Robust Multi-User Decode-and-Forward Relaying with Full-Duplex Operation

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***Abstract*—In this paper we propose a design framework for a multi-user full-duplex (FD) relaying system, which operates with decode-and-forward (DF) protocol. Our study starts with defining a system model which encompasses the limits of a FD system to overcome its own loopback self-interference. Afterwards we present a fair design strategy for the defined system focusing on the case where perfect channel state information (CSI) is available at the relay. Furthermore, we generalize our solution to the case with erroneous CSI following the wost-case enhancement approach. In the end the proposed methods are evaluated via numerical simulations and the obtained gains are observed.**

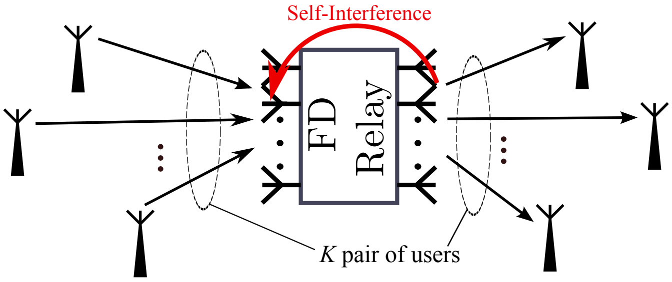

Fig. 1. The system under investigation. K pairs of nodes are communication with the help of a full-duplex one-way relay with decode-and-forward protocol.

I. Introduction

The tempting idea of full-duplex (FD) communications, as the ability to establish two directions of communication at the same time and frequency, has been long considered to be infeasible due to the inherent self-interference. In theory, since each node is aware of its own transmitted signal, the interference from the loopback path can be estimated and suppressed. However, in practice this procedure is challenging due to the high strength of the self-interference channel, limited channel state information (CSI) precision, as well as the inaccuracies in the Rx and Tx chains (e.g., power amplifier nonlinearity, oscillator phase noise, limited analog to digital convertor (ADC) and digital to analog convertor (DAC) precision, ...). These sources of inaccuracy, while being ignorable in many classic Half-Duplex (HD) communication schemes, can be harmful for FD operation. The reason stems in the fact that due to the proximity of transmit (Tx) and receive (Rx) antennas on the same node, the aforementioned interference is passing through a much stronger channel compared to the desired. Recently, via specialized designs, [1]–[8] have provided an adequate level of isolation between Tx and Rx directions to facilitate a FD communication. A common idea of these approaches is the accurate attenuation of main interference components in RF (prior to down-conversion), so that the remaining self-interference can be correctly processed in the effective dynamic range of the ADC and further attenuated in the digital domain. The reported result in [6] promises the suppression of self-interference down to the receiver noise floor for short distance scenarios throughout the bandwidth of 80 MHz. Hence investigating the possible gains by applying FD capability on the classic HD scenarios is becoming more promising. As an interesting use case, [9]–[14] have studied the FD gains and methodologies for scenarios of multi-hop wireless communication. As it has been shown, the majority of FD relaying scenarios while benefiting from the low-delay and efficient nature of FD relays, remain compatible with the HD operation of end users. As the main contribution in this work, we extend our previous work [14] to the case that a full-duplex decode-and-forward (DF) relay is shared among multiple pairs of communicating users. We start our work by defining a system model in Section II incorporating the FD decode-and-forward (DF) operation in presence of channel knowledge inaccuracy. The limits of a FD system to overcome its loopback self-interference is modeled following recent works [9], [15]. In Section III we define our optimization strategy and enhance the system performance assuming the availability of perfect CSI at the relay. In Section IV we expand our solution in Section III for the scenario that perfect channel knowledge is not available. The performance of the defined system operation is evaluated in Section V and the effectiveness of the obtained transmit strategies are evaluated via numerical simulations.

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Notations: Throughout this paper, The rank of a matrix, expectation and trace are denoted by $\mathrm{rank}(\cdot)$, $\mathbb{E}(\cdot)$, $\mathrm{Tr}(\cdot)$, respectively. The $\mathrm{vec}(\cdot)$ operator stacks the elements of a matrix into a vector. The set of all positive semi-definite matrices with Hermitian symmetry is denoted by $\mathcal{H}$.

II. System Model

We investigate a scenario where K pairs of single antenna HD users (K sources and K destinations) communicate via a one-way FD relay with N transmit and N receive antennas (Fig. 1). The relay is operating in DF mode and the direct paths between the end users are assumed to be ignorable. Channels are following the uncorrelated Rayleigh flat-fading model where $\boldsymbol{h}_{\mathrm{sr},k} \in \mathbb{C}^N$ denotes the channel between the k-th source and the relay, $\boldsymbol{h}_{\mathrm{rd},k}^{\mathrm{T}} \in \mathbb{C}^{1\times N}$ denotes the channel between the relay and k-th destination and $\boldsymbol{H}_{\mathrm{rr}} \in \mathbb{C}^{N\times N}$ is the self-interference channel. We denote the variance of the channel coefficients in the corresponding paths as $\rho_{\mathrm{sr},k}, \rho_{\mathrm{rd},k}, \rho_{\mathrm{rr}} \in \mathbb{C}^N$. Recent works [3], [16] show that the accurate estimation of communication and interference channels are particularly important for a functional FD system in order to establish a successful communication and self-interference subtraction. This stems from the simple fact that for a FD device, any CSI inaccuracy in the loopback path results in the inaccurate interference estimation. We denote the estimated versions of the defined channels as $\hat{\boldsymbol{h}}_{\mathrm{sr},k}, \hat{\boldsymbol{h}}_{\mathrm{rd},k}, \hat{\boldsymbol{H}}_{\mathrm{rr}}$, and the corresponding estimation errors as $\boldsymbol{\delta}_{\mathrm{sr},k}, \boldsymbol{\delta}_{\mathrm{rd},k}, \boldsymbol{\Delta}_{\mathrm{rr}}$, respectively. Similar to [17] we follow the so-called *deterministic model* for the estimation error which defines a feasible error region in the form of ellipsoid for each path:

$$\mathcal{D}_{\mathrm{sr},k} \triangleq \left\{ \boldsymbol{\delta}_{\mathrm{sr},k} : \ \boldsymbol{\delta}_{\mathrm{sr},k}^{\mathrm{H}} \boldsymbol{T}_{\mathrm{sr},k} \boldsymbol{\delta}_{\mathrm{sr},k} \leq \xi_{\mathrm{sr},k}^2 \right\}, \ \ \forall k \in \mathbb{F}_K, \quad (1)$$

$$\mathcal{D}_{\mathrm{rd},k} \triangleq \left\{ \boldsymbol{\delta}_{\mathrm{rd},k} : \ \boldsymbol{\delta}_{\mathrm{rd},k}^{\mathrm{T}} \boldsymbol{T}_{\mathrm{rd},k} \boldsymbol{\delta}_{\mathrm{rd},k}^{*} \leq \xi_{\mathrm{rd},k}^2 \right\}, \ \ \forall k \in \mathbb{F}_K, \quad (2)$$

$$\mathcal{D}_{\mathrm{rr}} \triangleq \left\{ \boldsymbol{\Delta}_{\mathrm{rr}} : \ \mathrm{Tr}\left(\boldsymbol{\Delta}_{\mathrm{rr}} \boldsymbol{T}_{\mathrm{rr}} \boldsymbol{\Delta}_{\mathrm{rr}}^{\mathrm{H}} \right) \leq \xi_{\mathrm{rr}}^2 \right\}, \ \ \forall k \in \mathbb{F}_K, \quad (3)$$

