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Exercise 11
- Proposed Solution -

Friday, February 3, 2017

Solution of Problem 1

a)

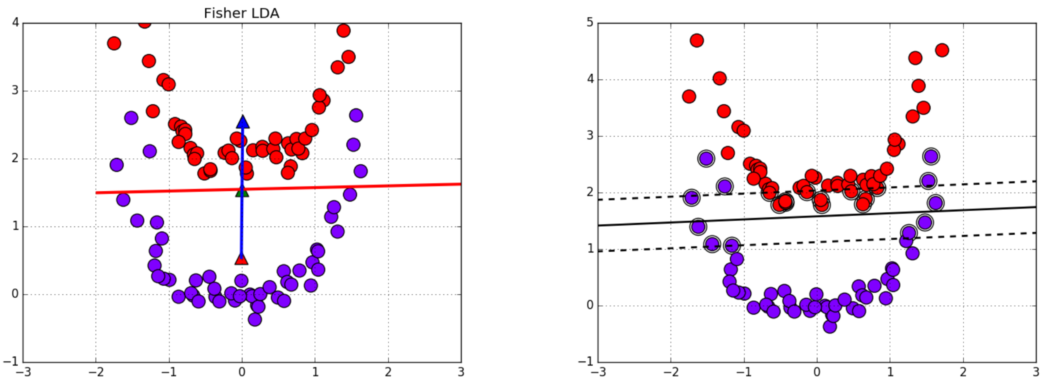

Figure 1: Non-linear classes: SVM versus LDA

b) The Gaussian kernel is given by $\exp(-\frac{\|\mathbf{x}-\mathbf{z}\|^2}{2\sigma^2})$. The following figures give the output for two different choices of σ^2.

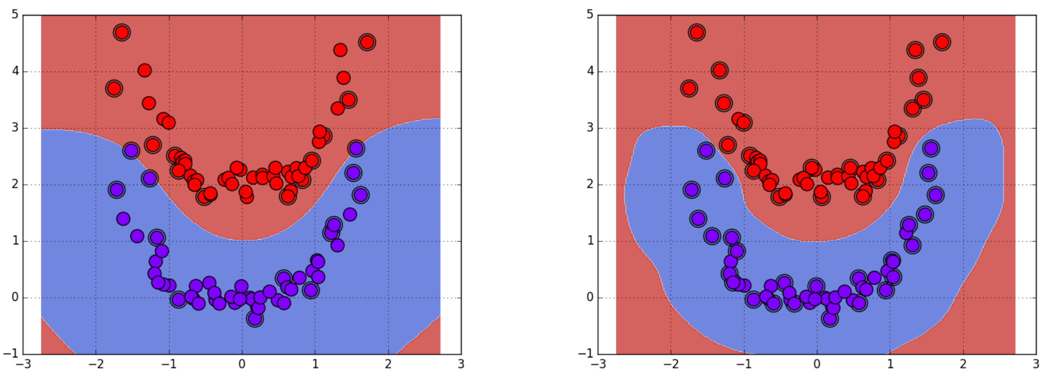

Figure 2: Two non-separable classes with Gaussian SVM, $\frac{1}{2\sigma^2} = 0.7, 5$

c)

