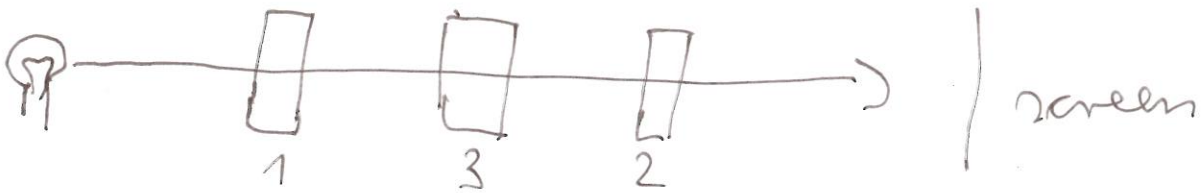
14. Quantum Cryptography

Quantum cryptography is closely related to quantum computing. There exists an efficient alg. for factoring large numbers (Shor, 1994, s. Tr & Wash. 2nd p 460 ff), ready to use once a powerful quantum computer exists. This would endanger many of the presently used cryptographic protocols and alg.

In parallel, quantum cryptography was developed to ensure physically secure transmission, particularly secure against quantum computing facilities. Quantum cryptography is based on quantum effects, not easily accessible for non-physicists. Quantum mechanics is a difficult subject with concepts where everyday experiences are not applicable.

We need particles like electrons or photons that we are able to observe. Photons make up light which is easily observable. They serve best for explaining the basic principles of quantum cryptography.

14.1 A quantum experiment



polarized filters

Filter 1 : Vertically polarized
" 2 : Horizontally "
Filter 3 : Diagonally "

Polarization of photons is described by a complex unit vector.

(a, b) of length $|a|^2 + |b|^2 = 1$

Choose a basis $|\uparrow\rangle$, $|\rightarrow\rangle$ (notation from physics)

Measurement postulate of quantum mechanics

Given a device for measuring polarization with basis $|\uparrow\rangle$ $|\rightarrow\rangle$
A photon with polarization $a|\uparrow\rangle + b|\rightarrow\rangle$ is measured as $|\uparrow\rangle$ with probability $|a|^2$ and as $|\rightarrow\rangle$ with probability $|b|^2$.
Measuring will change the state to the result of the measurement.

Model for the experiment

Photon with random polarization
filter 1 with $|\uparrow\rangle$, $|\rightarrow\rangle$

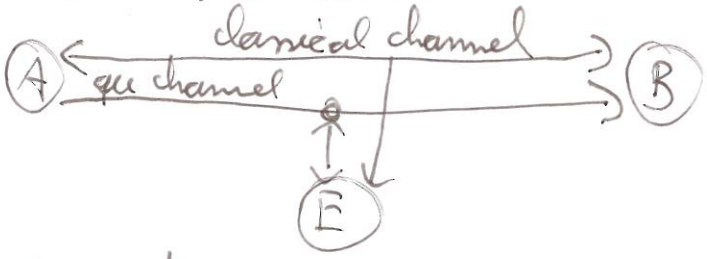
- measured as $|\uparrow\rangle$ with prob $1/2$, has pol. $|\uparrow\rangle$, pass through
- measured as $|\rightarrow\rangle$ with prob $1/2$, have pol. $|\rightarrow\rangle$, reflected

Filter 2 (without 3) with basis $|\rightarrow\rangle$, $|\uparrow\rangle$ left, no photon pass
Filter 3 in between 1 and 2 with basis $|\nearrow\rangle$, $|\nwarrow\rangle$:
Photon will pass filter 1 with prob $1/2$
There pass filter 3 with prob $\frac{1}{2}$
There pass filter 2 with prob $1/2$
Intensity: $1/8$ of the original

14.2 Quantum key exchange

Choose an orthonormal basis $|0\rangle$, $|1\rangle$ of a 2-dim complex vector space.
Each unit vector is called a quantum bit (qubit), written as
$a|0\rangle + b|1\rangle$ s.t $|a|^2 + |b|^2 = 1$
The probability of observing a qubit in state $|0\rangle$ is $|a|^2$

A and B want to exchange a sequence of bits. They use a classical channel and a quantum channel (one which transmits photons without altering the polarization)
Eve has access to both



System parameters

Alice and Bob use two bases
$B_1 = \{|\uparrow\rangle, |\rightarrow\rangle\}$ (rectilinear, +)
$B_2 = \{|\nearrow\rangle, |\nwarrow\rangle\}$ (diagonal, x)

Encryption

Alice selects randomly B_1 or B_2
If she chooses B_1 (+) she encodes
0 as $|\uparrow\rangle$ (vertically polarized photon)
1 as $|\rightarrow\rangle$ (horizontally " ")
If she chooses B_2 (x) she encodes
0 as $|\nearrow\rangle$ (diag, NE pol. ph.)
1 as $|\nwarrow\rangle$ (diag: NW " ")

Decryption

1. Bob measures the polarization of received photons randomly with B_1 or B_2, keeps the result secret.
2. B tells A over the classical channel which bases he has chosen
3. A tells B which bases are correct

A and B will agree on approx. half the amount of bits A has sent. These bits are used as key for the one-time pad, AES

Example

Alice:

Bits:	0	1	1	1	0	0	1	0	...
Random bases:	+	×	+	+	×	×	+	×	...
qubit (photon)	$\uparrow$	$\nearrow$	$\rightarrow$	$\rightarrow$	$\nearrow$	$\nearrow$	$\rightarrow$	$\nearrow$	...

Bob:

Random bases:	×	×	×	+	×	+	+	×
Bits:		c		c	c		c	c
Correct:		1		1	0		1	0

Security is based on physical phenomena. If Eve observes the photons from A, she will change the state, hence introducing add. errors

Actual implementations work over a distance of 100km using commercial fiber optical cables (Sep '15)