2(x) = -x^{\gamma_2} r_2(x) \bmod (x^n - 1). \tag{10}$$

The parameters $\gamma_i \in \mathbb{N}$ are included to enable an internal cyclic shift within the relays R_i. The change of sign at relay R_2 is essential to provide the complementary signals for IN.

2) DL-phase: As both relays forward four messages each, there are eight submessages received at each destination T_j, $j \in \mathcal{K}$, per dimension k:

$$t_j^{[k]} = \sum_{i=1}^{4} W_i^{[k-\delta_{1i}-\gamma_1-\eta_{j1}]} - W_i^{[k-\delta_{2i}-\gamma_2-\eta_{j2}]}. \tag{11}$$

With $\boldsymbol{\Gamma} = \mathrm{diag}(x^{\gamma_1}, -x^{\gamma_2})$, this is compactly expressed by:

$$\boldsymbol{t}^{\mathsf{T}} = \boldsymbol{E}\boldsymbol{\Gamma}\boldsymbol{D}\boldsymbol{u}^{\mathsf{T}} \bmod (x^n - 1). \tag{12}$$

Those submessages $\boldsymbol{w}_i$ that are back-propagated from the relays to their original transceiver T_i during the DL-phase are called *back-propagated self-interference* [9], [10]. Since the transceivers T_j know their own signals transmitted in the previous UL-phase a priori, they can completely cancel their corresponding self-interference. By taking such a cancellation into account, the received signal (11) at T_j yields:

$$t_j^{[k]} = \sum_{i=1, i \neq j}^{4} W_i^{[k-\delta_{1i}-\gamma_1-\eta_{j1}]} - W_i^{[k-\delta_{2i}-\gamma_2-\eta_{j2}]}. \tag{13}$$

Note that the self-interference is forwarded by both relays.

We further demand that the *inter-user interference* caused by the undesired transceivers in $\mathcal{I}_i$ is neutralized. The essential idea of IN is to combine two identical inter-user interference signals with complementary signs within the same dimension k, such that their sum is zero [14]. Thence, the *interference-neutralization conditions* for all interfering pairs, with $i \in \mathcal{K}$, $j \in \mathcal{I}_i$, are:

$$\delta_{1i} + \gamma_1 + \eta_{j1} \equiv \delta_{2i} + \gamma_2 + \eta_{j2} \pmod n. \tag{14}$$

This concept is also illustrated in Fig. 2.

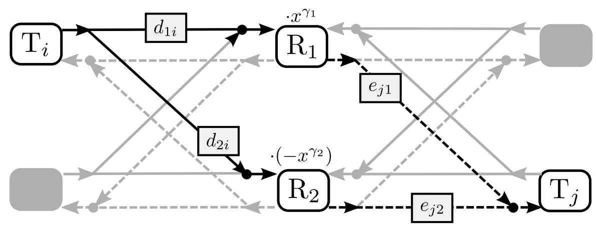

Fig. 2. The interference-neutralization conditions in (14) demand that the identical signals transmitted by T_i coincide at each undesired transceivers T_j with complementary signs, so that interference is "erased over the air".

However, dedicated submessages may not be neutralized and must remain decodable. Accordingly, the *no-signal-neutralization conditions* for all dedicated pairs, with $i \in \mathcal{K}$, $j \in \mathcal{D}_i$, are:

$$\delta_{1i} + \gamma_1 + \eta_{j1} \not\equiv \delta_{2i} + \gamma_2 + \eta_{j2} \pmod n. \tag{15}$$

Altogether, assuming that self-interference is removed and the conditions (14) and (15) hold, the transceivers T_j only receive a superposition of two dedicated submessages per dimension k:

$$t_j^{[k]} = W_i^{[k-\delta_{1i}-\gamma_1-\eta_{j1}]} - W_i^{[k-\delta_{2i}-\gamma_2-\eta_{j2}]}, j \in \mathcal{D}_i. \tag{16}$$