where $\mathbb{F}_K$ is the index set of all communicating pairs and $\boldsymbol{T}_{\mathrm{sr},k}, \boldsymbol{T}_{\mathrm{rd},k}, \boldsymbol{T}_{\mathrm{rr}} \in \mathbb{C}^{N \times N}$ are positive-definite matrices with Hermitian symmetry which shape the error's feasibility region. The values $\xi_{\mathrm{sr},k}, \xi_{\mathrm{rd},k}, \xi_{\mathrm{rr}} \in \mathbb{R}^+$ define the radius of the error feasibility region and are dependent on the quality of estimation process. The benefits of using this model for similar use cases is justified in [18] and [19]. The relay node continuously receives the transmitted signal from the source nodes while dealing with the loopback interference signal from its own Tx front-end:

$$\begin{aligned} \boldsymbol{r}_{\mathrm{in}} &= \sum_{\forall k \in \mathbb{F}_K} \left(\boldsymbol{h}_{\mathrm{sr},k} \sqrt{P_k} \cdot s_k \right) \quad + \quad \boldsymbol{H}_{\mathrm{rr}} \boldsymbol{r}_{\mathrm{out}} + \boldsymbol{n}_{\mathrm{r}} \\ &= \sum_{\forall k \in \mathbb{F}_K} \left(\hat{\boldsymbol{h}}_{\mathrm{sr},k} + \boldsymbol{\delta}_{\mathrm{sr},k} \right) \sqrt{P_k} \cdot s_k \quad + \quad \underbrace{\boldsymbol{\Delta}_{\mathrm{rr}} \boldsymbol{r}_{\mathrm{out}},}_{\mathrm{residual\ self-inerference}} \\ &\qquad + \boldsymbol{n}_{\mathrm{r}} + \underbrace{\hat{\boldsymbol{H}}_{\mathrm{rr}} \boldsymbol{r}_{\mathrm{out}},}_{\mathrm{estimated\ self-interference}} \end{aligned} \quad (4)$$

where $\boldsymbol{r}_{\mathrm{in}}, \boldsymbol{r}_{\mathrm{out}} \in \mathbb{C}^N$ are the received and transmitted signals from the relay node, respectively. The transmit data symbol and power of the k-th source node are respectively denoted as $s_k \in \mathbb{C}, \ P_k \in \mathbb{R}^+$ where $\mathbb{E}\{s_k s_k^*\} = 1$. The zero-mean circularly symmetric complex Gaussian (ZMCSCG) noise on the relay node with variance σ_{nr}^2 is denoted as $\boldsymbol{n}_{\mathrm{r}} \in \mathbb{C}^N$. While recent cancellation techniques (e.g., [6] for WiFi 802.11.ac) provide suppression of the self-interference signal down to the receiver noise floor, the offered methods are not yet successful for high power communications. Hence, following [9], [15] and [20] we assume that the *known* part of the self-interference can be subtracted if its power does not exceed the functional range of the receiver, applying cancellation methods in the receiver side [2]–[6]. Hence we have

$$\boldsymbol{r}_{\mathrm{in,supp}} = \sum_{\forall k \in \mathbb{F}_K} \left(\boldsymbol{h}_{\mathrm{sr},k} \cdot \sqrt{P_k} \cdot s_k \right) + \boldsymbol{n}_{\mathrm{r}} + \boldsymbol{\Delta}_{\mathrm{rr}} \boldsymbol{r}_{\mathrm{out}}, \quad (5)$$

$$\mathbb{E}\left\{ \left\| \hat{\boldsymbol{H}}_{\mathrm{rr}} \boldsymbol{r}_{\mathrm{out}} \right\|_2^2 \right\} \leq P_{\mathrm{int}}, \quad (6)$$

where $\boldsymbol{r}_{\mathrm{in,supp}}$ is the interference-suppressed version of the received signal and P_{int} defines the system interference power constraint which ensures successful receiver operation in facing with loopback interference. Afterwards, the transmitted symbol from the each source node is decoded at the relay, applying a linear fusion filter $\boldsymbol{f}_k \in \mathbb{C}^N$. Although linear reception strategies are not necessarily optimal, they are simple to implement and provide an efficient design strategy:

$$\hat{s}_k = \mathcal{P}\{m_k\} \ \ (\forall k \in \mathbb{F}_K), \quad (7)$$

where $m_k := \boldsymbol{f}_k^{\mathrm{H}} \cdot \boldsymbol{r}_{\mathrm{in,supp}}$ is the resulting decision variable on the relay's receiver-end corresponding to k-th communicating pair. The maximum likelihood detection operator and the decoded symbol are respectively represented as $\mathcal{P}\{\cdot\}, \ \hat{s}_k, \ \forall k \in \mathbb{F}_K$. The decoded symbols in the relay are then amplified by the corresponding relay amplification vectors ($\boldsymbol{w}_k \in \mathbb{C}^N, \ \forall k \in \mathbb{F}_K$) and constitute the relay's transmit signal:

$$\boldsymbol{r}_{\mathrm{out}} = \sum_{\forall k \in \mathbb{F}_K} \boldsymbol{w}_k \hat{s}_k. \quad (8)$$

Finally, the destination nodes receive and decode the transmitted signal from the relay

$$y_k = \boldsymbol{h}_{\mathrm{rd},k}^{\mathrm{T}} \boldsymbol{r}_{\mathrm{out}} + n_{\mathrm{d},k}, \ \ \forall k \in \mathbb{F}_K, \quad (9)$$

where $y_k, n_{\mathrm{d},k} \in \mathbb{C}$ respectively represent the received signal and additive ZMCSCG noise signal with variance σ_{nd}^2 at the destination corresponding to the k-th user pair. In addition to the defined interference power constraint, relay node is limited by its maximum allowed transmit power:

$$\mathbb{E}\left\{ \|\boldsymbol{r}_{\mathrm{out}}\|_2^2 \right\} \leq P_{\mathrm{max}}^{\mathrm{R}}, \quad (10)$$

where $P_{\mathrm{max}}^{\mathrm{R}}$ defines the maximum allowed Tx power in the relay. In the following parts of this paper, we investigate the optimal set of relaying parameters ($\boldsymbol{f}_k, \boldsymbol{w}_k, \ \forall k \in \mathbb{F}_K$), which results in the fair enhancement of communication quality for all pairs.

III. A Semi-Definite Relaxation Framework for Fair Performance Optimization with Perfect CSI

In this section we present a semi-definite relaxation (SDR) framework in order to optimize the system performance assuming that perfect CSI is available at the relay. For each of the communicating pairs, due to applying linear reception and precoding filters ($\boldsymbol{f}_k, \boldsymbol{w}_k, \ \forall k \in \mathbb{F}_K$), the contribution of other communicating pairs are treated as interference on the resulting decision variables ($m_k, y_k, \forall k \in \mathbb{F}_K$). As the quality measure of each individual link we evaluate the Signal-to-Interference-plus-Noise-Ratio (SINR) for the communication between source to relay ($\mathrm{SINR}_{\mathrm{sr},k}$) and relay to destination ($\mathrm{SINR}_{\mathrm{rd},k}$) where k represents the index of the communicating pair. Following the ideas in [3], [14] we assume that the end-to-end communication quality is bounded by the quality of the weakest intermediate link and define

$$\mathrm{SINR}_{\mathrm{effective},k} := \min\Big(\mathrm{SINR}_{\mathrm{sr},k}, \mathrm{SINR}_{\mathrm{rd},k} \Big), \ \ \forall k \in \mathbb{F}_K, \quad (11)$$

