Prof. Dr. Anke Schmeink, Ehsan Zandi, Yulin Hu

Tutorial 13

Monday, January 18, 2016

Problem 1. (General Barriers) The log barrier is based on the approximation of the indicator function $I_{-}(u)$ with the logaritmic function $-(1/t)\log(-u)$ (Section 7.2.1 in the lecture notes). We can also construct barriers from other approximations, which in turn yield generalizations of the central path and barrier method. Let $h : \mathbb{R} \longrightarrow \mathbb{R}$ be a twice differentiable, closed, increasing convex function with $\mathbf{dom}\ h = \mathbb{R}_{<0}$. One such function is $h(u) = \log(-u)$; another example is $h(u) = -1/u$ (for $u < 0$). Now consider the convex optimization problem (without equality constraints, for simplicity)

$$\begin{array}{ll} \text{minimize} & f_0(x) \\ \text{subject to} & f_i(x) < 0, \quad i = 1, \ldots, s, \end{array}$$

where f_i are twice differentiable. We define the h-barrier for this problem as

$$\Phi_h(x) = \sum_{i=1}^{s} h(f_i(x)),$$

with domain $\{x \mid f_i(x) < 0, i = 1, \ldots, s\}$. When $h(u) = -\log(-u)$, this is the usual logarithmic barrier; when $h(u) = -1/u$, Φ_h is called the inverse barrier. We define the h-central path as

$$x^*(t) = \text{argmin}\ t f_0(x) + \Phi_h(x),$$

where t>0 is a parameter.

a) Explain why $t f_0(x) + \Phi_h(x)$ is convex in x, for each $t > 0$.

b) Show how to construct a dual feasible $\boldsymbol{\lambda}$ from $x^*(t)$. Find the associated duality gap.

c) For what functions h does the duality gap found in part (b) depend only on t and s?

Problem 2. (Branch-and-bound algorithm for a 0-1 linear program)

a) A network operator can offer $n \in \mathbb{N}$ different services to its customers with revenues $c_1, \ldots, c_n \in \mathbb{R}$ corresponding to each service. Each service requires a certain bandwidth $v_1, \ldots, v_n \in \mathbb{R}$ within the frequency band available to the network operator, whose width is given as $B \in \mathbb{R}$. A service can at most be offered to one customer. Formulate the optimization problem which maximizes the revenue as an integer linear programming problem.

b) Solve the knapsack problem by using branch-and-bound algorithm for $n = 3$, and $c_i = v_i$ for $1 \leq i \leq 3$, where $c_1 = c_2 = 2$, $c_3 = 3$ and $B = 6$.