Using the vectorized notation, this yields $t_j(x) = (\boldsymbol{X}\boldsymbol{C}_j)\boldsymbol{w}_j^{\mathsf{T}}$ with coefficient matrix $\boldsymbol{C}_j = (c_{j,lm})_{0 \le l,m \le n-1}$ with row index l and column index m and $\boldsymbol{X} = \mathrm{diag}(x^0, x^1, \ldots, x^{n-1})$. For $i \in \mathcal{D}_j$, $\boldsymbol{C}_j$ is a *circulant matrix* with two non-zero bands:

$$c_{j,lm} = \begin{cases} 1 & \text{, if } m - l \equiv \delta_{1i} + \gamma_1 + \eta_{j1} \pmod n, \\ -1 & \text{, if } m - l \equiv \delta_{2i} + \gamma_2 + \eta_{j2} \pmod n, \\ 0 & \text{, else.} \end{cases} \tag{17}$$

The received submessages are decodable by *linear decoding* if $\det(\boldsymbol{C}_j) \neq 0$ holds. We can decompose the two-way $2 \times 2 \times 2$ relay-interference channel into four (unidirectional) relay-interference channels:

1) The dedicated links are $T_1 \to T_3$ and $T_2 \to T_4$.
2) The dedicated links are $T_3 \to T_1$ and $T_4 \to T_2$.
3) The dedicated links are $T_1 \to T_3$ and $T_4 \to T_2$.
4) The dedicated links are $T_3 \to T_1$ and $T_2 \to T_4$.

In [14, Lem. 1], we have already shown that Cyclic IN is not possible if all transmitting users allocate n submessages in n dimensions in a unidirectional $2 \times 2 \times 2$ relay-interference channel. Thus, as none of the four contained unidirectional relay-interference channels supports Cyclic IN for the case of n submessages per user, the two-way relay-interference channel does not either.

B. Asymptotic Cyclic Interference Neutralization

Anyhow, in order to enable Cyclic IN with linear decoding, we propose an *asymptotic* Cyclic IN scheme that generalizes the asymptotic Cyclic IN scheme in [14] to the given $2 \times 2 \times 2$ two-way relay-interference channel:

1) UL-phase: The transceivers $\mathrm{T}_1, \mathrm{T}_3$ transmit n submessages whereas $\mathrm{T}_2, \mathrm{T}_4$ only transmit $n - 1$ submessages, discarding the submessages $W_2^{[\tau_2]}, W_4^{[\tau_4]}$ for the pair of parameters $\tau_2, \tau_4 \in \{1, \dots, n-1\}$:

$$u_i(x) = \sum_{k=0}^{n-1} W_i^{[k]} x^k, \;\; i = 1, 3, \tag{18}$$

$$u_i(x) = \sum_{k=0, k\neq\tau_i}^{n-1} W_i^{[k]} x^k, \;\; i = 2, 4. \tag{19}$$

The submessages per dimension k received at the two relays correspond to (8), except for the cases with $j \in \mathcal{R}$, $m \in \{2, 4\}$:

$$r_j^{[\tau_m + \delta_{jm}]} = \sum_{i=1, i\neq m}^{4} W_i^{[\tau_m + \delta_{jm} - \delta_{ji}]}. \tag{20}$$

We choose the parameters τ_2, τ_4 such that

$$\kappa_2 := \tau_2 + \delta_{22} \equiv \tau_4 + \delta_{24} \pmod n \tag{21}$$

holds, i. e., both the discarded submessages will affect exactly one dimension κ_2 at receiver R_2. Accordingly, we define $\kappa_{12} \equiv \tau_2 + \delta_{12} \pmod n$ and $\kappa_{14} \equiv \tau_4 + \delta_{14} \pmod n$ to describe the dimensions at R_1 that are affected by the discarded submessages from T_2 and T_4. Due to the interference-neutralization conditions, the dimensions of these discarded submessages are also aligned at the relay R_1. To show this, we consider (14) for $i \in \{2, 4\}$ and $j = 1$:

$$\delta_{12} + \gamma_1 + \eta_{11} \equiv \delta_{22} + \gamma_2 + \eta_{12} \pmod n,$$
$$\delta_{14} + \gamma_1 + \eta_{11} \equiv \delta_{24} + \gamma_2 + \eta_{12} \pmod n.$$