where $\mathrm{SINR}_{\mathrm{effective},k}$ is the perceived (effective) end-to-end link quality corresponding to the k-th user pair. In the current work, we are aiming at enhancing the communicating link qualities in a fair fashion. In the other words, we aim at achieving the optimum system parameters which maximize the minimum $\mathrm{SINR}_{\mathrm{effective},k}$ among the communicating users. The

$$\mathrm{SINR}_{\mathrm{sr},k} = \frac{P_k \boldsymbol{f}_k^{\mathrm{H}} \left(\hat{\boldsymbol{h}}_{\mathrm{sr},k} + \boldsymbol{\delta}_{\mathrm{sr},k}\right) \left(\hat{\boldsymbol{h}}_{\mathrm{sr},k} + \boldsymbol{\delta}_{\mathrm{sr},k}\right)^{\mathrm{H}} \boldsymbol{f}_k}{\boldsymbol{f}_k^{\mathrm{H}} \left(\sigma_{\mathrm{nr}}^2 \boldsymbol{I}_N + \boldsymbol{\Delta}_{\mathrm{rr}} \left(\sum_{k\in\mathbb{F}_K} \boldsymbol{w}_k \boldsymbol{w}_k^{\mathrm{H}}\right) \boldsymbol{\Delta}_{\mathrm{rr}}^{\mathrm{H}} + \sum_{\forall l \in \mathbb{K}_k} P_l \left(\hat{\boldsymbol{h}}_{\mathrm{sr},l} + \boldsymbol{\delta}_{\mathrm{sr},l}\right)\left(\hat{\boldsymbol{h}}_{\mathrm{sr},l} + \boldsymbol{\delta}_{\mathrm{sr},l}\right)^{\mathrm{H}}\right) \boldsymbol{f}_k}, \tag{13}$$

$$\mathrm{SINR}_{\mathrm{rd},k} = \frac{\left(\hat{\boldsymbol{h}}_{\mathrm{rd},k} + \boldsymbol{\delta}_{\mathrm{rd},k}\right)^{\mathrm{T}} \boldsymbol{w}_k \boldsymbol{w}_k^{\mathrm{H}} \left(\hat{\boldsymbol{h}}_{\mathrm{rd},k} + \boldsymbol{\delta}_{\mathrm{rd},k}\right)^*}{\sigma_{\mathrm{nd}}^2 + \sum_{\forall l \in \mathbb{K}_k,} \left(\hat{\boldsymbol{h}}_{\mathrm{rd},k} + \boldsymbol{\delta}_{\mathrm{rd},k}\right)^{\mathrm{T}} \boldsymbol{w}_l \boldsymbol{w}_l^{\mathrm{H}} \left(\hat{\boldsymbol{h}}_{\mathrm{rd},k} + \boldsymbol{\delta}_{\mathrm{rd},k}\right)^*}, \ \mathbb{F}_K := \{1, \cdot\cdot, K\}, \ \mathbb{K}_k := \mathbb{F}_K \setminus k. \tag{14}$$

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corresponding optimization problem can be hence formulated as

$$\begin{aligned} \max_{\substack{\boldsymbol{f}_k, \boldsymbol{w}_k, t, \\ \forall k \in \mathbb{F}_K}} \quad & t \\ \text{s.t.} \quad & \mathbb{E}\left\{\|\boldsymbol{H}_{\mathrm{rr}} \cdot \boldsymbol{r}_{\mathrm{out}}\|_2^2\right\} \leq P_{\mathrm{int}}, \ \mathbb{E}\left\{\|\boldsymbol{r}_{\mathrm{out}}\|_2^2\right\} \leq P_{\max}^{\mathrm{R}}, \\ & \min\Big(\mathrm{SINR}_{\mathrm{sr},k}, \mathrm{SINR}_{\mathrm{rd},k}\Big) \geq t, \ \ \forall k \in \mathbb{F}_K, \end{aligned} \tag{12}$$

where the values $\mathrm{SINR}_{\mathrm{sr},k}, \mathrm{SINR}_{\mathrm{rd},k}$ respectively represent the resulting SINR for the source to relay and relay to destination paths. The auxiliary variable $t \in \mathbb{R}^+$ represents an end-to-end link quality which holds for all of the active communicating pairs. In the following we propose a design strategy applying semi-definite relaxation (SDR) as the main optimization framework.

A. SDR Optimization Framework

The general formulations for $\mathrm{SINR}_{\mathrm{sr},k}, \mathrm{SINR}_{\mathrm{rd},k}$ are provided in (13) and (14), incorporating the CSI estimation errors. We denote the index set of all communicating pairs as $\mathbb{F}_K$ and the index set of all communicating pairs except the index k as $\mathbb{K}_k$. For the case with availability of perfect CSI, our problem turns into

$$\begin{aligned} \max_{\substack{\boldsymbol{f}_k, \boldsymbol{w}_k, t, \\ \forall k \in \mathbb{F}_K}} \quad & t \\ \text{s.t.} \quad & \mathrm{Tr}\left(\sum_{k \in \mathbb{F}_K} \boldsymbol{w}_k \boldsymbol{w}_k^{\mathrm{H}}\right) \leq P_{\max}, && \text{(15a)} \\ & \mathrm{Tr}\left(\left(\sum_{k \in \mathbb{F}_K} \boldsymbol{w}_k \boldsymbol{w}_k^{\mathrm{H}}\right) \boldsymbol{H}_{\mathrm{rr}}^{\mathrm{H}} \boldsymbol{H}_{\mathrm{rr}}\right) \leq P_{\mathrm{int}}, && \text{(15b)} \\ \forall k \in \mathbb{F}_K : \quad & \frac{\mathrm{Tr}\left(\boldsymbol{h}_{\mathrm{rd},k}^* \boldsymbol{h}_{\mathrm{rd},k}^{\mathrm{T}} \boldsymbol{w}_k \boldsymbol{w}_k^{\mathrm{H}}\right)}{\sigma_{\mathrm{nd}}^2 + \mathrm{Tr}\left(\sum_{l \in \mathbb{K}_k} \boldsymbol{h}_{\mathrm{rd},k}^* \boldsymbol{h}_{\mathrm{rd},k}^{\mathrm{T}} \boldsymbol{w}_l \boldsymbol{w}_l^{\mathrm{H}}\right)} \geq t, && \text{(15c)} \\ \forall k \in \mathbb{F}_K : \quad & \frac{\mathrm{Tr}\left(P_k \boldsymbol{h}_{\mathrm{sr},k} \boldsymbol{h}_{\mathrm{sr},k}^{\mathrm{H}} \boldsymbol{f}_k \boldsymbol{f}_k^{\mathrm{H}}\right)}{\mathrm{Tr}\left(\left(\sigma_{\mathrm{nr}}^2 \boldsymbol{I}_N + \sum_{l \in \mathbb{K}_k} P_l \boldsymbol{h}_{\mathrm{sr},l} \boldsymbol{h}_{\mathrm{sr},l}^{\mathrm{H}}\right) \boldsymbol{f}_k \boldsymbol{f}_k^{\mathrm{H}}\right)} \geq t, && \text{(15d)} \end{aligned}$$

where (15a), (15b) respectively represent the relay's transmit and interference power constraints and (15c), (15d) are formulating the resulting SINR in the source to relay path as well as the relay to destination paths. It is worth mentioning that due to the perfect estimation in this scenario the self-interference signal is accurately subtracted, given the self-interference power does not exceed the functional receiver range (15b). As first step, we observe that if a value of t is feasible, every smaller t will be feasible as well (and vise-versa). Hence, we can apply a bi-section search over t which turns our problem into a feasibility check in each step. By defining $\boldsymbol{W}_k := \boldsymbol{w}_k \boldsymbol{w}_k^{\mathrm{H}}, \ \boldsymbol{F}_k := \boldsymbol{f}_k \boldsymbol{f}_k^{\mathrm{H}}, \ \forall k \in \mathbb{F}_K$, and relaxing the rank-1 constraint from $\boldsymbol{F}_k, \boldsymbol{W}_k$ our feasibility check problem turns into:

$$\begin{aligned} & \text{find} \ \ \boldsymbol{F}_k \in \mathcal{H}, \boldsymbol{W}_k \in \mathcal{H}, \forall k \in \mathbb{F}_K \\ & \text{s.t.} \ \ \mathrm{Tr}\left(\sum_{k \in \mathbb{F}_K} \boldsymbol{W}_k\right) \leq P_{\max}, \\ & \mathrm{Tr}\left(\left(\sum_{k \in \mathbb{F}_K} \boldsymbol{W}_k\right) \bar{\boldsymbol{H}}_{\mathrm{rr}}\right) \leq P_{\mathrm{int}}, \\ & \forall k \in \mathbb{F}_K : \mathrm{Tr}\left(\boldsymbol{H}_{\mathrm{rd},k}\left(\boldsymbol{W}_k - t\sum_{l \in \mathbb{K}_k} \boldsymbol{W}_l\right)\right) - t\sigma_{\mathrm{nd}}^2 \geq 0, \\ & \forall k \in \mathbb{F}_K : \mathrm{Tr}\left(\left(\boldsymbol{H}_{\mathrm{sr},k} - t\left(\sigma_{\mathrm{nr}}^2 \boldsymbol{I}_N + \sum_{l \in \mathbb{K}_k} \boldsymbol{H}_{\mathrm{sr},l}\right)\right) \boldsymbol{F}_k\right) \geq 0, \end{aligned} \tag{16}$$

where $\boldsymbol{H}_{\mathrm{rd},k} := \boldsymbol{h}_{\mathrm{rd},k}^* \boldsymbol{h}_{\mathrm{rd},k}^{\mathrm{T}}, \boldsymbol{H}_{\mathrm{sr},k} := P_k \boldsymbol{h}_{\mathrm{sr},k} \boldsymbol{h}_{\mathrm{sr},k}^{\mathrm{H}}, \ k \in \mathbb{F}_K$, and $\bar{\boldsymbol{H}}_{\mathrm{rr}} := \boldsymbol{H}_{\mathrm{rr}}^{\mathrm{H}} \boldsymbol{H}_{\mathrm{rr}}$. The above feasibility check clearly follows a semi-definite programming (SDP) structure and hence can be determined within a polynomial time. Nevertheless, SDP structure does not in general provide an optimal rank-1 solution. Popular rank-1 approximation and randomization techniques [21] have been developed to handle the rank-constraint optimization problems with SDP structure. Fortunately, for the certain structure of (16) as a complex-valued SDP with $2K+1$ constraints and with $2K$ positive semi-definite variables with Hermitian symmetry ($\boldsymbol{F}_k, \boldsymbol{W}_k \in \mathcal{H}, \ \forall k \in \mathbb{F}_K$), we can always achieve an optimal rank-1 solution due to the Theorem 3.2 in [22] and results of [23]. The similar argumentation regarding rank-constraint optimization can be also found in [9], [14]. The introduced bi-section search steps with the defined feasibility check should be continued until required resolution of optimal t is achieved. Given the optimal rank-1 solutions for $\boldsymbol{F}_k, \boldsymbol{W}_k \ \forall k \in \mathbb{F}_K$ we obtain

$$\boldsymbol{f}_k^\star = \boldsymbol{F}_k^{\frac{1}{2}}, \ \ \boldsymbol{w}_k^\star = \boldsymbol{W}_k^{\frac{1}{2}}, \ \ \forall k \in \mathbb{F}_K \tag{17}$$

where $\boldsymbol{f}_k^\star, \boldsymbol{w}_k^\star \ \forall k \in \mathbb{F}_K$ are the optimal relaying parameters for the scenario with the availability of perfect CSI.

IV. Semi-Definite Relaxation for Robust Design with Imperfect CSI

In order to incorporate the CSI inaccuracy in our design, we follow the works in [16], [17], [19] and choose the worst-case enhancement as our design strategy. In the other words, we present a design which improves the guaranteed link quality among all communicating links ($\min_k \text{SINR}_{\text{effective},k}$), which is preserved for all feasible channel estimation errors (1)-(3). The similar strategy for a scenario with single communicating pair has been presented in [14]. The corresponding optimization problem can be then formulated as

$$\begin{aligned}\max_{\substack{\boldsymbol{f}_k,\boldsymbol{w}_k,t,\\ \forall k\in\mathbb{F}_K}} \min_{\substack{\boldsymbol{\delta}_{\text{sr},k},\boldsymbol{\delta}_{\text{rd},k},\boldsymbol{\Delta}_{\text{rr}}\\ \forall k\in\mathbb{F}_K}} \; & t \\ \text{s.t.} \;\; & \text{SINR}_{\text{sr},k} \geq t, \;\; \text{SINR}_{\text{rd},k} \geq t, \;\; \forall k \in \mathbb{F}_K, \\ & \mathbb{E}\left\{ \left\| \hat{\boldsymbol{H}}_{\text{rr}} \cdot \boldsymbol{r}_{\text{out}} \right\|_2^2 \right\} \leq P_{\text{int}}, \\ & \mathbb{E}\left\{ \left\| \boldsymbol{r}_{\text{out}} \right\|_2^2 \right\} \leq P_{\max}^{\text{R}}, \\ \forall k \in \mathbb{F}_K : \;\; & \boldsymbol{\delta}_{\text{sr},k} \in \mathcal{D}_{\text{sr},k}, \;\; \boldsymbol{\delta}_{\text{rd},k} \in \mathcal{D}_{\text{rd},k}, \;\; \boldsymbol{\Delta}_{\text{rr}} \in \mathcal{D}_{\text{rr}},\end{aligned} \tag{18}$$

where the expressions for $\text{SINR}_{\text{sr},k}, \text{SINR}_{\text{rd},k}$ are provided in (13) and (14), and $\mathcal{D}_{\text{sr},k}, \mathcal{D}_{\text{rd},k}, \mathcal{D}_{\text{rr}}$ represent the set of feasible CSI errors defined in (1), (2), (3). The variable t represents the guaranteed SINR in all links which is preserved for all feasible channel estimation errors. Similar to Section III we apply bisection search on values of t which turns our problem into a feasibility check in each iteration. Due to our worst-case design approach, a value of t is feasible if and only if every individual problem constraints ($2K + 2$ constraints in total) hold for all feasible CSI inaccuracies. This is clear since if a feasible CSI error results in violation of any of the problem constraints, the corresponding t (guaranteed link quality) will not hold in the worst-case scenario. Hence we constitute the following feasibility check:

$$\text{find} \;\; \boldsymbol{f}_k, \boldsymbol{w}_k, \;\; \forall k \in \mathbb{F}_K$$

$$\text{s.t.} \;\; \left(\min_{\boldsymbol{\delta}_{\text{sr},k}\in\mathcal{D}_{\text{sr},k},\, \boldsymbol{\delta}_{\text{rd},k}\in\mathcal{D}_{\text{rd},k},\, \boldsymbol{\Delta}_{\text{rr}}\in\mathcal{D}_{\text{rr}}} \text{SINR}_{\text{sr},k} \right) \geq t, \; \forall k \in \mathbb{F}_K, \tag{19a}$$

