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Exercise 8

Friday, January 13, 2017

Problem 1. *(ML Discriminant Rule versus Linear Discriminant Analysis)* In this problem, we work with the first datasets (available online in the webpage as 2classPub.txt and 3classPub.txt). In each dataset, the rows are sequence of numbers. The last number is the label or the class indicator while the previous numbers are entries of each data point. One dataset consists of two classes while the other one consists of three classes.

a) (2-Classes) For the dataset with two classes, Figure 1, find the linear discriminant rule and maximum likelihood discriminant rule. Explain each step.

Find out how this rule divides the space into disjoint regions corresponding to each class.

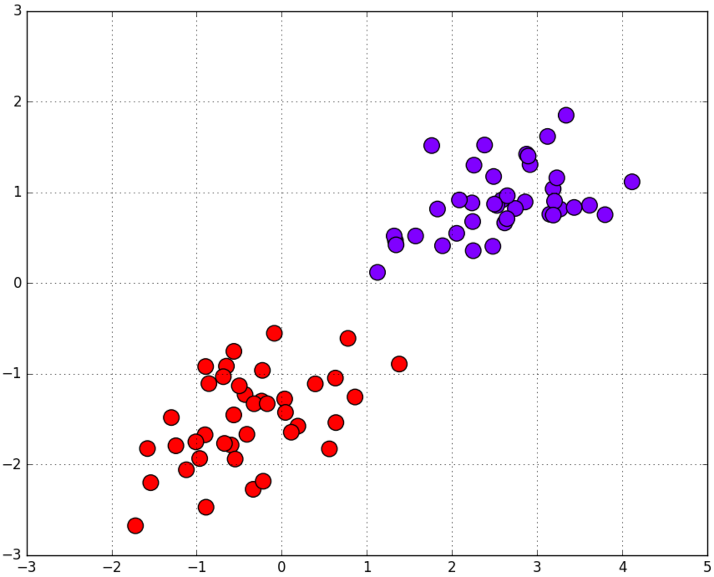

Figure 1: Dataset with two classes

b) (3-Classes) For the dataset with three classes, Figure 2, find the linear discriminant rule and maximum likelihood discriminant rule. Explain each step. Find out how this rule divides the space into disjoint regions corresponding to each class.

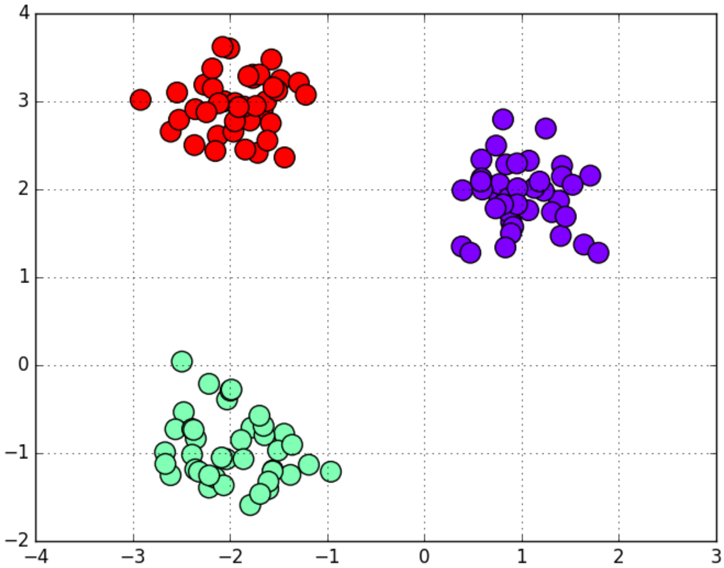

Figure 2: Dataset with three classes

Problem 2. *(ML Discriminant Rule versus Linear Discriminant Analysis)* In this problem, we work with the second datasets (available online in the webpage as 2classPubII.txt and 3classPubII.txt) In each dataset, the rows are sequence of numbers. The last number is the label or the class indicator while the previous numbers are entries of each data point. One dataset consists of two classes while the other one consists of three classes.

a) (2-Classes) For the dataset with two classes, Figure 3, find the linear discriminant rule and maximum likelihood discriminant rule. Explain each step.

Find out how this rule divides the space into disjoint regions corresponding to each class.

b) (3-Classes) For the dataset with three classes, Figure 4, find the linear discriminant rule and maximum likelihood discriminant rule. Explain each step. Find out how this rule divides the space into disjoint regions corresponding to each class.