By substituting γ_1, we easily obtain:

$$\delta_{12} - \delta_{14} \equiv \delta_{22} - \delta_{24} \pmod n. \tag{22}$$

It follows from (21) and (22) that $\tau_2 + \delta_{12} \equiv \tau_4 + \delta_{14} \pmod n$ holds, i. e., we may set $\kappa_1 := \kappa_{12} \equiv \kappa_{14} \pmod n$.

2) DL-phase: Relay R_1 forwards its received polynomial $r_1(x)$ according to (9). Relay R_2 forwards only $n-1$ dimensions of the received polynomial $r_2(x)$ and discards $r_2^{[\kappa_2]}$:

$$v_2(x) = -x^{\gamma_2} \sum_{k=0, k\neq\kappa_2}^{n-1} r_2^{[k]} x^k \bmod (x^n - 1). \tag{23}$$

Let $\sigma_{ji} = \kappa_i + \gamma_i + \eta_{ji}$. The received dimensions at the destinations T_j are as given in (16). The following cases result from the discarded coefficients, and self-interference cancellation for $j \in \mathcal{G}_{13}, i \in \mathcal{D}_j$:

$$\begin{aligned} t_j^{[\sigma_{j1}]} = \;& W_i^{[\sigma_{j1} - \delta_{1i} - \gamma_1 - \eta_{j1}]} - W_i^{[\sigma_{j1} - \delta_{2i} - \gamma_2 - \eta_{j2}]} \\ & - W_2^{[\sigma_{j1} - \delta_{22} - \gamma_2 - \eta_{j2}]} \\ & + W_4^{[\sigma_{j1} - \delta_{14} - \gamma_1 - \eta_{j1}]} - W_4^{[\sigma_{j1} - \delta_{24} - \gamma_2 - \eta_{j2}]}, \end{aligned} \tag{24}$$

$$\begin{aligned} t_j^{[\sigma_{j2}]} = \;& W_i^{[\sigma_{j1} - \delta_{1i} - \gamma_1 - \eta_{j1}]} - W_i^{[\sigma_{j2} - \delta_{2i} - \gamma_2 - \eta_{j2}]} \\ & + W_2^{[\sigma_{j1} - \delta_{12} - \gamma_1 - \eta_{j1}]} \\ & + W_4^{[\sigma_{j1} - \delta_{14} - \gamma_1 - \eta_{j1}]} - W_4^{[\sigma_{j1} - \delta_{24} - \gamma_2 - \eta_{j2}]}, \end{aligned} \tag{25}$$

and for $j \in \mathcal{G}_{24}, i \in \mathcal{D}_j$:

$$\begin{aligned} t_j^{[\sigma_{j1}]} = \;& W_1^{[\sigma_{j1} - \delta_{11} - \gamma_1 - \eta_{j1}]} - W_1^{[\sigma_{j1} - \delta_{21} - \gamma_2 - \eta_{j2}]} \\ & - W_i^{[\sigma_{j1} - \delta_{2i} - \gamma_2 - \eta_{j2}]} \\ & + W_3^{[\sigma_{j1} - \delta_{13} - \gamma_1 - \eta_{j1}]} - W_3^{[\sigma_{j1} - \delta_{23} - \gamma_2 - \eta_{j2}]}, \end{aligned} \tag{26}$$

$$\begin{aligned} t_j^{[\sigma_{j2}]} = \;& W_1^{[\sigma_{j2} - \delta_{11} - \gamma_1 - \eta_{j1}]} + W_i^{[\sigma_{j2} - \delta_{1i} - \gamma_1 - \eta_{j1}]} \\ & + W_3^{[\sigma_{j2} - \delta_{13} - \gamma_1 - \eta_{j1}]}. \end{aligned} \tag{27}$$