$$\left(\min_{\boldsymbol{\delta}_{\text{sr},k}\in\mathcal{D}_{\text{sr},k},\, \boldsymbol{\delta}_{\text{rd},k}\in\mathcal{D}_{\text{rd},k},\, \boldsymbol{\Delta}_{\text{rr}}\in\mathcal{D}_{\text{rr}}} \text{SINR}_{\text{rd},k} \right) \geq t, \; \forall k \in \mathbb{F}_K, \tag{19b}$$

$$\mathbb{E}\left\{ \left\| \hat{\boldsymbol{H}}_{\text{rr}} \cdot \boldsymbol{r}_{\text{out}} \right\|_2^2 \right\} \leq P_{\text{int}}, \;\; \mathbb{E}\left\{ \left\| \boldsymbol{r}_{\text{out}} \right\|_2^2 \right\} \leq P_{\max}^{\text{R}}, \tag{19c}$$

where (19a) and (19b) examine the worst case CSI error conditions corresponding to source to relay and relay to destination paths, respectively. As the first simplification, we observe that the components of $\boldsymbol{\Delta}_{\text{rr}}$ are only effective in the denominators of the $\text{SINR}_{\text{sr},k}$ values. It can be proved that for the communicating pair with index k, the worst-case $\boldsymbol{\Delta}_{\text{rr}}$ occurs on the boarder of $\mathcal{D}_{\text{rr}}$ and can be consequently achieved via:

$$\begin{aligned}\max_{\boldsymbol{\Delta}_{\text{rr}}} \;\; & \boldsymbol{f}_k^{\text{H}} \boldsymbol{\Delta}_{\text{rr}} \boldsymbol{C} \boldsymbol{\Delta}_{\text{rr}}^{\text{H}} \boldsymbol{f}_k \\ \text{s.t.} \;\; & \text{Tr}\left(\boldsymbol{\Delta}_{\text{rr}} \boldsymbol{T}_{\text{rr}} \boldsymbol{\Delta}_{\text{rr}}^{\text{H}} \right) = \xi_{\text{rr}}^2,\end{aligned} \tag{20}$$

where $\boldsymbol{C} := \sum_{k\in\mathbb{F}_K} \boldsymbol{w}_k \boldsymbol{w}_k^{\text{H}} = \sum_{k\in\mathbb{F}_K} \boldsymbol{W}_k$, and the term $\boldsymbol{f}_k^{\text{H}} \boldsymbol{\Delta}_{\text{rr}} \boldsymbol{C} \boldsymbol{\Delta}_{\text{rr}}^{\text{H}} \boldsymbol{f}_k$ represents the resulting interference component in the denominator of (13). It is worth mentioning that since $\text{rank}(\boldsymbol{C}) \neq 1$ in general, our previous norm argumentation in [14], (12-14) is not valid for the above problem. Following the variable definition as $\boldsymbol{\Delta}_{\text{rr}}^{'} := \boldsymbol{\Delta}_{\text{rr}} \boldsymbol{T}_{\text{rr}}^{\frac{1}{2}}$, $\boldsymbol{\delta}_{\text{rr}}^{'} := \text{vec}\left(\boldsymbol{\Delta}_{\text{rr}}^{'} \right)$ and $\bar{\boldsymbol{C}} := \boldsymbol{T}_{\text{rr}}^{-\frac{1}{2}} \boldsymbol{C}^{\frac{1}{2}}$ we can equivalently formulate (15) as

$$\begin{aligned}\max_{\boldsymbol{\Delta}_{\text{rr}}^{'}} \;\; & \boldsymbol{f}_k^{\text{H}} \boldsymbol{\Delta}_{\text{rr}}^{'} \bar{\boldsymbol{C}} \bar{\boldsymbol{C}}^{\text{H}} \boldsymbol{\Delta}_{\text{rr}}^{'\,\text{H}} \boldsymbol{f}_k \\ \text{s.t.} \;\; & \boldsymbol{\delta}_{\text{rr}}^{'\,\text{H}} \boldsymbol{\delta}_{\text{rr}}^{'} = \xi_{\text{rr}}^2.\end{aligned} \tag{21}$$

Applying some basic matrix operations, the homogeneous form of (21) can be written as

$$\max_{\boldsymbol{\delta}_{\text{rr}}^{'}} \;\; \frac{\boldsymbol{\delta}_{\text{rr}}^{'\,\text{H}} \boldsymbol{A} \boldsymbol{\delta}_{\text{rr}}^{'}}{\boldsymbol{\delta}_{\text{rr}}^{'\,\text{H}} \boldsymbol{\delta}_{\text{rr}}^{'}} \xi_{\text{rr}}^2, \quad \text{s.t.} \;\; \boldsymbol{\delta}_{\text{rr}}^{'\,\text{H}} \boldsymbol{\delta}_{\text{rr}}^{'} = \xi_{\text{rr}}^2, \tag{22}$$

where $\boldsymbol{A} := \left(\bar{\boldsymbol{C}}^{\text{T}} \otimes \boldsymbol{f}_k^{\text{H}} \right)^{\text{H}} \left(\bar{\boldsymbol{C}}^{\text{T}} \otimes \boldsymbol{f}_k^{\text{H}} \right)$. The above problem holds the famous Rayleigh quotient structure with the known optimal objective value as $\lambda_{\max}\{\boldsymbol{A}\}\, \xi_{\text{rr}}^2$, where $\lambda_{\max}\{\cdot\}$ represents the maximum-eigenvalue operator. Applying similar matrix relations as (487), (494) in [24] we conclude:

$$\begin{aligned}\lambda_{\max}\{\boldsymbol{A}\}\, \xi_{\text{rr}}^2 &= \lambda_{\max}\left\{ \left(\bar{\boldsymbol{C}}^{\text{T}} \otimes \boldsymbol{f}_k^{\text{H}} \right)^{\text{H}} \left(\bar{\boldsymbol{C}}^{\text{T}} \otimes \boldsymbol{f}_k^{\text{H}} \right) \right\} \xi_{\text{rr}}^2 \\ &= \lambda_{\max}\left\{ \left(\bar{\boldsymbol{C}}^{*} \bar{\boldsymbol{C}}^{\text{T}} \right) \otimes \left(\boldsymbol{f}_k \otimes \boldsymbol{f}_k^{\text{H}} \right) \right\} \xi_{\text{rr}}^2 \\ &= \lambda_{\max}\{\boldsymbol{C}\}\, \lambda_{\max}\{\boldsymbol{F}_k\}\, \xi_{\text{rr}}^2.\end{aligned} \tag{23}$$