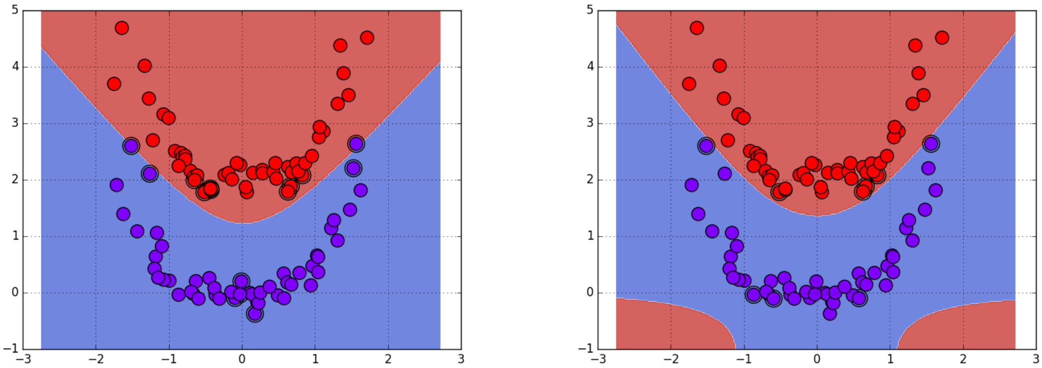

Figure 3: Two non-separable classes with polynomials of degree 2 and 3

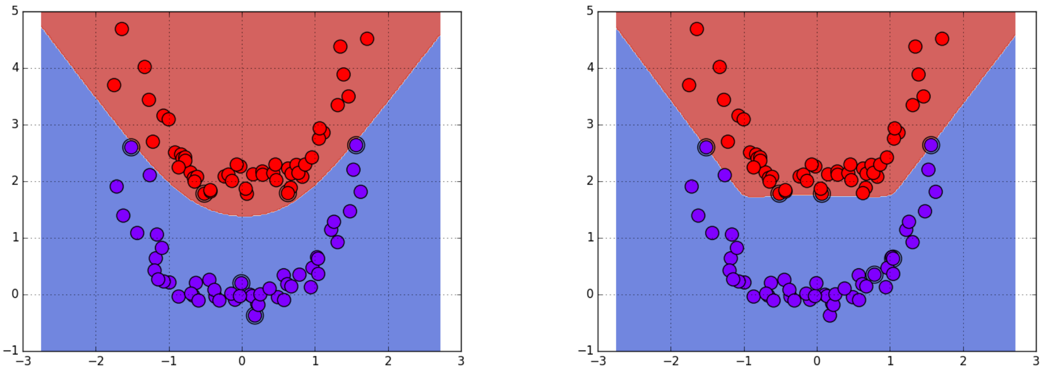

Figure 4: Two non-separable classes with polynomials of degree 4 and 40

Solution of Problem 2

If a Kernel is given by $K(\mathbf{x}, \mathbf{z}) = (\mathbf{x}^T\mathbf{z} + c)^d$, we have:

$$
\begin{aligned}
(\mathbf{x}^T\mathbf{z} + c)^d &= (\sum_{i=1}^{n} x_i z_i + c)^d = \sum \beta(\alpha_1, \dots, \alpha_{n+1})(x_1 z_1)^{\alpha_1} \dots (x_n z_n)^{\alpha_n} c^{\alpha_{n+1}} \\
&= \sum \left(\sqrt{c^{\alpha_{n+1}} \beta(\alpha_1, \dots, \alpha_{n+1})} x_1^{\alpha_1} \dots x_n^{\alpha_n}\right) \left(\sqrt{c^{\alpha_{n+1}} \beta(\alpha_1, \dots, \alpha_{n+1})} z_1^{\alpha_1} \dots z_n^{\alpha_n}\right)
\end{aligned}
$$

where the sum is taken over all $(\alpha_1, \dots, \alpha_{n+1}) \in \mathbb{N}^{n+1}$ such that $\sum_{i=1}^{n+1} \alpha_i = d, \alpha_i \in \mathbb{N}$. The number of all these monomials are given by the number of answers to the above equation which is $\binom{n+d}{d}$. Therefore the feature map can be considered as:

$$
\phi(\mathbf{x}) = \left(\sqrt{c^{\alpha_{n+1}} \beta(\alpha_1, \dots, \alpha_{n+1})} x_1^{\alpha_1} \dots x_n^{\alpha_n}\right)_{\sum_{i=1}^{n+1} \alpha_i = d, \alpha_i \in \mathbb{N}} \in \mathbb{R}^{\binom{n+d}{d}}.
$$