By further including the interference-neutralization conditions from (14), the equations (24) and (25) reduce to:

$$t_j^{[\sigma_{j1}]} = W_i^{[\sigma_{j1} - \delta_{1i} - \gamma_1 - \eta_{j1}]}, i \neq j \in \mathcal{G}_{13}, \tag{28}$$

$$t_j^{[\sigma_{j2}]} = W_i^{[\sigma_{j2} - \delta_{2i} - \gamma_2 - \eta_{j2}]}, i \neq j \in \mathcal{G}_{13}. \tag{29}$$

By definition of σ_{ji} and by condition (14), we observe that (26) and (27) coincide for each $j \in \mathcal{G}_{13}$.

According simplifications also apply to (26) and (27):

$$t_j^{[\sigma_{j1}]} = -W_i^{[\sigma_{j1} - \delta_{2i} - \gamma_2 - \eta_{j2}]}, i \neq j \in \mathcal{G}_{24}, \tag{30}$$

$$\begin{aligned} t_j^{[\sigma_{j2}]} = \;& W_i^{[\sigma_{j2} - \delta_{1i} - \gamma_1 - \eta_{j1}]} + W_1^{[\sigma_{j2} - \delta_{11} - \gamma_1 - \eta_{j1}]} \\ & + W_3^{[\sigma_{j2} - \delta_{13} - \gamma_1 - \eta_{j1}]}, i \neq j \in \mathcal{G}_{24}. \end{aligned} \tag{31}$$

The following theorem generalizes the (unidirectional) Cyclic IN scheme in [14, Thm. 2] to the present two-way case.

Theorem 1. *Asymptotic Cyclic Interference Neutralization on the $2 \times 2 \times 2$ full-duplex two-way relay-interference channel achieves $\frac{4n-2}{n}$ DoF if all the following conditions hold:*

(a) *backpropagated self-interference is cancelled at each T_i,*
(b) *the separability conditions* (14) *and* (15) *hold,*
(c) *and the number of signalling dimensions is $n \geq 2$.*

Proof: The n dedicated submessages received at T_j, $j \in \mathcal{G}_{13}$, are described by (16) and by the exceptions in (28) and (29). Now, the corresponding coefficient matrices $\boldsymbol{C}_j$ have almost the same structure as (17) except that the single entry in row σ_{j1} and column $\sigma_{j1} - \delta_{1i} - \gamma_1 - \eta_{j1}$ is zero. In this case, all n submessages at T_j with $j \in \mathcal{G}_{13}$ are decodable since $\det(\boldsymbol{C}_j) = 1$ holds as in [14] for the unidirectional case.

The $n-1$ dedicated submessages at T_j, $j \in \mathcal{G}_{24}$, are also decodable. In this case, it suffices to consider a reduced $(n-1) \times (n-1)$ coefficient matrices $\widetilde{\boldsymbol{C}}_j$ since only $n-1$ submessages per transceiver must be decoded. Moreover, the interference in the remaining dimension is not neutralized anyway. In particular, the entry in row σ_{j2} and column with $W_i^{[\tau_i]}$, $j \in \mathcal{D}_i, j \neq i \in \mathcal{G}_{24}$ is discarded. Then, $\det(\widetilde{\boldsymbol{C}}_j) = 1$ for $j \in \mathcal{G}_{13}$ as analogously shown in [14].