In order to further simplify (23) we recall from (13) that the values of $\text{SINR}_{\text{sr},k}$ are homogeneous with respect to $\boldsymbol{f}_k \;\forall k \in \mathbb{F}_K$. Furthermore there is no effect of $\boldsymbol{f}_k$ in the values of $\text{SINR}_{\text{rd},k}$. Hence without loss of generality, from now on we assume $\|\boldsymbol{f}_k\|_2 = 1$ and consequently $\text{Tr}(\boldsymbol{F}_k) = 1$ and $\lambda_{\max}\{\boldsymbol{F}_k\} = 1 \;\forall k \in \mathbb{F}_K$. This simplifies (23) into $\lambda_{\max}\{\boldsymbol{C}\}\, \xi_{\text{rr}}^2$ which is a convex function on $\boldsymbol{C}$ and consequently a jointly-convex function on $\boldsymbol{W}_k, \;\forall k \in \mathbb{F}_K$. This concludes our study for the worst-case effect of $\boldsymbol{\Delta}_{\text{rr}}$ in (13). As another part of CSI error, we investigate the effects of $\boldsymbol{\delta}_{\text{sr},k}$ on the resulting value of $\text{SINR}_{\text{sr},k}$. It is clear that there is no effect of $\boldsymbol{\delta}_{\text{sr},k}$ in the relay-to-destination path. For the communicating pair with index k, the worst-case $\boldsymbol{\delta}_{\text{sr},k}$ is characterized by its contribution in the nominator of $\text{SINR}_{\text{sr},k}$ and can be formulated via following optimization problem:

$$\begin{aligned}\min_{\boldsymbol{\delta}_{\text{sr},k}} \;\; & P_k \boldsymbol{f}_k^{\text{H}} \left(\hat{\boldsymbol{h}}_{\text{sr},k} + \boldsymbol{\delta}_{\text{sr},k} \right) \left(\hat{\boldsymbol{h}}_{\text{sr},k} + \boldsymbol{\delta}_{\text{sr},k} \right)^{\text{H}} \boldsymbol{f}_k, \\ \text{s.t.} \;\; & \boldsymbol{\delta}_{\text{sr},k} \in \mathcal{D}_{\text{sr},k},\end{aligned} \tag{24}$$

which is a known problem [17] and can be equivalently formulated as

$$\begin{aligned}\max_{b} \;\; & b \\ \text{s.t.} \;\; & \forall\, \boldsymbol{\delta}_{\text{sr},k} \mid \boldsymbol{\delta}_{\text{sr},k}^{\text{H}} \boldsymbol{T}_{\text{sr},k} \boldsymbol{\delta}_{\text{sr},k} \leq \xi_{\text{sr},k}^2 \Rightarrow \\ & P_k \left(\hat{\boldsymbol{h}}_{\text{sr},k} + \boldsymbol{\delta}_{\text{sr},k} \right)^{\text{H}} \boldsymbol{F}_k \left(\hat{\boldsymbol{h}}_{\text{sr},k} + \boldsymbol{\delta}_{\text{sr},k} \right) \geq b.\end{aligned} \tag{25}$$

The above structure has been studied in the context of robust system optimization in [16]–[18]. The key components of dealing with such problems are the so-called S-procedure [25] as a powerful tool to deal with robust quadratic problems as well as the famous Schurs complement [26]. The equivalent form of (25) as a feasibility check over value of b can be written as

$$\begin{aligned} &\text{find} \;\; \boldsymbol{F}_k, \boldsymbol{Z}_{\text{sr},k}, \mu_{\text{sr},k} \\ &\text{s.t.} \;\; \text{Tr}\left(\left(\boldsymbol{F}_k - \boldsymbol{Z}_{\text{sr},k} \right) \hat{\boldsymbol{H}}_{\text{sr},k} \right) - \xi_{\text{sr},k}^2 \cdot \mu_{\text{sr},k} \geq \frac{b}{P_k}, \\ &\quad \left[\begin{array}{cc} \boldsymbol{Z}_{\text{sr},k} & \boldsymbol{F}_k \\ \boldsymbol{F}_k & \boldsymbol{F}_k + \mu_{\text{sr},k} \boldsymbol{T}_{\text{sr},k} \end{array} \right] \geq 0, \\ &\quad \boldsymbol{F}_k \in \mathcal{H}, \; \mu_{\text{sr},k} \geq 0, \end{aligned} \tag{26}$$

where $\hat{\boldsymbol{H}}_{\text{sr},k} := \hat{\boldsymbol{h}}_{\text{sr},k} \hat{\boldsymbol{h}}_{\text{sr},k}^{\text{H}}, \; \forall k \in \mathbb{F}_K$ and $\mu_{\text{sr},k} \in \mathbb{R}, \; \boldsymbol{Z}_{\text{sr},k} \in \mathbb{C}N \times N$ are auxiliary variables. The formulation in (26) provides a SDP approach to examine the worst case behavior of the CSI error on the desired source to relay path. On the other hand, we can also study the effect of $\boldsymbol{\delta}_{\text{sr},l}, \; l \in \mathbb{K}_k$, for the k-th communication pair (the worst-case effect of the interfering channels in source to relay path). According to (13), the worst-case $\boldsymbol{\delta}_{\text{sr},l}, \; l \in \mathbb{K}_k$ is the one that maximizes the inter-pair communication interference which appears in the denominator of $\text{SINR}_{\text{sr},k}$. The corresponding optimization problem for any $l, \; l \neq k$ ($\forall l \in \mathbb{K}_k$) is

$$\begin{aligned} &\max_{\boldsymbol{\delta}_{\text{sr},l}} \; P_l \boldsymbol{f}_k^{\text{H}} \left(\hat{\boldsymbol{h}}_{\text{sr},l} + \boldsymbol{\delta}_{\text{sr},l} \right) \left(\hat{\boldsymbol{h}}_{\text{sr},l} + \boldsymbol{\delta}_{\text{sr},l} \right)^{\text{H}} \boldsymbol{f}_k, \\ &\text{s.t.} \;\; \boldsymbol{\delta}_{\text{sr},l} \in \mathcal{D}_{\text{sr},l}. \end{aligned} \tag{27}$$

We formulate the objective in (27) as

$$P_l \left(\left| \boldsymbol{f}_k^{\text{H}} \hat{\boldsymbol{h}}_{\text{sr},l} + \boldsymbol{f}_k^{\text{H}} \boldsymbol{\delta}_{\text{sr},l} \right| \right)^2 \leq P_l \left(\left| \boldsymbol{f}_k^{\text{H}} \hat{\boldsymbol{h}}_{\text{sr},l} \right| + \left| \boldsymbol{f}_k^{\text{H}} \boldsymbol{\delta}_{\text{sr},l} \right| \right)^2, \tag{28}$$

where (28) holds due to the triangular inequality. At this point we observe that the feasibility of $\boldsymbol{\delta}_{\text{sr},l}$ is invariant to a scalar phase rotation since it does not affect the value of $\boldsymbol{\delta}_{\text{sr},l}^{\text{H}} \boldsymbol{T}_{\text{sr},l} \boldsymbol{\delta}_{\text{sr},l}$ in (1). Hence at the optimality of (27) the equality holds in (28), due to the results of triangular inequality. By defining $\boldsymbol{\delta}_{\text{sr},l}^{'} := \boldsymbol{T}_{\text{sr},l}^{\frac{\text{H}}{2}} \boldsymbol{\delta}_{\text{sr},l}$ we can equivalently write (27) as

$$\max_{\boldsymbol{\delta}_{\text{sr},l}^{'}} P_l \left(\left| \boldsymbol{f}_k^{\text{H}} \hat{\boldsymbol{h}}_{\text{sr},l} \right| + \left| \boldsymbol{f}_k^{\text{H}} \boldsymbol{T}_{\text{sr},l}^{-\frac{\text{H}}{2}} \boldsymbol{\delta}_{\text{sr},l}^{'} \right| \right)^2, \; \text{s.t.} \; \|\boldsymbol{\delta}_{\text{sr},l}^{'}\|_2 = \xi_{\text{sr},l}, \tag{29}$$

which has the known structure and results in $\boldsymbol{\delta}_{\text{sr},l} = \frac{\boldsymbol{T}_{\text{sr},l}^{-1} \boldsymbol{f}_k}{\|\boldsymbol{T}_{\text{sr},l}^{-\frac{1}{2}} \boldsymbol{f}_k\|_2} \xi_{\text{sr},l}$ at the optimality of (27). The corresponding optimal objective value in (27) can be then calculated via basic matrix operations as