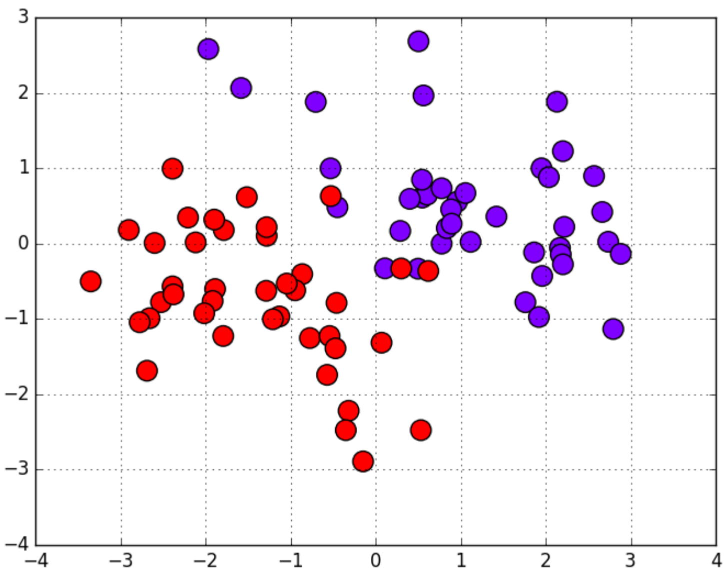

Figure 3: Dataset with two classes

Problem 3. *(Maximum Likelihood Clustering)* Suppose that $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n$ are n samples from g populations, each with Gaussian distribution $N_p(\boldsymbol{\mu}_k, \boldsymbol{\Sigma})$. The corresponding densities are:

$$f_k(\mathbf{u}) = \frac{1}{(2\pi)^{p/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\left\{-\frac{1}{2}(\mathbf{u} - \boldsymbol{\mu}_k)^T \boldsymbol{\Sigma}^{-1} (\mathbf{u} - \boldsymbol{\mu}_k)\right\}, \mathbf{u} \in \mathbb{R}^p.kl = 1, \ldots, g.$$

a) Define the cluster analysis problem as maximization of log-likelihood function and write down the respective optimization problem.

b) Given clustering of samples $S_1, \ldots, S_g$, find ML-estimation of $\boldsymbol{\Sigma}$.

c) Show that if $\boldsymbol{\Sigma}$ is unknown, the ML-cluster analysis is equivalent to the following optimization problem:

$$\min_{S_1,\ldots,S_g} \det(\mathbf{W})$$

where

$$\mathbf{W} = \sum_{k=1}^{g} \sum_{i \in S_k} (\mathbf{x}_i - \overline{\mathbf{x}}_k)(\mathbf{x}_i - \overline{\mathbf{x}}_k)^T.$$

d) If $\boldsymbol{\Sigma}$ is known, show that ML-cluster analysis is equivalent to the following optimization problem:

$$\min_{S_1,\ldots,S_g} \operatorname{tr}(\mathbf{W}\boldsymbol{\Sigma}^{-1}).$$

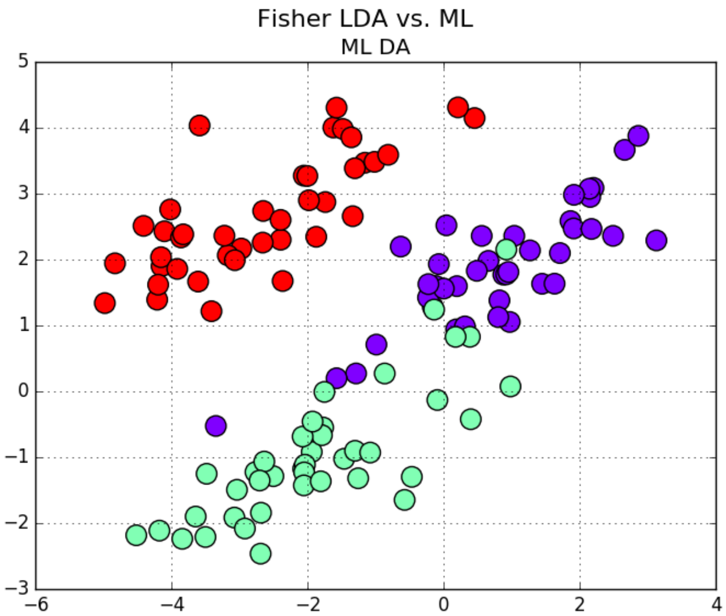

Figure 4: Dataset with three classes