By considering the derivation of (22), we observe that the proposed interference-neutralization conditions demand a particular symmetry of the considered channel. We subsume the symmetry for all analogous cases by the parameters $\alpha_1, \alpha_2, \beta_1, \beta_2 \in \mathbb{N}$:

$$\alpha_1 := \delta_{11} - \delta_{21} \equiv \delta_{13} - \delta_{23} \pmod n, \tag{32}$$

$$\alpha_2 := \delta_{12} - \delta_{22} \equiv \delta_{14} - \delta_{24} \pmod n, \tag{33}$$

$$\beta_1 := \eta_{11} - \eta_{12} \equiv \eta_{31} - \eta_{32} \pmod n, \tag{34}$$

$$\beta_2 := \eta_{21} - \eta_{22} \equiv \eta_{41} - \eta_{42} \pmod n. \tag{35}$$

Using this parameterization, the interference-neutralization conditions yield:

$$\alpha_1 + \gamma_1 \equiv \gamma_2 - \beta_2 \pmod n, \tag{36}$$

$$\alpha_2 + \gamma_1 \equiv \gamma_2 - \beta_1 \pmod n, \tag{37}$$

and the no-signal-neutralization conditions are:

$$\alpha_1 + \gamma_1 \not\equiv \gamma_2 - \beta_1 \pmod n, \quad (38)$$
$$\alpha_2 + \gamma_1 \not\equiv \gamma_2 - \beta_2 \pmod n. \quad (39)$$

Substituting (36) and (37) into (38) and (39) yields in both cases:

$$\alpha_1 - \alpha_2 \not\equiv \beta_2 - \beta_1 \pmod n. \quad (40)$$

Valid matrices that fulfill these simplified conditions clearly exist if $n \geq 2$ as demanded by (c).

Altogether, $4n-2$ submessages are conveyed interference-free over n dimensions. In the asymptotic limit, the Cyclic IN scheme achieves $\lim_{n\to\infty} \frac{4n-2}{n} = 4$ DoF on the two-way $2\times2\times2$ relay-interference channel. □

IV. Discussion

In contrast to our work on the Cyclic IN scheme on the (unidirectional) relay-interference channel [14], the present Cyclic IA scheme can not be translated to a corresponding *Aligned IN* scheme in exactly the same manner.

The main problem, we encouter is that the signals from the transceivers T_2 and T_4 aligned at R_1 are exactly the same ones aligned at R_2, since $\kappa_{12} \equiv \kappa_{14} \pmod n$ holds as given in Section III. Although this problem is not an issue for the cyclic polynomial channel representation or the related linear deterministic channel representation, we conjecture that this is an overconstrained problem for the Aligned IN framework [5] based on spatial IA [1].

Just recently, it has been shown that the cut-set upper bound of $4M$ DoF is achievable, but under the condition that the relays are equipped with a greater number of $N > \frac{4}{3}M$ antennas by using the particular IN scheme provided in [10]. An interesting problem that remains to be solved is whether an Aligned IN scheme is capable to achieve the cut-set upper bound of $4M$ DoF on the related Gaussian MIMO channel model with only $M = N$ antennas at the relays.

V. Conclusions

We combine the concepts of *Cyclic Interference Neutralization* and of *two-way relaying* and apply them to the $2\times2\times2$ two-way relay-interference channel. It is shown that the presented scheme asymptotically achieves the cut-set upper bound of 4 degrees of freedom for a given symmetry of the channel matrix.

We utilize a conceptual channel model that describes the signals in terms of cyclically shifted polynomials in a polynomial ring. The channel access of the proposed Cyclic Interference Neutralization scheme is described by an uplink-phase and a downlink-phase which correspond to the first and second hop of the interference neutralization scheme of the unidirectional $2\times2\times2$ relay-interference channel but also to the *multiple-access-phase* and the *broadcast-phase* of current two-way relaying schemes with single relays. In order to obtain an interference-free communication, we presume that the *interference-neutralization conditions* and the *no-signal-neutralization conditions* hold. Furthermore, each transceiver must be capable to cancel self-interference which is backpropagated from the relays.

We point out in our discussion, that the presented Cyclic IN scheme is not yet fully applicable to the corresponding counter-part of the Gaussian channel model. Nonetheless, the proposed IN scheme is dedicated to address conceptual problems and highlight further opportunities for the two-way $2\times2\times2$ relay-interference channel.

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