$$\begin{aligned} V_{k,l}^{(\text{sr})} :=& P_l \text{Tr}\left(\left(\hat{\boldsymbol{H}}_{\text{sr},l} + \xi_{\text{sr},l}^2 \boldsymbol{T}_{\text{sr},l}^{-1} \right) \boldsymbol{F}_k \right) \\ &+ 2\xi_{\text{sr},l} P_l \left\| \hat{\boldsymbol{h}}_{\text{sr},l}^{\text{H}} \boldsymbol{F}_k \boldsymbol{T}_{\text{sr},l}^{-\frac{\text{H}}{2}} \right\|_2, \end{aligned} \tag{30}$$

where $V_{k,l}^{(\text{sr})}$ is the resulting objective value in (27) corresponding to worst-case CSI error. The significance of the presented structures in (30) and (26) is the fact that they follow a convex structure, relating the worst-case individual link quality ($\text{SINR}_{\text{sr},k}, \forall k \in \mathbb{F}_K$) to the corresponding design parameters ($\boldsymbol{F}_k, \boldsymbol{W}_k, \forall k \in \mathbb{F}_K$). This concludes our worst-case study for CSI errors in the source to relay paths. As we can see from (14), studying the effect of worst-case CSI errors on the rely to destination path is rather simpler since for the destination user with index k, the only effective CSI error is $\boldsymbol{\delta}_{\text{sr},k}$. Nevertheless the resulting error components in the denominator and the nominator of (14) can not be treated independently as they belong to the same channel. The feasibility check in (19b) for the value of t can be written as

$$\begin{aligned} &\text{find} \;\; \boldsymbol{W}_k, \;\; \forall k \in \mathbb{F}_K \\ &\text{s.t.} \;\; \forall \; \boldsymbol{\delta}_{\text{rd},k} \mid \boldsymbol{\delta}_{\text{rd},k}^{\text{T}} \boldsymbol{T}_{\text{rd},k} \boldsymbol{\delta}_{\text{rd},k}^{*} \leq \xi_{\text{rd},k}^2 \Rightarrow \\ &\quad \left(\hat{\boldsymbol{h}}_{\text{rd},k} + \boldsymbol{\delta}_{\text{rd},k} \right)^{\text{T}} \boldsymbol{Q}_k \left(\hat{\boldsymbol{h}}_{\text{rd},k} + \boldsymbol{\delta}_{\text{rd},k} \right)^{*} - t\sigma_{\text{nd}}^2 \geq 0, \end{aligned} \tag{31}$$

where $\boldsymbol{Q}_k := \left(\boldsymbol{W}_k - t \sum_{l \in \mathbb{K}_k} \boldsymbol{W}_l \right)$. Above feasibility check holds the similar structure as in [16]–[18] and can be equivalently written as

$$\begin{aligned} &\text{find} \;\; \boldsymbol{W}_k, \boldsymbol{Z}_{\text{rd},k}, \mu_{\text{rd},k}, \forall k \in \mathbb{F}_K \\ &\text{s.t.} \;\; \text{Tr}\left(\left(\boldsymbol{Q}_k - \boldsymbol{Z}_{\text{rd},k} \right) \hat{\boldsymbol{H}}_{\text{rd},k} \right) - \xi_{\text{rd},k}^2 \cdot \mu_{\text{rd},k} \geq t\sigma_{\text{nd}}^2, \\ &\quad \left[\begin{array}{cc} \boldsymbol{Z}_{\text{rd},k} & \boldsymbol{Q}_k \\ \boldsymbol{Q}_k & \boldsymbol{Q}_k + \mu_{\text{rd},k} \boldsymbol{T}_{\text{rd},k} \end{array} \right] \geq 0, \\ &\quad \boldsymbol{Q}_k \in \mathcal{H}, \; \mu_{\text{rd},k} \geq 0, \end{aligned} \tag{32}$$

where $\hat{\boldsymbol{H}}_{\text{rd},k} := \hat{\boldsymbol{h}}_{\text{rd},k}^{*} \hat{\boldsymbol{h}}_{\text{rd},k}^{\text{T}}$ and $\mu_{\text{rd},k} \in \mathbb{R}, \; \boldsymbol{Z}_{\text{rd},k} \in \mathbb{C}^{N \times N}$ are axillary variables. By collecting the results from (26), (30), (32) and by relaxing the rank-1 constraint of $\boldsymbol{F}_k, \boldsymbol{W}_k, \forall k \in \mathbb{F}_K$, similar to (16), in each step of our bi-section search we obtain a feasibility check for the value of t as

$$\begin{aligned} &\text{find} \;\; \boldsymbol{F}_k, \boldsymbol{W}_k, \boldsymbol{Z}_{\text{sr},k}, \boldsymbol{Z}_{\text{rd},k}, \gamma_{\text{sr},k}, \mu_{\text{sr},k}, \mu_{\text{rd},k} \;\; \forall k \in \mathbb{F}_K \\ &\text{s.t.} \;\; \text{Tr}\left(\sum_{k \in \mathbb{F}_K} \boldsymbol{W}_k \right) \leq P_{\max}, \; \text{Tr}\left(\left(\sum_{k \in \mathbb{F}_K} \boldsymbol{W}_k \right) \bar{\boldsymbol{H}}_{\text{rr}} \right) \leq P_{\text{int}}, \\ &\quad \forall k \in \mathbb{F}_K : \\ &\quad \sigma_{\text{nr}}^2 + \lambda_{\max} \left\{ \sum_{\forall k \in \mathbb{F}_K} \boldsymbol{W}_k \right\} \xi_{\text{rr}}^2 + \sum_{l \in \mathbb{K}_k} V_{k,l}^{(\text{sr})} \leq \gamma_{\text{sr},k}, \\ &\quad \text{Tr}\left(\left(\boldsymbol{F}_k - \boldsymbol{Z}_{\text{sr},k} \right) \hat{\boldsymbol{H}}_{\text{sr},k} \right) - \xi_{\text{sr},k}^2 \cdot \mu_{\text{sr},k} \geq t\gamma_{\text{sr},k}/P_k, \\ &\quad \text{Tr}\left(\left(\boldsymbol{Q}_k - \boldsymbol{Z}_{\text{rd},k} \right) \hat{\boldsymbol{H}}_{\text{rd},k} \right) - \xi_{\text{rd},k}^2 \cdot \mu_{\text{rd},k} \geq t\sigma_{\text{nd}}^2, \\ &\quad \left[\begin{array}{cc} \boldsymbol{Z}_{\text{rd},k} & \boldsymbol{Q}_k \\ \boldsymbol{Q}_k & \boldsymbol{Q}_k + \mu_{\text{rd},k} \boldsymbol{T}_{\text{rd},k} \end{array} \right] \geq 0, \\ &\quad \left[\begin{array}{cc} \boldsymbol{Z}_{\text{sr},k} & \boldsymbol{F}_k \\ \boldsymbol{F}_k & \boldsymbol{F}_k + \mu_{\text{sr},k} \boldsymbol{T}_{\text{sr},k} \end{array} \right] \geq 0, \\ &\quad \boldsymbol{Q}_k = \left(\boldsymbol{W}_k - t \sum_{l \in \mathbb{K}_k} \boldsymbol{W}_l \right), \\ &\quad \boldsymbol{F}_k \in \mathcal{H}, \; \boldsymbol{W}_k \in \mathcal{H}, \\ &\quad \text{Tr}\left(\boldsymbol{F}_k \right) = 1, \end{aligned} \tag{33}$$

where $\gamma_{\text{sr},k}, \mu_{\text{sr},k}, \mu_{\text{rd},k} \in \mathbb{R}^{+}$, and $\boldsymbol{Z}_{\text{sr},k}, \boldsymbol{Z}_{\text{rd},k} \in \mathbb{C}^{N \times N} \; \forall k \in \mathbb{F}_K$ are auxiliary variables. Despite the crowded look, the above feasibility check (33) over values of t has a convex structure and can be solved in a polynomial time. Nevertheless, the resulting matrices $\boldsymbol{F}_k, \boldsymbol{W}_k, \; \forall k \in \mathbb{F}_K$ are not rank-1 matrices in general. Hence a rank-1 approximation on the resulting matrices is applied using the results of randomization theory [21]. The intended linear fusion filters and relay precoding vectors are obtained as

$$\boldsymbol{f}_k^{\star} = \boldsymbol{F}_k^{\star \frac{1}{2}}, \;\; \boldsymbol{w}_k^{\star} = \boldsymbol{W}_k^{\star \frac{1}{2}}, \; \forall k \in \mathbb{F}_K \tag{34}$$

TABLE I. UNLESS STATED OTHERWISE THE FOLLOWING VALUES ARE SET IN OUR SIMULATIONS.

Parameters	Value
$\boldsymbol{T}_{\mathrm{sr},k} = \boldsymbol{T}_{\mathrm{rd},k} = \boldsymbol{T}_{\mathrm{rr}},\ \forall k \in \mathbb{F}_K$	$\boldsymbol{I}_N$
$\rho_{\mathrm{sr},k},\ \forall k \in \mathbb{F}_K$	$-10\mathrm{dB}$
$\rho_{\mathrm{rd},k},\ \forall k \in \mathbb{F}_K$	$-10\mathrm{dB}$
ρ_{rr}	$0\mathrm{dB}$
$P_k = P_{\max} = P^{\mathrm{R}}_{\max},\ \forall k \in \mathbb{F}_K$	$1W$
β	1
K	2
N	3

where $\boldsymbol{F}_k^\star, \boldsymbol{W}_k^\star\ \ \forall k \in \mathbb{F}_K$ are the corresponding rank-1 matrices, and $\boldsymbol{f}_k^\star, \boldsymbol{w}_k^\star\ \forall k \in \mathbb{F}_K$ denote the obtained values for $\boldsymbol{f}_k, \boldsymbol{w}_k$.

Hence by applying the semi-definite-relaxation framework, In this section we have presented a multi-user FD relaying strategy, while incorporating the effects of inaccurate channel estimation. In Section V we evaluate the presented design strategy via numerical simulations.

V. Simulation Results

In this section we evaluate the performance of the introduced methods via Monte Carlo simulations. We follow the defined system setup and channel model as described in Section II and average our results over multiple realizations. The comparison is made between the performance of the equivalent HD system with equal number of Tx and Rx chains as well as the same CSI error condition as the FD setup (legend: HD), the derived robust design with FD setup in Section IV (legend: FD-Robust), and the non-robust FD design where channel inaccuracy is not taken into account for the design of relaying parameters (legend: FD-NonRobust). As the comparison metric we choose the worst-case communication rate which maintains for all feasible CSI error conditions in all links. In Fig.2 the effect of self-interference suppression capability ($\beta := \frac{P_{\mathrm{int}}}{P_{\max}}$) is been studied. As it can be observed for small values of CSI error and high suppression capability, the FD system is capable of achieving nearly twice of rate compared to an equivalent HD system. Nevertheless, performance of the FD scheme decreases dramatically as β decreases. Fig.3 illustrates the extremely destructive effect of the CSI error ($\xi_{\mathrm{sr},k} = \xi_{\mathrm{rd},k} = \xi_{\mathrm{rr}} := \xi,\ \forall k \in \mathbb{F}_K$) in the FD operation. This verifies our expectation since any inaccuracy of the self-interference channel directly results in the residual interference in the receiver which is not suppressible, since it is not known. As it can be observed, the gains of FD compared to HD setup disappears as CSI error increases. Furthermore, while the proposed (robust) design improves the quality in medium range of CSI error, the obtained gains disappear for very high, or very low level of CSI error. Unless stated otherwise, the values of Table I have been chosen for system parameters in our simulations.

VI. Conclusion

In this paper we have presented a semi-definite relaxation framework to address the design of a multi-user relaying system. The relay is operating with decode-and-forward protocol and is capable of effective self-interference cancellation

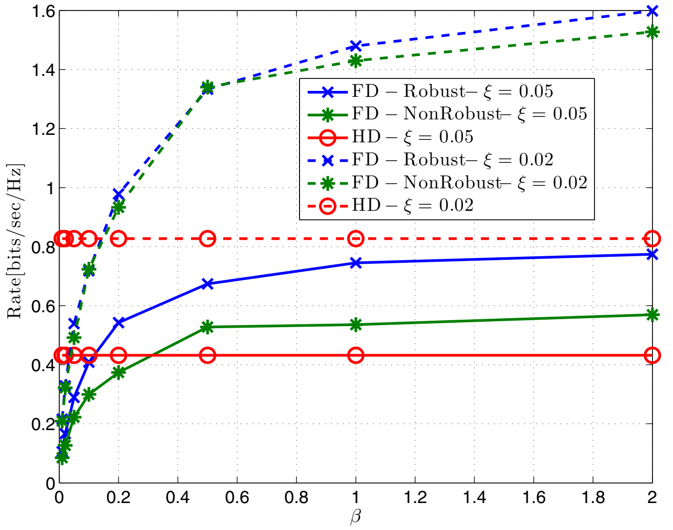

Fig. 2. Worst-case individual link quality [bits/sec/Hz] vs. β. The full-duplex relaying performance is highly dependent on suppression quality (β). SNR = 0 dB.

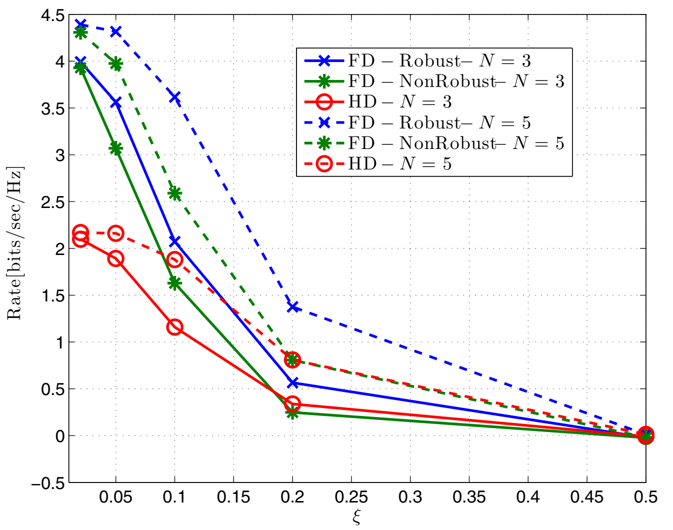

Fig. 3. Worst-case individual link quality [bits/sec/Hz] vs. $\xi := \xi_{\mathrm{sr},k} = \xi_{\mathrm{rd},k} = \xi_{\mathrm{rr}},\ \forall k \in \mathbb{F}_K$. The proposed design provides robustness against CSI inaccuracy. SNR = 20 dB.

(full-duplex operation). Our study encompasses scenarios of perfect and erroneous channel knowledge and provides a robust design framework to tackle the resulted degradation. As it has been shown, while benefits of the full-duplex operation is degraded due to the effects of inaccurate channel estimation, our proposed design provide a level of robustness for the resulting system performance. In the end we should once again note the sensitivity of the FD system to the loopback CSI error which requires dedicated estimation intervals and specialized solutions.

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